

## Outline

### Uncertainty in Knowledge Representation : Beyond Probabilistic Reasoning

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#### 1 INCOMPLETENESS AND GRADUALNESS

The lecture starts with a short typology of imperfect information. The stress is first put on incomplete information, its forms and the basic reasoning principles. It is shown that classical logic and interval analysis are the main frameworks and rely on set-based representations of information. The key notions expressing uncertainty in the setting of incomplete information are the modalities of possibility (expressing plausibility) and necessity (expressing certainty). The talk will insist on the frequent confusion between the state of ignorance and a truth-value intermediary between true and false, showing that the uncertainty calculus induced by incomplete information cannot be truth-functional. Hence three-valued and, more generally, truth-functional many-valued logics are useless to account for uncertainty. The last part of the talk develops a case where many-valued logics are useful: the case of gradual predicates. Basics of fuzzy set theory and fuzzy logics in the narrow sense are provided.

#### 2. CONDITIONALS AND THE LIMITATIONS OF PROBABILITY

This lecture considers uncertainty in a more general setting. An important distinction is made between generic and singular pieces of information. This distinction lays bare the difference between the problem of plausible inference from generic knowledge in the presence of singular evidence so as to construct beliefs, and the problem of merging singular pieces of uncertain information. In the scope of modelling generic knowledge, the talk considers a notion of conditional representing if-then rules, whose probability is a conditional probability. A genuine three-valued logic of conditionals is described, which provides a semantics for the preferential logic of conditional assertions by Kraus, Lehmann and Magidor. The association of propositional logic and the three-valued logic of conditionals enables foundations for the plausible inference problem to be defined in agreement with probabilistic reasoning.

Next, we provide a general framework for representing uncertainty using set-functions, probability measures being a special case. We show why the Bayesian assumption that a unique probability distribution can represent any uncertain situation is debatable. Especially we discuss Ellsberg paradox and conclude that any good framework for uncertain reasoning should lay bare the distinction between two sources of uncertainty, that is, variability and incompleteness.

### 3. PLAUSIBLE INFERENCE IN THE ORDINAL SETTING: THE BRIDGE BETWEEN NON-MONOTONIC REASONING AND POSSIBILITY THEORY

This lecture considers ordinal representations of uncertainty by means of confidence relations among events. It considers the important particular case of possibility orderings, first proposed by David Lewis, and their representations by means of possibility measures mapping on an ordinal scale. Possibility theory is the natural generalisation of the uncertainty calculus of propositional logic to the case where some situations are more plausible than others. The nature of possibilistic reasoning is shown to be that of jumping to normal conclusions in the presence of incomplete information. This principle captures the essence of non-monotonic exception-tolerant reasoning as studied by Pearl and Lehmann in the early nineties. Basics of possibilistic logic are presented and the properties of inconsistency-tolerant inference laid bare, the key property being that of rational monotony. The problem of plausible inference from generic knowledge in the presence of singular evidence is reconsidered in this framework.

### 4 IMPRECISE PROBABILITY AND BELIEF FUNCTIONS

This last lecture presents a generic numerical framework for reasoning under incomplete statistical information. The case of imprecise probability popularized by Walley is outlined, and the distinction between plausible inference (focusing on a reference class) and revision is laid bare. An important special case is the framework of random sets, that subsumes numerical possibility theory. Random sets can either model imprecise statistical information (following Dempster) or uncertain evidence (following Shafer and Smets) like unreliable testimonies. The latter view is instrumental for posing the problem of merging uncertain evidence. In this framework, several conditioning rules are exhibited, especially one addressing the problem of plausible inference, the other (called Dempster's rule of conditioning) being tailored to the merging of uncertain evidence with a sure piece of singular information.