Preservation under Substructures modulo Bounded Cores

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We investigate a model-theoretic property that generalizes the classical notion of preservation under substructures. We call this property preservation under substructures modulo bounded cores, and present a syntactic characterization via Σ_2^0 sentences for properties of arbitrary structures definable by FO sentences. As a sharper characterization, we further show that the count of existential quantifiers in the Σ_2^0 sentence equals the size of the smallest bounded core. We also present our results on the sharper characterization for special fragments of FO and also over special classes of structures. We present a (not FO-definable) class of finite structures for which the sharper characterization fails, but for which the classical Łos-Tarski preservation theorem holds. As a fallout of our studies, we obtain combinatorial proofs of the Łos-Tarski theorem for some of the aforementioned cases.