MTH 412: Stochastic Processes
SE 359: Applied Stochastic Processes
Assignment No. 2: Markov Chains-I

1. For a homogeneous MC $\{X_n : n = 0, 1, 2, \ldots\}$, show that $P(X_{n+m} = j | X_n = i)$ does not depend on $n \in \{0, 1, \ldots\}$, where $i, j \in S$ and m is a positive integer.

2. Show that $\{X_n : n = 0, 1, 2, \ldots\}$ is a MC if, and only if, $P(X_n = j | X_0 = i_0, X_1 = i_1, \ldots, X_{n-1} = i_{n-1}) = P(X_n = j | X_{n-1} = i_{n-1})$, $\forall n \geq 1$ and $\forall i_0, i_1, \ldots, i_{n-1}, j \in S$.

3. Consider a MC with state space $S = \{1, 2, 3, 4, 5\}$ and transition probability matrix

$$P = \begin{bmatrix} \frac{1}{2} & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}.$$

   Determine $\lim_{n\to\infty} p_{13}^{(4n+r)}$, for $r = 0, 1, 2, 3$.

4. Let P be a transition probability matrix of a MC. Argue that if, for some positive integer r, $P^{(r)}$ has all positive entries, then so does $P^{(n)}$, for all $n \geq r$.

5. Prove that if number of states in a MC is M and if state j can be reached from state i, then it can be reached in M steps or less.

6. Consider a MC $\{X_n : n = 0, 1, 2, \ldots\}$ with state space $S = \{0, 1, 2\}$ and transition probability matrix

$$P = \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \\ 1 & 0 & 0 \end{bmatrix}.$$

   Let $f(0) = 0$ and $f(1) = f(2) = 1$. If $Y_n = f(X_n), n = 0, 1, 2, \ldots$, is $\{Y_n : n = 0, 1, 2, \ldots\}$ a MC? Comment.

7. Suppose that coin 1 has probability 0.7 of coming up heads and coin 2 has probability 0.6 of coming up heads. If the coin flipped today comes up heads, then we select coin 1 to flip tomorrow and if it comes up tails, then we select coin 2 to flip tomorrow. If the coin initially flipped is equally likely to be coin 1 or coin 2, then what is the probability that the coin flipped on the third day after the initial flip is coin 1?