Module 1: Elements of Statistical Inference

- $\underline{X} = (X_1, \ldots, X_n)$: a random sample (collection of i.i.d. r.v.s) from a population (collection of values of certain characteristics of a set of units/individuals) having an unknown d.f. $F \in \mathcal{P}$ where $\mathcal{P}$ has at least two elements.

- $g(F)$: a function defined on $\mathcal{P}$ and taking values in $\mathbb{R}^k$ (the k-dimensional Euclidean space)
  - ↳ a population parameter of interest.

- Statistical Inference problems: Based on the data $\underline{X}$, make inferences (conclusions) about $g(F)$.

- Statistical Inference
  - Estimation [goal is to estimate $g(F)$ based on data $\underline{X}$]
  - Testing of hypothesis [goal is to test validity of certain hypothesis about $g(F)$ against a given alternative hypothesis]

- In parametric statistical inference problems

$$\mathcal{P} = \{F_\theta : \theta \in \Theta\}$$

where for each $\theta \in \Theta$, form of F_θ is known but $\theta \in \Theta$ is unknown (e.g., $F_\theta \equiv$ df of $N(\theta, 1)$ =

$\Phi(x-\theta)$. Here we write $g(F) \equiv g(\theta)$, $\theta \in \Theta$ and

Θ: parametric space of $\underline{\theta}$.

$\mathcal{X}$: Sample Space

$\mathcal{A}$: Action Space (set of allowable actions)

Generally in an estimation problem

$\mathcal{A}$ = set of possible values of $g(F)$ or $g(\theta)$,

and in hypothesis testing problems

$\mathcal{A} = \{0, 1\}$

where taking action 0 (1) means accept (reject) the given hypothesis.