Cooperative Communication in Spatially Modulated MIMO systems

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Introduction

Spatial modulation (SM)\(^1\) has been recently gaining popularity due to its ability to enable MIMO communication with fewer transmit RF chains.

SM systems can be combined with STBC to obtain a substantial improvement in performance due to the additional transmit diversity and enhanced spectral efficiency\(^2\).

- In the STBC-SM scheme, both STBC symbols and the indices of the transmit antennas from which these symbols are transmitted, carry information.

- The end-to-end reliability of the STBC-SM system can be further enhanced by exploiting the cooperative diversity along with spatial diversity.


System Model 1

Figure: Selective DF based SM MIMO-STBC Cooperative System.
Kronecker channel model is employed to model MIMO channel matrices with receive/transmit antenna correlation as\(^3\),

\[
\begin{align*}
H_{SD} &= R_d^{\frac{1}{2}} \tilde{H}_{SD} R_s^{\frac{T}{2}}, \\
H_{SR} &= R_r^{\frac{1}{2}} \tilde{H}_{SR} R_s^{\frac{T}{2}}, \\
H_{RD} &= R_d^{\frac{1}{2}} \tilde{H}_{RD} R_r^{\frac{T}{2}},
\end{align*}
\]

where

- \(R_s, R_d\) and \(R_r\) denote the spatial correlation matrices at the source, destination and relay nodes respectively.
- The coefficients of \(\tilde{H}_{SD}, \tilde{H}_{SR}\) and \(\tilde{H}_{RD}\) are distributed as \(\mathcal{CN}(0, \delta_{sd}^2), \mathcal{CN}(0, \delta_{sr}^2)\) and \(\mathcal{CN}(0, \delta_{rd}^2)\) respectively.

The received codeword matrices $\mathbf{Y}_{SD} \in \mathbb{C}^{N_d \times T}$, $\mathbf{Y}_{SR} \in \mathbb{C}^{N_r \times T}$ at the destination and relay respectively in the first phase are given as,

$$
\mathbf{Y}_{SD} = \sqrt{\frac{P_0}{N_a}} \mathbf{H}_{SD} \mathbf{X} + \mathbf{W}_{SD},
$$

$$
\mathbf{Y}_{SR} = \sqrt{\frac{P_0}{N_a}} \mathbf{H}_{SR} \mathbf{X} + \mathbf{W}_{SR},
$$

where

- $\mathbf{X} \in \mathbb{C}^{N_s \times T}$ denotes the transmitted codeword,
- $P_0$ is the transmit power at the source,
- $N_a$ denotes number of active antennas at the transmitter.

Entries of matrices $\mathbf{W}_{SD} \in \mathbb{C}^{N_d \times T}$ and $\mathbf{W}_{SR} \in \mathbb{C}^{N_r \times T}$ are symmetric complex Gaussian with variance $\eta_0$. 
The received codeword matrix $Y_{RD} \in \mathbb{C}^{Nd \times T}$ at the destination, in the second phase, is given as,

$$Y_{RD} = \sqrt{\frac{P_1}{N_a}} H_{RD} X + W_{RD}, \quad (2)$$

where

- $P_1$ is the transmit power at the relay,
- Entries of matrix $W_{RD} \in \mathbb{C}^{Nd \times T}$ is symmetric complex Gaussian with variance $\eta_0/2$ per dimension.
The average PEP bound at the destination for the cooperative SM MIMO-STBC system is given by

\[
P_e \leq \sum_{i=1}^{\lvert C \rvert} \Pr(X_i) \sum_{j=1, j \neq i}^{\lvert C \rvert} P_e(X_i \rightarrow X_j),
\]  

where the end-to-end PEP for the error event \(X_i \rightarrow X_j\) is

\[
P_e(X_i \rightarrow X_j) = P_{S \rightarrow D}(X_i \rightarrow X_j) \times P_{S \rightarrow R}^e(X_i)
\]

\[
+ P_{S \rightarrow D, R \rightarrow D}(X_i \rightarrow X_j) \times (1 - P_{S \rightarrow R}^e(X_i)),
\]

and the average error probability at the relay is

\[
P_{S \rightarrow R}^e(X_i) = \sum_{k=1, k \neq i}^{\lvert C \rvert} P_{S \rightarrow R}(X_i \rightarrow X_k).
\]
The PEP for the error event corresponding to the transmitted codeword $X_i \in C$ being confused for codeword $X_k \in C$ at the relay, where $k \neq i$, conditioned on $H_{SR}$, is given as\(^4\),

$$
P_{S \to R}(X_i \to X_k|H_{SR}) = Q\left(\sqrt{\frac{P_0 \|H_{SR}(X_i - X_k)\|_F^2}{2N_a \eta_0}}\right),$$

$$
= \frac{1}{\pi} \int_0^{\pi/2} \exp\left(\frac{-P_0 \|H_{SR}(X_i - X_k)\|_F^2}{4N_a \eta_0 \sin^2 \theta}\right) d\theta,
$$

---

The average PEP at the relay can now be obtained as,

$$P_{S \rightarrow R}(\mathbf{X}_i \rightarrow \mathbf{X}_k) = \mathbb{E}_{\mathbf{H}_{SR}} \left\{ \frac{1}{\pi} \int_0^{\pi/2} \exp \left( -\frac{P_0 \| \mathbf{H}_{SR}(\mathbf{X}_i - \mathbf{X}_k) \|^2_F}{4N_a \eta_0 \sin^2 \theta} \right) d\theta \right\},$$

$$= \frac{1}{\pi} \int_0^{\pi/2} \phi_{ik}^{\| \mathbf{H}_{SR} \Delta_{ik} \|^2_F} \left( \frac{-P_0}{4N_a \eta_0 \sin^2 \theta} \right) d\theta,$$

where

- $\phi_{ik}^{\| \mathbf{H}_{SR} \Delta_{ik} \|^2_F}(s)$ denotes the moment generating function of $\| \mathbf{H}_{SR}(\mathbf{X}_i - \mathbf{X}_k) \|^2_F = \| \mathbf{H}_{SR} \Delta_{ik} \|^2_F$, where $\Delta_{ik} = \mathbf{X}_i - \mathbf{X}_k$. 
The moment generating function of \( \| H_{SR} \Delta_{ik} \|_F^2 \) can be obtained as \(^5\),

\[
\phi_{\| H_{SR} \Delta_{ik} \|_F^2}^i(k)(s) = \prod_{a=1}^{\gamma_{ik}^a} \prod_{b=1}^{\gamma_{ik}^*} (1 - s \mu_{ik}^a \lambda_{ik}^* \delta_{SR}^2)^{-1},
\]

where

- \( \mu_{ik}^a \) is the \( a \)th eigenvalue of the matrix \( \Delta_{ik} \Delta_{ik}^H R_s \),
- \( \lambda_{ik}^* \) is the \( b \)th eigenvalue of the correlation matrix \( R_r \),
- \( \gamma_{ik}^a \) denotes the rank of the matrix \( \Delta_{ik} \Delta_{ik}^H R_s \),
- \( \gamma_{ik}^* \) denotes the rank of the correlation matrix \( R_r \).

Pair-wise Error Probability (PEP) Analysis

Using $\phi^i_k = \frac{\gamma^i_k}{\|H_{SR} \Delta^i_k\|_F^2}(s) = \prod_{a=1}^{\gamma^i_a} \prod_{b=1}^{\gamma^i_b} (1 - s \mu^i_a \lambda^*_b \delta^2_{SR})^{-1}$, the average error probability at the relay can be solved as,

$$P_{S\rightarrow R}(X_i \rightarrow X_k) = \frac{1}{\pi} \int_0^{\pi/2} \prod_{a=1}^{\gamma^i_a} \prod_{b=1}^{\gamma^i_b} \left(1 + \frac{P_0 \mu^i_a \lambda^*_b \delta^2_{SR}}{4N_a \eta_0 \sin^2 \theta}\right)^{-1} d\theta$$

$$= G(v(\theta))$$

where

- The quantity $G(v(\theta))$ is defined as $G(v(\theta)) = \frac{1}{\pi} \int_0^{\pi/2} \frac{1}{v(\theta)} d\theta$. 

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Following a similar approach, the average PEP of confusion event $X_i \rightarrow X_j$ at the destination when the relay decodes all the symbols erroneously, can be solved as,

$$P_{S \rightarrow D}(X_i \rightarrow X_j) = G \left( \prod_{a=1}^{\gamma_a^i} \prod_{b=1}^{\gamma_b} \left( 1 + \frac{P_0 \mu_a^i \lambda_b \delta_{SD}^2}{4N_a \eta_0 \sin^2 \theta} \right) \right), \quad (6)$$

where

- $\mu_a^i$ is the $a^{th}$ eigenvalue of the matrix $\Delta_{ij} \Delta_{ij}^H R_s$,
- $\lambda_b$ is the $b^{th}$ eigenvalue of the correlation matrix $R_d$,
- $\gamma_a^i$ denotes the rank of the matrix $\Delta_{ij} \Delta_{ij}^H R_s$,
- $\gamma_b$ denotes the rank of the correlation matrix $R_d$. 
The average PEP of the error event $X_i \rightarrow X_j$ at the destination when the relay decodes all the symbols correctly, is given as,

$$P_{S \rightarrow D, R \rightarrow D}(X_i \rightarrow X_j)$$

$$= \mathbb{E}_{H_{SD}, H_{RD}} \left\{ \frac{1}{\pi} \int_{0}^{\pi/2} \exp \left( -\frac{P_0 \| H_{SD} \Delta_{ij} \|^2_F - P_1 \| H_{RD} \Delta_{ij} \|^2_F}{4N_a \eta_0 \sin^2 \theta} \right) d\theta \right\}$$

$$= G \left( \prod_{a=1}^{\gamma^i_j} \prod_{b=1}^{\gamma^b} \left( 1 + \frac{P_0 \mu^i_j \lambda_b \delta^2_{SD}}{4N_a \eta_0 \sin^2 \theta} \right) \prod_{p=1}^{\gamma^i_p} \prod_{q=1}^{\gamma^q} \left( 1 + \frac{P_1 \mu^i_p \lambda_q \delta^2_{RD}}{4N_a \eta_0 \sin^2 \theta} \right) \right),$$

where
- $\mu^i_p$ is the $p^{th}$ eigenvalue of the matrix $\Delta_{ij} \Delta_{ij}^H R_r$,
- $\lambda_q$ represents the $q^{th}$ eigenvalue of the correlation matrix $R_d$,
- $\gamma^i_j$ denotes the rank of the matrix $\Delta_{ij} \Delta_{ij}^H R_r$,
- $\gamma^i_q$ denotes the rank of the correlation matrix $R_d$. 

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Cooperative Communication in SM MIMO systems
Using the fact that at high SNR $1 - P_{e_S}^{R}(X_i) \approx 1,$

$$1 + \frac{P_0 \mu_a^j \lambda_b \delta^2_{SD}}{4 N_a \eta_0 \sin^2 \theta} \approx \frac{P_0 \mu_a^i \lambda_b \delta^2_{SD}}{4 N_a \eta_0 \sin^2 \theta},$$

$$1 + \frac{P_0 \mu_a^j \lambda_q \delta^2_{RD}}{4 N_a \eta_0 \sin^2 \theta} \approx 1,$$ the asymptotic PEP is given as,

$$P_e \approx \sum_{i=1}^{\mid C \mid} \sum_{j=1}^{\mid C \mid} \left[ \sum_{k=1, k \neq i}^{\mid C \mid} A_1 \left( \frac{P}{\eta_0} \right)^{-\left( \gamma_a^j \gamma_b + \gamma_a^i \gamma_b^* \right)} + A_2 \left( \frac{P}{\eta_0} \right)^{-\left( \gamma_a^i \gamma_b + \gamma_p^j \gamma_q \right)} \right] \approx C_1 \left( \frac{P}{\eta_0} \right)^{-\left( \gamma_a^j \gamma_b + \gamma_a \gamma_b^* \right)} + C_2 \left( \frac{P}{\eta_0} \right)^{-\left( \gamma_a \gamma_b + \gamma_p \gamma_q \right)},$$

(7)

where

- $\gamma_a^j = rank(\Delta_{ij} \Delta_{ij}^H R_s) = \gamma_a$ and $\gamma_p^j = rank(\Delta_{ij} \Delta_{ij}^H R_r) = \gamma_p.$
- This assumption is true if spatial correlation matrices have full rank and all the codeword difference-matrices have equal rank.
Diversity Order Analysis II

- The diversity order of the above system is
  \[
  - \lim_{\frac{P}{\eta_0} \to \infty} \frac{\log(P_e)}{\log(P_{\eta_0})} = \gamma_a \gamma_b + \min\{\gamma_a \gamma_b^*, \gamma_p \gamma_q\}
  \]

- For uncorrelated systems, the diversity order is
  \[
  \gamma_a \mathcal{N}_d + \min\{\gamma_a \mathcal{N}_r, \gamma_p \mathcal{N}_d\},
  \]
  where \(\gamma_a = \gamma_p\) denotes the rank of the matrix \(\Delta_{ij}\Delta_{ij}^H\).

- This result shows that both the performance of the system and the resulting diversity order depend on the correlation.

- If the correlation matrix at any of the nodes is not full rank, the diversity order of the system decreases.
The convex optimization problem for optimal source-relay power allocation can be formulated as,

$$
\min \left\{ \frac{\alpha}{\gamma a \gamma b + \gamma a \gamma b^*} + \frac{\beta}{a_0 a_1} \right\},
$$

s.t. \quad a_0 + a_1 = 1,

(8)

where

$$
\alpha = C_1 |a_0=1 \left( \frac{P}{\eta_0} \right)^{-(\gamma a \gamma b + \gamma a \gamma b^*)}
$$

and

$$
\beta = C_2 |a_0=1, a_1=1 \left( \frac{P}{\eta_0} \right)^{-(\gamma a \gamma b + \gamma p \gamma q)}.
$$
Theorem (Optimal Power Allocation)

One non-negative zero of the polynomial equation below

\[ \alpha(\gamma_a \gamma_b + \gamma a \gamma^*_b)(1-a_0)^{1+\gamma_p \gamma_q} - \beta(\gamma_a \gamma_b + \gamma_p \gamma_q) a_0^{1+\gamma a \gamma^*_b} + \beta(\gamma_a \gamma_b) a_0^{\gamma a \gamma^*_b} = 0, \]

determines the optimal power factor \( a_0^* \) at the source and hence, the optimal powers at the source and relay are \( P_0^* = a_0^*P \) and \( P_1^* = (1 - a_0^*)P \) for a given total cooperative power budget \( P \).
Simulation Result I: PEP Performance

Figure: Comparison of the PEP performance of the proposed scheme with that of existing works for unity average channel gains.


Simulation Result II: PEP Performance

Figure: PEP performance of the proposed SM MIMO-STBC based selective DF cooperative scheme over correlated Rayleigh fading links.
Simulation Result III: PEP Performance

Figure: PEP Performance of the cooperative SM MIMO-STBC system using optimal/equal power allocation for various channel conditions.

Scenario I:
$\delta_{sr}^2 = \delta_{sd}^2 = \delta_{rd}^2 = 1$.

Scenario II:
$\delta_{sr}^2 = 100, \delta_{sd}^2 = \delta_{rd}^2 = 1$.

Scenario III:
$\delta_{sr}^2 = \delta_{sd}^2 = 1, \delta_{rd}^2 = 100$. 
Conclusion

- Closed form analytical expressions have been derived for the end-to-end PEP, diversity order and optimal power allocation for a selective DF based SM MIMO-STBC cooperative system considering receive/transmit antenna correlation.

- It has been observed that the end-to-end performance of selective DF based SM MIMO-STBC cooperative systems is superior in comparison to that of STBC, STBC-SM and STBC based cooperative systems.
Thank You