Performance Analysis of MIMO-OSTBC based Selective DF Cooperative Wireless System with Node Mobility and Channel Estimation Errors

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Overview

1. System Model
2. Statistical Characteristics
3. Performance Metrics
4. Numerical Results
Figure: MIMO-OSTBC based selective DF cooperative system.

The received codeword matrices $\mathbf{Y}_{SD}(k) \in \mathbb{C}^{N_d \times T}$, $\mathbf{Y}_{SR}(k) \in \mathbb{C}^{N \times T}$ at the destination and relay respectively are given as,

$$
\mathbf{Y}_{SD}(k) = \sqrt{\frac{P_0}{NR_c}} \mathbf{H}_{SD}(k) \mathbf{X}(k) + \mathbf{W}_{SD}(k), \quad (1)
$$

$$
\mathbf{Y}_{SR}(k) = \sqrt{\frac{P_0}{NR_c}} \mathbf{H}_{SR}(k) \mathbf{X}(k) + \mathbf{W}_{SR}(k), \quad (2)
$$

where

- $\mathbf{X}(k)$ denotes the transmitted codeword,
- $R_c$ denotes the rate of the OSTBC,
- $P_0$ is the transmit power at the source,
- Entries of matrices $\mathbf{W}_{SD}(k) \in \mathbb{C}^{N_d \times T}$ and $\mathbf{W}_{SR}(k) \in \mathbb{C}^{N \times T}$ are symmetric complex Gaussian with variance $\eta_0$. 
The received codeword matrix $Y_{RD}(k) \in \mathbb{C}^{N_d \times T}$ at the destination, in the second phase, is given as,

$$Y_{RD}(k) = \sqrt{\frac{P_1}{NR_c}} H_{RD}(k) X(k) + W_{RD}(k).$$

(3)

where

- $P_1$ is the transmit power at the relay,
- Entries of matrix $W_{RD}(k) \in \mathbb{C}^{N_d \times T}$ is symmetric complex Gaussian with variance $\eta_0/2$ per dimension.
Each of the links are time-selective in nature and can be modeled using first order Autoregressive (AR) process as,

\[ H_i(k) = \rho_i H_i(k - 1) + \sqrt{1 - \rho_i^2} E_i(k), \]  

where

- \( i \in \{SD, SR, RD\} \),
- The quantities \( \rho_{SD}, \rho_{SR} \) and \( \rho_{RD} \), are the correlation parameters for the SD, SR and RD links,
- The matrices \( E_{SD}, E_{SR} \) and \( E_{RD} \) are the time-varying components of the SD, SR and RD links whose entries can be modeled as zero mean circularly symmetric complex Gaussian with variances \( \sigma_{e_{SD}}^2, \sigma_{e_{SR}}^2 \) and \( \sigma_{e_{RD}}^2 \) respectively.

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Similar to the works\textsuperscript{3,4}, assume imperfect channel knowledge
\( \hat{H}_{SR}(1) = H_{SR}(1) + H_{\epsilon,SR}(1) \), \( \hat{H}_{SD}(1) = H_{SD}(1) + H_{\epsilon,SD}(1) \) and \( \hat{H}_{RD}(1) = H_{RD}(1) + H_{\epsilon,RD}(1) \), estimated only in the beginning of each frame.

The entries of the error matrices \( H_{\epsilon,SR}(1) \), \( H_{\epsilon,SD}(1) \) and \( H_{\epsilon,RD}(1) \) are circularly symmetric complex Gaussian with zero mean and variances \( \sigma_{\epsilon,SR}^2 \), \( \sigma_{\epsilon,SD}^2 \) and \( \sigma_{\epsilon,RD}^2 \) respectively.


\textsuperscript{4} Neeraj Varshney and Aditya K. Jagannatham, "Capacity Analysis for MIMO Beamforming Based Cooperative Systems over Time-Selective Links with Full SNR/One-Bit feedback based Path Selection and Imperfect CSI,"Accepted in 2016 IEEE Wireless Communications and Networking Conference (WCNC), 3-6 April 2016.
Using AR model, one can obtain the expression for $H_{SD}(k)$ as,

$$H_{SD}(k) = \rho_{SD}^{k-1}H_{SD}(1) + \sqrt{1 - \rho_{SD}^2} \sum_{i=1}^{k-1} \rho_{SD}^{k-i-1}E_{SD}(i),$$

$$= \rho_{SD}^{k-1}\hat{H}_{SD}(1) - \rho_{SD}^{k-1}H_{\epsilon,SD}(1)$$

$$+ \sqrt{1 - \rho_{SD}^2} \sum_{i=1}^{k-1} \rho_{SD}^{k-i-1}E_{SD}(i). \quad (5)$$
Substituting now the above expression for $H_{SD}(k)$ in (1), the received codeword matrix $Y_{SD}(k)$ can be expressed as,

$$Y_{SD}(k) = \sqrt{\frac{P_0}{NR_c}} \rho_{SD}^{k-1} \hat{H}_{SD}(1)X(k) - W_{SD}^l(k)$$

$$+ W_{SD}^M(k) + W_{SD}(k),$$  \hspace{1cm} (6)

where

1. $W_{SD}^l(k) = \sqrt{\frac{P_0}{NR_c}} \rho_{SD}^{k-1} H_{\epsilon,SD}(1)X(k)$ denotes the noise component arising due to imperfect channel estimates

2. $W_{SD}^M(k) = \sqrt{\frac{P_0(1-\rho_{SD}^2)}{NR_c}} \sum_{i=1}^{k-1} \rho_{SD}^{k-i-1} E_{SD}(i)X(k)$ denotes the noise component arising due to node mobility.
The effective instantaneous SNR at the destination node for each symbol can be obtained using (6) as,

$$\gamma_{SD}(k) = C_{SD}(k) ||\hat{H}_{SD}(1)||^2,$$

(7)

where

$$C_{SD}(k) = \frac{\tilde{\gamma}_{SD}\rho_{SD}^{2(k-1)}}{NR_c \left(1 + \frac{\tilde{\gamma}_{SD}}{NR_c} \rho_{SD}^{2(k-1)} \tilde{\sigma}_{\varepsilon_{SD}}^2 + \frac{\tilde{\gamma}_{SD}}{NR_c} \left(1 - \rho_{SD}^{2(k-1)} \right) \tilde{\sigma}_{\varepsilon_{SD}}^2 \right)},$$

and $$\tilde{\gamma}_{SD} = \frac{P_0}{\eta_0}.$$

The quantities $$\tilde{\sigma}_{\varepsilon_{SD}}^2 = N_a \sigma_{\varepsilon_{SD}}^2$$ and $$\tilde{\sigma}_{\varepsilon_{SD}}^2 = N_a \sigma_{\varepsilon_{SD}}^2$$, where $$N_a$$ is the number of non-zero symbol transmissions per codeword instant.
The effective instantaneous SNRs at the relay and destination nodes corresponding to the transmission over the SR and RD links as,

\[ \gamma_{SR}(k) = C_{SR}(k) ||\hat{H}_{SR}(1)||^2, \quad \text{and} \quad \gamma_{RD}(k) = C_{RD}(k) ||\hat{H}_{RD}(1)||^2, \]

where

\[ C_{SR}(k) = \frac{\tilde{\gamma}_{SR} \rho_{SR}^{2(k-1)}}{NR_c \left( 1 + \frac{\tilde{\gamma}_{SR}}{NR_c} \rho_{SR}^{2(k-1)} \tilde{\sigma}_{\epsilon_{SR}}^2 + \frac{\tilde{\gamma}_{SR}}{NR_c} \left( 1 - \rho_{SR}^{2(k-1)} \right) \tilde{\sigma}_{\epsilon_{SR}}^2 \right)}, \]

\[ C_{RD}(k) = \frac{\tilde{\gamma}_{RD} \rho_{RD}^{2(k-1)}}{NR_c \left( 1 + \frac{\tilde{\gamma}_{RD}}{NR_c} \rho_{RD}^{2(k-1)} \tilde{\sigma}_{\epsilon_{RD}}^2 + \frac{\tilde{\gamma}_{RD}}{NR_c} \left( 1 - \rho_{RD}^{2(k-1)} \right) \tilde{\sigma}_{\epsilon_{RD}}^2 \right)}, \]

and \( \tilde{\gamma}_{SR} = \frac{P_0}{\eta_0}, \quad \tilde{\gamma}_{RD} = \frac{P_1}{\eta_0}. \)
Statistical Characteristics

PDF and CDF of various SNR quantities $\gamma_{SD}(k)$, $\gamma_{SR}(k)$ & $\gamma_{RD}(k)$

$$f_\gamma(x) = \frac{\Lambda^\Theta x^{\Theta-1}}{\Gamma(\Theta)} e^{-\Lambda x}, \quad F_\gamma(x) = \frac{\gamma(\Theta, \Lambda x)}{\Gamma(\Theta)}, \quad (8)$$

where

1. $\Theta = NN_d$, $\Lambda = \frac{1}{C_{SD}(k)\delta_{sd}^2}$ and $\tilde{\delta}_{sd}^2 = \delta_{sd}^2 + \sigma_{\epsilon SD}^2$ for SD link.
2. $\Theta = N^2$, $\Lambda = \frac{1}{C_{SR}(k)\delta_{sr}^2}$ and $\tilde{\delta}_{sr}^2 = \delta_{sr}^2 + \sigma_{\epsilon SR}^2$ for SR link.
3. $\Theta = NN_d$, $\Lambda = \frac{1}{C_{RD}(k)\delta_{rd}^2}$ and $\tilde{\delta}_{rd}^2 = \delta_{rd}^2 + \sigma_{\epsilon RD}^2$ for RD link.
The per-frame average PEP bound at the destination for the selective DF based MIMO-OSTBC cooperative system is given by

\[ P_e \leq \frac{1}{N_b} \sum_{k=1}^{N_b} \sum_{i=1}^{\left| \mathcal{C} \right|} P_e(X_0(k) \rightarrow X_i(k)), \quad (9) \]

where

\[ P_e(X_0(k) \rightarrow X_i(k)) = P_{S \rightarrow D}(X_0(k) \rightarrow X_i(k)) \times P_{S \rightarrow R}^{(k)} \]

\[ + P_{S \rightarrow D,R \rightarrow D}(X_0(k) \rightarrow X_i(k)) \times \left(1 - P_{S \rightarrow R}^{(k)}\right), \]

and

\[ P_{S \rightarrow R}^{(k)} \leq \sum_{X_j(k) \in \mathcal{C}, \ X_j(k) \neq X_0(k)} P_{S \rightarrow R}(X_0(k) \rightarrow X_j(k)). \]
Pair-wise Error Probability (PEP) II

\[
P_{S \rightarrow R}(X_0(k) \rightarrow X_j(k)) = \mathbb{E}_{\gamma_{SR}(k)} \left\{ \frac{1}{\pi} \int_0^{\pi/2} \exp \left( -\frac{\lambda_j^2 \gamma_{SR}(k)}{4 \sin^2 \theta} \right) d\theta \right\},
\]

\[
= G \left( \left( 1 + \frac{\lambda_j^2 C_{SR}(k) \delta_{sr}^2}{4 \sin^2 \theta} \right) \right)^{N^2}, \quad (10)
\]

where the function \( G(\nu(\theta)) \) is defined as \( G(\nu(\theta)) = \frac{1}{\pi} \int_0^{\pi/2} \frac{1}{\nu(\theta)} d\theta \) [?]

Similarly,

\[
P_{S \rightarrow D}(X_0(k) \rightarrow X_i(k)) = G \left( \left( 1 + \frac{\lambda_i^2 C_{SD}(k) \delta_{sd}^2}{4 \sin^2 \theta} \right) \right)^{N N_d}. \quad (11)
\]
Pair-wise Error Probability (PEP) III

\[
P_{S \rightarrow D, R \rightarrow D}(X_0(k) \rightarrow X_i(k))
\]

\[
= \mathbb{E}_{\gamma_{SD}(k), \gamma_{RD}(k)} \left\{ \frac{1}{\pi} \int_{0}^{\pi/2} \exp \left( -\frac{\lambda_i^2 (\gamma_{SD}(k) + \gamma_{RD}(k))}{4 \sin^2 \theta} \right) d\theta \right\},
\]

\[
= G \left( \left( 1 + \frac{\lambda_i^2 C_{SD}(k) \delta^2_{sd}}{4 \sin^2 \theta} \right)^{NN_d} \right) \left( 1 + \frac{\lambda_i^2 C_{RD}(k) \delta^2_{rd}}{4 \sin^2 \theta} \right)^{NN_d}.
\]

(12)
The per-frame average outage probability $\bar{P}_{out}(\gamma_0)$ at the destination for the selective DF based MIMO-OSTBC cooperative system is given by,

$$\bar{P}_{out}(\gamma_0) = \frac{1}{N_b} \sum_{k=1}^{N_b} \bar{P}^{(k)}_{out}(\gamma_0)$$ (13)

where $\bar{P}^{(k)}_{out}(\gamma_0) = \mathbb{E}_\gamma \left\{ Pr(\gamma_{SD}(k) \leq \gamma_0) Pr(\gamma_{SR}(k) \leq \gamma_0) ight. +\left. Pr(\gamma_{SD}(k) + \gamma_{RD}(k) \leq \gamma_0) Pr(\gamma_{SR}(k) > \gamma_0) \right\}$,

$$= F_{\gamma_{SD}(k)}(\gamma_0) F_{\gamma_{SR}(k)}(\gamma_0) + F_{\gamma_{SD}(k)+\gamma_{RD}(k)}(\gamma_0) \times (1 - F_{\gamma_{SR}(k)}(\gamma_0)),$$ (14)
CDF $F_{\gamma_{SD}(k) + \gamma_{RD}(k)}(\gamma_0)$ is given by

$$
F_{\gamma_{SD}(k) + \gamma_{RD}(k)}(\gamma_0) = \frac{1}{\Gamma(2NN_d)(C_{SD}(k)\tilde{\delta}_{sd}^2)^{NN_d}(C_{RD}(k)\tilde{\delta}_{rd}^2)^{NN_d}} \times \sum_{n=0}^{K} \frac{NN_d^{(n)}}{2NN_d^{(n)} n!}(C_{RD}(k)\tilde{\delta}_{rd}^2)^{2NN_d+n} \left( \frac{1}{C_{RD}(k)\tilde{\delta}_{rd}^2} - \frac{1}{C_{SD}(k)\tilde{\delta}_{sd}^2} \right)^n \\
\times \gamma \left( 2NN_d + n, \frac{\gamma_0}{C_{RD}(k)\tilde{\delta}_{rd}^2} \right)^{2NN_d+n},
$$

(15)

where $N_d^0 = 1$, $N_d^{(n)} = N_d(N_d + 1) \cdots (N_d + n - 1)$. 

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The convex optimization problem for optimal source-relay power allocation can be formulated as,

$$\min_{P_0, P_1} \left\{ C_1 \left( \frac{1}{P_0} \right)^{N^2+NN_d} + C_2 \left( \frac{1}{P_0} \right)^{NN_d} \left( \frac{1}{P_1} \right)^{NN_d} \right\},$$

s.t. \( P_0 + P_1 = P. \)

where \( C_1 = \frac{(\gamma_0 \eta_0 NR_c)^{N^2+NN_d}}{N^2!(NN_d)!(\delta_{sr}^2)^{N^2}(\delta_{sd}^2)^{NN_d}} \) and \( C_2 = \frac{(\gamma_0 \eta_0 NR_c)^{2NN_d}}{(2NN_d)!(\delta_{sr}^2)^{NN_d}(\delta_{rd}^2)^{NN_d}}. \)
Theorem (Optimal Power Allocation)

One non-negative zero of the polynomial equation below

\[
\zeta(N, N_d)(P - P_0)^{NN_d+1} - P_0^{N^2+1} + P_0^{N^2}(P - P_0) = 0, \quad (16)
\]

determines the optimal power factor \(P_0^*\) at the source and hence, the optimal power factor \(P_1^*\) at the relay is \(P_1^* = P - P_0^*\) for a given total cooperative power budget \(P\).

where

\[
\zeta(N, N_d) = \begin{cases} 
\frac{2(2N_2)!((\delta_{rd}^2/N^2)}{(N^2!)^2(\delta_{sr}^2/N^2)^2} \quad \text{for } N_d = N, \\
\frac{N_2!(NN_d)!NN_d(\delta_{rd}^2)^{NN_d}(\gamma_0\eta_0NR_c)^{N^2-NN_d}}{(N^2+NN_d)(2NN_d)!((\delta_{rd}^2/N^2)^{NN_d}} \quad \text{for } N > N_d, \\
\frac{N_2!(NN_d)!NN_d(\delta_{sr}^2)^{NN_d}}{(N^2+NN_d)(2NN_d)!((\delta_{rd}^2/N^2)^{NN_d}} \quad \text{for } N < N_d.
\end{cases}
\]
Simulation Result I: PEP Performance

Figure: Per-block average PEP, asymptotic floor, simulated PEP for a Alamouti-coded selective DF-MIMO cooperative communication with $N=N_{d}=2$, $N_{b}=20$, channel gains $\delta_{sd}^{2}=\delta_{sr}^{2}=\delta_{rd}^{2}=1$ and error variances $\sigma_{e}^{2}=0.1$, $\sigma_{\epsilon}^{2}=0.01$. 
Simulation Result II: PEP Performance

Figure: Per-block average PEP, asymptotic floor, simulated PEP for a Alamouti-coded selective DF-MIMO cooperative communication with $N = N_d = 2$, $N_b = 20$, channel gains $\delta_{sd}^2 = \delta_{sr}^2 = \delta_{rd}^2 = 1$ and error variances $\sigma_e^2 = 0.01$, $\sigma_\epsilon^2 = 0$. 

Either source or destination is mobile, i.e., $\rho_{rd} = 1, \rho_{sd} = \rho_{sr} = 0.9724$ or $\rho_{sr} = 1, \rho_{sd} = \rho_{rd} = 0.9724$

Only relay is mobile, i.e., $\rho_{sd} = 1, \rho_{sr} = \rho_{sr} = 0.9724$

All nodes are static, i.e., $\rho_{sd} = \rho_{sr} = \rho_{sr} = 1$
Simulation Result III: Outage Performance

Figure: Simulated outage performance with equal power i.e., $\frac{P_0}{P} = \frac{P_1}{P} = \frac{1}{2}$ and optimal power allocation.
**Simulation Result IV: Outage Performance**

![Graph showing simulated outage performance with equal power and optimal power allocation.](image)

**Figure:** Simulated outage performance with equal power i.e., $\frac{P_0}{P} = \frac{P_1}{P} = \frac{1}{2}$ and optimal power allocation.

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Performance Analysis with Node Mobility and Imperfect CSI
Thank You