Optimal Power Allocation for Decode-and-Forward Based Mixed MIMO-RF/FSO Cooperative Systems

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June 14, 2016
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Motivation

- Mixed RF/FSO relaying systems have generated a significant research interest due to their ability to connect a large number of RF users to last-mile access network.
  - because of the features offered by the FSO technology, such as rapid deployment time, low cost, license-free bandwidths, and higher data-rates.
  - However, the performance of FSO systems is highly vulnerable to the unpredictable weather conditions.
  - In addition, the end-to-end data rate of RF/FSO systems is limited by the low speed RF link.

- In order to overcome this limitation, multiple-input multiple-output (MIMO) technology can be employed.
- On the other hand, the end-to-end performance can be significantly enhanced using the optimal power allocation framework.
System Model I

Figure: Schematic diagram of the DF based mixed RF/FSO cooperative system.
System Model II

The transmission of vector $\mathbf{x} \in \mathbb{C}^{N_s \times 1}$ from $S$ to $R$ is given as

$$\mathbf{y}_{SR} = \sqrt{\frac{P_0}{N_s}} \mathbf{H}_{SR} \mathbf{x} + \mathbf{w}_{SR},$$

(1)

where

- $\mathbf{y}_{SR} \in \mathbb{C}^{N_r \times 1}$ denotes the corresponding received vector at the relay node.
- $P_0$ is the available transmit power at the source.
- $\mathbf{H}_{SR} \in \mathbb{C}^{N_r \times N_s}$ is the SR MIMO channel matrix, where entries are assumed as independent zero mean circularly symmetric complex Gaussian random variables with variance $\delta_{SR}^2$.
- $\mathbf{w}_{SR} \in \mathbb{C}^{N_r \times 1}$ is the noise vector, having symmetric complex Gaussian entries with variance $\eta_0/2$ per dimension.
Now, employing the zero-forcing receiver, the instantaneous SNR for the $i^{th}$ symbol corresponding to the source-relay link is given as

$$\gamma_{SR}^i = \frac{P_0}{N_s N_0 \left[ (H_{SR}^H H_{SR})^{-1} \right]_{i,i}},$$

where $i = 1, 2, \cdots, N_s$ and $H_{SR}^H H_{SR}$ is a $N_s \times N_s$ complex Wishart distributed matrix with $N_r$ degrees of freedom.

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The probability density function (PDF) and cumulative distribution function (CDF) of $\gamma_{SR}^{i}$ can be written as:

$$f_{\gamma_{SR}^{i}}(x) = \left( \frac{1}{C_{SR} \delta_{SR}^{2}} \right)^{N_{r} - N_{s} + 1} \frac{x^{N_{r} - N_{s}} \exp \left( - \frac{x}{C_{SR} \delta_{SR}^{2}} \right)}{\Gamma(N_{r} - N_{s} + 1)},$$

(3)

$$F_{\gamma_{SR}^{i}}(x) = \gamma \left( N_{r} - N_{s} + 1, \frac{x}{C_{SR} \delta_{SR}^{2}} \right) \frac{1}{\Gamma(N_{r} - N_{s} + 1)},$$

(4)

where $C_{SR} = \frac{P_{0}}{N_{s} N_{0}}$, $\Gamma(\cdot)$ denotes the Gamma function, and $\gamma(\cdot, \cdot)$ denotes the incomplete Gamma function.$^{3}$

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The relay decodes the received $N_s$ bits using ZF receiver and forwards them to the destination over a FSO link.

The optical channel is modeled as

$$H_{RD} = H_l \times H_a \times H_p,$$

where

- $H_l$ accounts for path loss,
- $H_a$ represents the atmospheric turbulence-induced fading, modeled using the versatile Gamma-Gamma distribution, and
- $H_p$ represents the misalignment fading (pointing errors).

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The corresponding PDF of the channel between relay and destination is given by

\[ f_{HRD}(h) = \frac{\alpha\beta \xi^2}{A_0 H_l \Gamma(\alpha) \Gamma(\beta)} G_{3,0}^{1,3} \left( \frac{\alpha\beta h}{A_0 H_l} \right) \xi^2 - 1, \alpha - 1, \beta - 1 \],

where

- \( A_0 \) is the fraction of the collected power at \( r = 0 \),
- \( r \) is the aperture radius,
- \( \xi = w_e / (2 \sigma_s) \), \( w_e \) is the equivalent beam-width radius, and \( \sigma_s \) is the standard deviation of the pointing error displacement at the receiver.
- \( \alpha \) and \( \beta \) are the large-scale and small-scale scintillation parameters, respectively, which depend on the Rylov variance \( \sigma_R^2 = 1.23 C_n^2 k^{7/6} L^{11/6} \). Here, \( k \) is the optical wave number, \( C_n^2 \) is the refractive index structure constant, and \( L \) is the link distance.
Now, employing a coherent optical receiver with heterodyne detection at the destination, the instantaneous SNR of the relay-destination link is given as

$$\gamma_{RD} \approx \frac{\mathcal{R}A}{q \triangle f} H_{RD},$$  \hspace{1cm} (7)

where

- $\mathcal{R}$ is the photodetector responsivity,
- $A$ is photodetector area,
- $q$ is the electronic charge, and
- $\triangle f$ denotes the noise equivalent bandwidth of the receiver.

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The PDF and CDF of $\gamma_{RD}$ are given as

$$f_{\gamma_{RD}}(x) = \frac{\xi^2}{x\Gamma(\alpha)\Gamma(\beta)} G^{3,0}_{1,3} \left( \frac{\alpha\beta}{\bar{\gamma}} \middle| \begin{array}{c} \xi^2 + 1 \\ \xi^2, \alpha, \beta \end{array} \right),$$

(8)

$$F_{\gamma_{RD}}(x) = \frac{\xi^2}{\Gamma(\alpha)\Gamma(\beta)} G^{3,1}_{2,4} \left( \frac{\alpha\beta}{\bar{\gamma}} \middle| \begin{array}{c} 1, \xi^2 + 1 \\ \xi^2, \alpha, \beta, 0 \end{array} \right),$$

(9)

where $\bar{\gamma} = \frac{\Re AH_j A_0}{q^2 \Delta f} \left( \frac{\xi^2}{1+\xi^2} \right)$ is the average SNR.
Theorem (Per-block average outage probability)

The Per-block average outage probability at the destination for the DF based mixed RF/FSO cooperative system is given by

\[ P_{\text{out}} = \frac{1}{N_s} \sum_{i=1}^{N_s} \left[ 1 - \exp\left( -\frac{\gamma_{\text{th}}}{C_{SR} \delta_{SR}^2} \right) \sum_{k=0}^{N_r-N_s} \frac{1}{k!} \left( \frac{\gamma_{\text{th}}}{C_{SR} \delta_{SR}^2} \right)^k \right] \times \left\{ 1 - \frac{\xi^2}{\Gamma(\alpha) \Gamma(\beta)} \frac{\alpha \beta}{\gamma^2} \gamma_{\text{th}} \left| G_{2,4}^{3,1} \left( \frac{\alpha \beta}{\gamma^2} \gamma_{\text{th}} \left| 1, \xi^2 + 1 \right| \xi^2, \alpha, \beta, 0 \right) \right\} \right] . \] (10)

where \( \gamma_{\text{th}} \) is the threshold SNR.
For the $i^{th}$ transmitted symbol, where $i \in \{1, 2, ... N_s\}$, the outage probability is given by

$$P_{out}^{(i)} = Pr(\min(\gamma_{SR}^i, \gamma_{RD}^i) \leq \gamma_{th}) = \mathcal{F}_{\gamma_{\min}^i}(\gamma_{th}),$$

(11)

Since, the links encounter fading independently, $\mathcal{F}_{\gamma_{\min}^i}(x)$ can be written as

$$\mathcal{F}_{\gamma_{\min}^i}(x) = 1 - Pr(\gamma_{SR}^i > x)Pr(\gamma_{RD} > x),$$

$$= \mathcal{F}_{\gamma_{SR}^i}(x) + \mathcal{F}_{\gamma_{RD}}(x) - \mathcal{F}_{\gamma_{SR}^i}(x)\mathcal{F}_{\gamma_{RD}}(x).$$

(12)

Now, using (11) and (12), the per-block average outage probability can be obtained as

$$P_{out} = \frac{1}{N_s} \sum_{i=1}^{N_s} \left[ \mathcal{F}_{\gamma_{SR}^i}(\gamma_{th}) + \mathcal{F}_{\gamma_{RD}}(\gamma_{th}) - \mathcal{F}_{\gamma_{SR}^i}(\gamma_{th})\mathcal{F}_{\gamma_{RD}}(\gamma_{th}) \right].$$
The per-block average error probability for the dual-hop mixed RF/FSO DF cooperative system is given as 

$$P_e = \frac{1}{N_s} \sum_{i=1}^{N_s} P_e^{(i)},$$

where the average BER $P_e^{(i)}$ for $i^{th}$ bit is given as,

$$P_e^{(i)} = \frac{1}{2\Gamma(p)\Gamma(N_r-N_s+1)} G_{1,2}^{1,2} \left( \frac{1}{C_{SR}\delta_{SR}^2 q} \left| \begin{array}{c} 1-p, 1 \\ N_r-N_s+1, 0 \end{array} \right. \right)$$

$$+ \frac{\xi^2}{2\Gamma(p)\Gamma(\alpha)\Gamma(\beta)} \sum_{k=0}^{N_r-N_s} \frac{q^p}{k! \left( C_{SR}\delta_{SR}^2 \right)^k \left( q + \frac{1}{C_{SR}\delta_{SR}^2} \right)^{p+k}}$$

$$\times G_{3,2}^{3,2} \left( \frac{\alpha\beta}{\bar{\gamma} \left( q + \frac{1}{C_{SR}\delta_{SR}^2} \right)} \left| \begin{array}{c} 1-p-k, 1, \xi^2+1 \\ \xi^2, \alpha, \beta, 0 \end{array} \right. \right).$$

(13)
The average BER for the $i^{th}$ bit based on the CDF of the end-to-end SNR can be written as

$$
\overline{P}_e^{(i)} = - \int_0^\infty P_e'(\epsilon|x) F_{\gamma_{\min}}(x) dx,
$$

where $P_e'(\epsilon|x)$ is the first order derivative of conditional error probability (CEP) for the given SNR.

For coherent and non-coherent binary modulation schemes, a unified expression for CEP is given as 

\[ P_e(\varepsilon|\gamma) = \frac{\Gamma(p, q\gamma)}{2\Gamma(p)} \]  

(15)

where

- \( \Gamma(\cdot, \cdot) \) is the complementary incomplete gamma function.
- The values of parameters \( p \) and \( q \) corresponding to different modulation schemes are as summarized in Table 1.

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### Error Rate Analysis IV

**Table:** Parameters $p$ and $q$ for binary modulation schemes

<table>
<thead>
<tr>
<th>Modulation Scheme</th>
<th>$p$</th>
<th>$q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coherent binary frequency shift keying</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>Coherent binary phase shift keying</td>
<td>0.5</td>
<td>1</td>
</tr>
<tr>
<td>Non-Coherent binary frequency shift keying</td>
<td>1</td>
<td>0.5</td>
</tr>
<tr>
<td>Differential binary phase shift keying</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
Differentiating $P_e(\epsilon | \gamma)$ and substituting in (14), the $\overline{P}_e^{(i)}$ can be simplified as

\[
\overline{P}_e^{(i)} = \frac{q^p}{2\Gamma(p)} \int_0^\infty e^{-qx} x^{p-1} F_{\gamma_{\min}}^i(x) dx.
\]  

(16)

Substituting the CDF, $F_{\gamma_{\min}}^i(\cdot)$, from (12) in (16), the expression for $\overline{P}_e^{(i)}$ can be written as

\[
\overline{P}_e^{(i)} = \left[ \int_0^\infty e^{-qx} x^{p-1} F_{\gamma_{SR}}^i(x) dx + \int_0^\infty e^{-qx} x^{p-1} F_{\gamma_{RD}}^i(x) dx \right] \frac{q^p}{2\Gamma(p)}.
\]  

(17)
Error Rate Analysis VI

Using the identity for $\gamma(\cdot, \cdot)$ in terms of the Meijer’s-G function [8.4.16.1]$^9$ with [2.24.3.1]$^{11}$, each integral in (17) can be solved to yield the final expression for $\overline{P_e}^{(i)}$ as,

$$
\overline{P_e}^{(i)} = \frac{1}{2\Gamma(p)\Gamma(N_r-N_s+1)} G_{2,2}^{1,2}
\begin{pmatrix}
1 \\
\frac{1}{C_{SR}\delta_{SR}^2} q
\end{pmatrix}
\left|
\begin{array}{c}
1-p, 1 \\
N_r-N_s+1, 0
\end{array}
\right|

+ \frac{\xi^2}{2\Gamma(p)\Gamma(\alpha)\Gamma(\beta)} \sum_{k=0}^{N_r-N_s} \frac{q^p}{k! \left( C_{SR}\delta_{SR}^2 \right)^k \left( q + \frac{1}{C_{SR}\delta_{SR}^2} \right)^{p+k}}

\times G_{3,4}^{3,2}
\begin{pmatrix}
\frac{\alpha \beta}{\bar{\gamma} \left( q + \frac{1}{C_{SR}\delta_{SR}^2} \right)}
\end{pmatrix}
\left|
\begin{array}{c}
1-p-k, 1, \xi^2 + 1 \\
\xi^2, \alpha, \beta, 0
\end{array}
\right|.

(18)

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At high-SNRs, the term corresponding to $k = 0$ in (18), dominates the summation. Further, using the identity for the Meijer’s-G function [18], one can obtain the high-SNR asymptotic approximation of BER as

$$\overline{P_e} \leq C_1 \left( \frac{N_0}{P} \right)^{N_r - N_s + 1} + C_2 \left( \frac{N_0}{P} \right)^{\xi^2} + C_3 \left( \frac{N_0}{P} \right)^{\alpha} + C_4 \left( \frac{N_0}{P} \right)^{\beta},$$  

(19)

where the terms $C_1, C_2, C_3$ and $C_4$ are given in the paper.

Using (19), one can readily observe that the net diversity of the considered mixed RF/FSO DF cooperative system is

$$d = \min\{N_r - N_s + 1, \xi^2, \alpha, \beta\}.$$  

(20)

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Further, considering a total power budget $P$, the convex optimization problem for optimal source-relay power allocation can be formulated as

$$\min_{P_0, P_1} \left\{ \tilde{C}_1 \left( \frac{1}{P_0} \right)^{N_r - N_s + 1} + \tilde{C}_2 \left( \frac{1}{P_1} \right)^{\xi^2} + \tilde{C}_3 \left( \frac{1}{P_1} \right)^{\alpha} + \tilde{C}_4 \left( \frac{1}{P_1} \right)^{\beta} \right\},$$

subject to

$$P_0 + P_1 = P,$$

where

- $\tilde{C}_1 = C_1 \left( a_0 N_0 \right)^{N_r - N_s + 1}$,
- $\tilde{C}_2 = C_2 \left( a_1 N_0 \right)^{\xi^2}$,
- $\tilde{C}_3 = C_3 \left( a_1 N_0 \right)^{\alpha}$,
- $\tilde{C}_4 = C_4 \left( a_1 N_0 \right)^{\beta}$,

and $a_0 = P_0 / P, a_1 = P_1 / P$ are the power factors at the source and relay respectively.
Theorem (Optimal Power Allocation)

For a scenario when $\beta$ is less than $\alpha$ and $\xi^2$, one non-negative zero of the polynomial equation below

$$P_0^{N_r-N_s+2} - \kappa_1(P - P_0)^{\beta+1} = 0,$$

(22)

determines the optimal power $P_0^*$ at the source and hence, the optimal power at the relay is $P_1^* = P - P_0^*$ for a given total cooperative power budget $P$, where $\kappa_1 = \frac{(N_r-N_s+1)\tilde{C}_1}{\beta\tilde{C}_4}$. 
Similarly, one can obtain the polynomial equations for the other scenarios as

\[ P_0^{N_r-N_s+2} - \kappa_1 (P - P_0)^{\kappa_2+1} = 0, \quad (23) \]

where

- \( \kappa_1 = \frac{(N_r-N_s+1)\tilde{C}_1}{\xi^2 \tilde{C}_2} \), \( \kappa_2 = \xi^2 \) for the scenario when \( \xi^2 < \alpha, \beta \).
- \( \kappa_1 = \frac{(N_r-N_s+1)\tilde{C}_1}{\alpha \tilde{C}_3} \), \( \kappa_2 = \alpha \) for the scenario when \( \alpha < \xi^2, \beta \).
Simulation Result I: Outage Performance

Figure: Outage performance of the DF based mixed RF/FSO cooperative system under various channel conditions.
Simulation Result II: Error rate Performance

Figure: BER performance of the DF based mixed RF/FSO co-operative system under various channel conditions.
Closed form analytical expressions have been derived for the end-to-end SNR outage probability, BER, diversity order and optimal power allocation for a DF based mixed MIMO-RF/FSO cooperative system.

It has been observed that the optimal power factors derived using the proposed framework, significantly improve the end-to-end performance of the MIMO-RF/FSO cooperative system.
Thank You