Analysis of greedy heuristic for finding small dominating set in Graph

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1 Review of [2]

Given a graph $G = (V, E)$, a Dominating Set $D$ of $G$ is a subset of $V$ such that for every node $v \in V$, either $v \in D$ or there exists a node $u \in D$ such that $(u, v) \in E$. The minimum dominating set problem, henceforth referred to as MDS, is to find a Dominating Set of minimum size. The paper establishes a relationship between the approximate dominating set size and dimensions of graph. The approximate algorithm that is used here is analog of standard algorithm analysed by chavatal for finding smallest set covers. The paper establishes the result that maximum cardinality of dominating set ($d_g$) satisfies $d_g \leq N + 1 - \sqrt{2M + 1}$.

Greedy algorithm chooses the vertex of maximum outdegree so as to maximise the number of points dominated by it and to construct the smaller instance of our original graph, algorithm removes the chosen vertex and it’s out edges. The authour first converted the graph into corresponding directed graph by inducing bothway edges for any edges and adding self-loops so as to analyse the number of edges reduction as we add any vertex to dominating set.

Let $E_i$ denote the number of edges coming from uncovered nodes at the end of $i$th iteration, then author establishes the relation between $E_{i+1}$ and $E_i$ and used the telescopic series summation method to obtain the final result.

Note that, chavatal [1] showed that $\frac{d_g}{\pi} \leq \sum_{i=1}^{\delta+1} \frac{1}{i}$, this result should not be viewed as analogous to result established by chavatal, the relationship established in this paper is between physical dimensions of graph and cardinality of dominating set.

References