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Place of possibility theory in transportation analysis

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Abstract

The transportation phenomena, as a manifestation of the complex human, social, economic, and political interactions, are filled with uncertainties. In order for the analysis of transportation to be scientifically credible, uncertainty must be accounted for properly. Traditionally, probability theory has been used as the only paradigm for dealing with uncertainty without much thought being given to its limits of application. In recent years, a systematized framework of uncertainty theory that handles different types of uncertainty has emerged. In this framework, possibility theory offers a useful way of handling the uncertain situations that often arise in transportation analysis, particularly when incomplete data and perception are involved. This paper describes possibility theory for its mathematical structure, and discusses the reasons why its use is justified for analysis of certain transportation problems. It is shown that the use of a particular theoretical framework depends on the type of information available and the nature of the predicate of the proposition. Probability theory is justified when the propensity of occurrence of well-defined events is the issue. Possibility theory, on the other hand, is justified when the information is partially perceived and evidence points to the nested sets. The dual measures of possibility theory, possibility and necessity, evaluate the truth, optimistically and conservatively. This paper advocates the use of a proper analysis framework that is consistent with the type of information. Such an attitude not only enhances the scientific credibility of the field but also allows the analyst to express how much is known and how much is not known honestly.

Keywords: Uncertainty modeling; Possibility theory; Probability theory; Fuzzy sets theory

1. Introduction

Uncertainty is an inseparable accomplice to the analysis of transportation systems. In most analysis, data is incomplete, approximate, or inconsistent; and the models or the knowledge bases are incomplete. After all, human behavior, the main focus of transportation analysis, exhibits considerable unexplained variations. Traditionally, in transportation engineering and planning, the aspect of uncertainty has been either left untouched, or if it is considered, dealt only within one paradigm, probability theory.

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One of the challenges that the transportation community is facing today is accountability. How to profess what we do know and what we do not know in an honest and logical manner, so that the public and decision-makers understand the risks and the limits of prediction, is a pressing issue. In this respect, we need to understand many facets of uncertainty that exist in the data and the analysis process, and use the appropriate analytical treatment for them.

In the last two decades, significant advances have been made in the field of systems science to systematize theories of uncertainty. The general framework for analyzing uncertain situations has been established in the context of set theory, evidence theory, and measure theory (Zimmermann, 2000; Klir, 1999; Klir and Wierman, 2000; Smets, 1998). Under this framework, the application domain of probability theory is defined clearly. The domains of other theories of uncertainty are also defined. Among them, possibility theory is considered very relevant to transportation analysis. However, its utility in transportation analysis has not been exploited enough so far.

This paper presents the essence of possibility theory, its concept, computation procedures, and utilities when handling incomplete information in the transportation analysis. The motivation behind this paper is that the analyst must recognize different types of uncertainty that are embedded in the problems, and that, for transportation research to be scientifically credible, proper analytical treatment must be applied to each type of uncertainty. Traditionally, in transportation engineering and planning, scientific rigor in dealing with uncertainty has been rather laissez-faire despite its abundance.

2. Uncertain situations and purpose of the paper

Uncertainty is considered in the context of finding the truth of "x is A", i.e., $x \in A$. Two classes of uncertainty exist. One, when x is known but the meaning of the word A is not clear; hence, the truth of "x is A" is uncertain. Two, when A is defined clearly but the information about x is incomplete; hence, the truth (yes or no) is uncertain. The former is often referred to as vagueness or fuzziness, and the latter is referred to as ambiguity.

Vagueness is associated with the lack of clarity of the definition of a class. Expressions, such as "a large traffic volume", and "a severe congestion", are in this category. These expressions lack clarity as to its limits, and the meaning depends on the context; nevertheless, these words express a particular situation often more effectively than the numerical data. Mathematically, the membership function of the fuzzy set is a way to represent the vague notion of the word, and the operations of fuzzy sets facilitate building the logical framework for reasoning with such vague natural language based descriptors.

An increasing number of papers have been published on the application of fuzzy theory to the transportation problems. Most of them, however, are about the use of the fuzzy rule base for inference and control. A noteworthy success of fuzzy control applied to transportation is the use of fuzzy rule base to control traffic signal timing in the City of Helsinki (Nittymaki, 1998).

Discussion on application of fuzzy set theory to transportation is found in a number of articles including Kikuchi and Pursula (1998), Teodorovic and Vukadinovic (1998), and Hoogendoorn et al. (1998). Books and papers related to fuzzy set theory and its application are many; among them, particularly relevant to this paper are Dubois and Prade (1980), Klir (1999), Klir and Folger (1988), Tanaka (1990), Klir and Yuan (1995), Klir et al. (1997), Pedrycz (1995), Yager and Filev (1994), and Zimmermann (1996).

Ambiguity is associated with the lack of clarity in information about x. If, for example, information about x is given by a probability distribution, then the truth of a statement "the estimated delay (x) is 30 min", depends on the form of the probability distribution of the estimated delay (x). If the statement is "the estimated delay is *approximately* 30 min", then uncertainty is compounded due to the fuzziness of the word *approximately* as well as the probability distribution of delay.

Traditionally, probability theory has been the only paradigm for analyzing uncertainty, in which the probability distribution is the information about x, and the truth is evaluated in terms of the probability measure. In the past two decades, however, the treatment of ambiguity has been systematized under the theory of evidence, which subsumes probability theory, possibility theory, and the Demspter–Shafer theory. A comprehensive description of the theory of evidence is found in Klir and Wierman (2000).

The primary purpose of this paper is to address the aspect of handling *ambiguity*. It is about the type of information on x and how to measure its support for a proposition, "x is A". The question is how one should assess and interpret the degree of truth of a proposition, when the information is incomplete and it is not necessarily related to stochastic events. We introduce possibility theory as an avenue to deal with such situations.

Relevance of this subject is found in many areas, particularly, in decision-making (both at the individual and the collective levels) when information is based on perception, approximate values, or estimated quantities (of cost, time, etc.). We compare probability theory and possibility theory in terms of the nature of information that each can handle. This is followed by a discussion on the nature of uncertainty that resides in many transportation problems, and a discussion of the situations where possibility theory framework is useful. Examples are also presented.

3. The proposition, "x is A": the nature of x and A

3.1. The proposition

The simplest form of a proposition is "x is A", where subject x refers to the condition of an object, and predicate A refers to an alternative. The following are examples of simple propositions that we typically encounter.

Example 1. Proposition: *Traffic condition of the freeway segment is level of service C* ($LOS = C$).

$x =$ the traffic condition of the freeway segment, say in terms of density; $A =$ the range of values of density defining LOS $= C$.

Example 2. Proposition: *If I leave now, the arrival time at the destination is* (*will be*) *within the desired arrival time*.

$x =$ one's estimated arrival time at the destination; $A =$ the desired arrival time (which may be an exact time or a range).

In general, we determine the truth of these propositions in terms of yes or no, possibility, or probability. In all cases the truth is evaluated by the degree that the information (evidence) about x supports A. Usually, determining the truth or falsehood of these statements is relevant because the degree of truth becomes the degree of support for a particular decision that follows; in the above examples, Example 1, if true, may lead to a decision "no need to improve the current design at this time". Or Example 2, if true, may lead to a decision "I should leave now", or "I should wait a little while". Finding the truth of a proposition is the precursor to a decision and the decision-making process.

3.2. Information about x

Traditionally in transportation analysis, occurrence of event, x, is treated as either a deterministic or a stochastic event, and hence, the assumed information about x is either a single value or a probability distribution. This is the case when one is dealing with purely random events, such as the number of vehicles arriving per unit time.

In many problems, however, x refers to non-random events, such as a *desired* quantity or an *estimated* value. Even for the event whose occurrence may be random, information about it may not be purely frequency based; it may be biased by memory, perception, or the guess of an expert. Information about x can be categorized into three types, Types 1, 2, and 3, by the manner in which pieces of information points to different alternatives. They are illustrated in Fig. 1.

Type 1 information: This type of information, as shown in Fig. 1, is called "conflicting" because each piece of evidence is in conflict with one another. This is the situation in a random phenomenon, in which each experiment yields a distinct outcome, such as the case of rolling a die. Based on the frequency (either empirical or theoretical) for each outcome, a frequency distribution is developed, and this distribution—the probability distribution—becomes the instrument to measure the truth of a proposition, x is A. Fig. 1 also shows a typical shape of probability distribution obtained from Type 1 information.

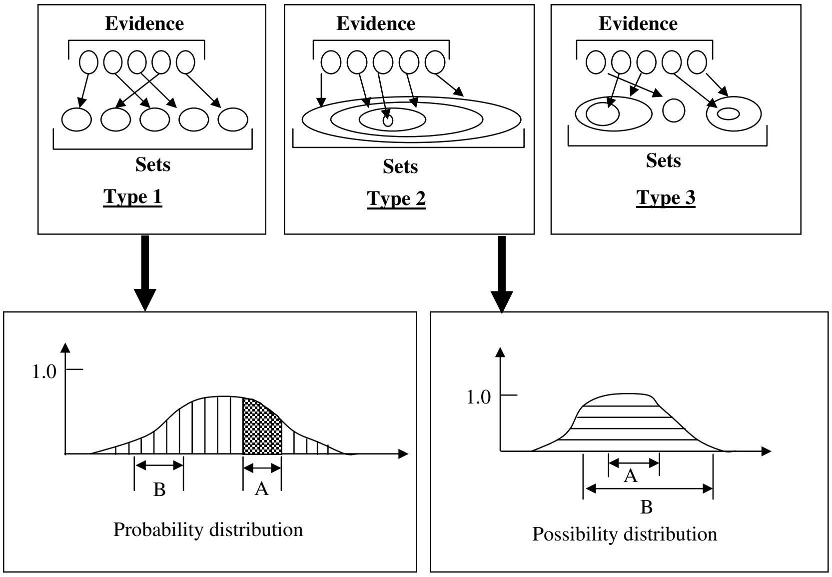


Fig. 1. The three types of evidential pattern.

Type 2 information: This type of information, as shown in Fig. 1, is called "consonant", because every piece of evidence points to a range of nested sets so that each piece of evidence agrees with (or reinforces) one another to a degree. This is a situation that one often encounters in the data that express acceptable or tolerable quantities (such as acceptable price or time), or desire. The distribution of this type of evidence is called the possibility distribution. The information is called possibilistic information.

Fig. 1 also depicts a possibility distribution. Individual pieces of evidence, which are represented by the horizontal lines, cover the intervals in the nested manner. Note that the maximum value of this distribution is 1. This pattern indicates that each evidential claim (shown as a range on the x-axis) agrees with one another to some degree; for example, the piece of evidence that supports set A also supports its superset, set B.

Type 3 information: This type is a mix of Type 1 and Type 2 evidential pattern, or a generalization of these two types. In this case, some pieces of evidence point to sets that are nested and some to mutually exclusive sets as seen in Type 1. This evidential pattern is studied in Demspter–Shafer theory (Shafer, 1976); because this is not the central topic of this paper, we do not discuss it further here.

3.3. Character of set A in "x is A"

The predicate A, of the proposition, x is A, is a description of a situation and can be viewed as a set representing either a single value, precise range, or a fuzzy set. An example when A is a precise range is the LOS classifications, as defined in the Highway Capacity Manual (Transportation Research Board, 2000). Another example is a range of values (or the limiting values) defined by regulations, e.g., air quality standards. Examples when A is a fuzzy set are found in the approximate values that describe desire and goal, or the tolerable quantity, such as the acceptable value of delay and the acceptable cost of travel.

4. Frameworks for determining the measure of truth of "x is A"

How information about x and the type of set A are combined dictates the analytical framework for determining the truth of "x is A". The types of information that we are concerned with in this paper are Type 1 (probabilistic information) and Type 2 (possibilistic information). The types of set for A that we are concerned with are the crisp set and the fuzzy set. Table 1 presents different frameworks that correspond to the combination of the type of information about x and the type of set A. Each case will be explained in the following sections.

Table 1
Classification of analysis frameworks and general expressions for computation

Type of information about x	Type of set (A)	
	Crisp set	Fuzzy set
Type 1 (conflicting) probabilistic information	Analysis framework is probability theory (see Eq. (1))	Analysis framework is probability theory (see Eq. (2))
Type 2 (consonant) possibilistic information	Analysis framework is possibility theory (see Eq. (4))	Analysis framework is possibility theory (see Eq. (5))

4.1. Measuring the truth of "x is A" with type 1 information for x

When the information is Type 1 or a probability distribution, truth is measured by summing all pieces of evidence that points towards individual elements of set A (as shown in Fig. 1). Summation (or addition) is possible, because under Type 1 evidential pattern, each piece of evidence points towards specific elements (or subsets of A) exclusively. How this summation is performed depends on the nature of set A.

4.1.1. When set A is a crisp set

The truth of the proposition, x is A, is evaluated by summing the weights of evidence supporting set A.

$$\text{Truth of "}x\text{ is }A\text{"} = \begin{cases} \sum_{\forall x \in A} \text{Prob}(x) & \text{when } x \text{ is discrete} \\ \int_{\forall x \in A} f(x)\,\mathrm{d}x & \text{when } x \text{ is continuous} \end{cases} \tag{1}$$

where Prob(x) and $f(x)$ are the probability density functions defining the evidence about x, when x is discrete, and when x is continuous, respectively.

4.1.2. When set A is a fuzzy set

The evidence pointing to a partial member (of set A) should count less towards determining the truth of "x is A" than the evidence pointing to a full member of the set A. Thus, in this case, a weighted sum of the evidence is used to measure the truth. The weight associated with the evidence pointing to an element is simply the membership grade of that element in the set A. This is a generalization of the idea used for crisp sets where every member had an equal membership of 1 in the set. Thus, in this case truth is measured as follows:

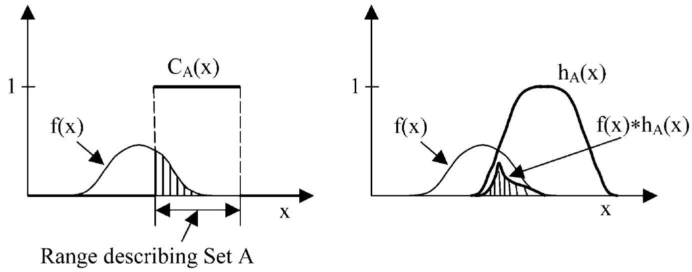
$$\text{Truth of "}x\text{ is }A\text{"} = \begin{cases} \sum_{\forall x} h_A(x) \cdot \text{Prob}(x) & \text{when } x \text{ is discrete} \\ \int_{\forall x} h_A(x) \cdot f(x)\,\mathrm{d}x & \text{when } x \text{ is continuous} \end{cases} \tag{2}$$

where $h_A(x)$ is a membership function of fuzzy set A. Note that, Eq. (1) can be obtained from Eq. (2) by replacing $h_A(x)$ with 1 for $\forall x \in A$, and 0 otherwise.

The two cases described above are illustrated (for the instances when x is continuous) in Fig. 2(a) and (b), respectively. The hashed areas in both the figures indicate the measured amount of the truth of "x is A".

4.2. Measuring the truth of "x is A" with type 2 information for x

For Type 2 information, it is difficult to determine the evidential support for a specific set A, because each piece of evidence does not necessarily support a particular set A exclusively, but it supports its complement as well (see Fig. 1). Thus, the measure of truth of "x is A" cannot be determined through summation as in the case of the probability measure (Type 1 information) explained earlier.



(a) Probability of a crisp event A **(b)** Probability of a fuzzy event A

Fig. 2. Illustration of computations presented in Eqs. (1) and (2).

With Type 2 information, the evidence is evaluated in two ways:

- Weight all pieces of evidence that point to set A, regardless of whether or not they point to Not A at the same time.
- Weight those pieces of evidence that point to set A only; this is done by not weighing the pieces of evidence that point to Not A.

The degree of truth obtained by the former approach (or way) is called the possibility measure. This measure symbolizes the optimistic view, because it accounts for any evidence as long as it supports set A, either partially or exclusively. The degree of truth obtained by the latter approach is called the necessity measure. It symbolizes the conservative view, because it accounts for only the evidence that points to the set and none others. The latter is a stricter way of measuring the truth than the former. In a way, possibility measure is similar to determining the truth of "x is A" when it is interpreted as "I *think* that x is A", while necessity is similar to determining the truth of "x is A" when it is interpreted as "I *have no doubt* that x is A". Thus, the value of possibility measure is always greater than or equal to the necessity measure.

The necessity measure, Nec(·), and the possibility measure, Poss(·), have the following dual relationship:

$$\text{Nec}(A) = 1 - \text{Poss}(\text{Not } A) \quad \text{and} \quad \text{Poss}(A) = 1 - \text{Nec}(\text{Not } A) \tag{3}$$

The above indicates that A is necessarily true (i.e., $\text{Nec}(A) = 1$) only when "Not A" is not possible. Incidentally, in probability theory, the counterpart to Eq. (3) is, $\text{Prob}(A) = 1 - \text{Prob}(\text{Not } A)$; Prob($A$) can be derived from the probability of its complement, Prob(Not A). This is not the case for possibility, because the weights of evidence for "A" and for "not A" are not additive. In the following, specific expressions for deriving possibility and necessity measures are presented when A is a crisp set, and when A is a fuzzy set, separately.

4.2.1. When set A is a crisp set

As stated earlier, two views, Poss(A) and Nec(A), need to be investigated when information about x is incomplete. For Poss(A), if any evidence points to set A, then it must be accounted for; for Nec(A), if there is evidence that points to "Not A", then it must be discounted. The general expression of Poss(A) and Nec(A) are as follows:

$$\begin{aligned}&\text{Truth of "}x\text{ is }A\text{" (optimisitically)} = \text{Poss}(A) = \underset{\forall x \in A}{\text{Max}}\, \pi(x)\\&\text{Truth of "}x\text{ is }A\text{" (conservatively)} = \text{Nec}(A) = 1 - \text{Poss}(\text{Not}A) = 1 - \underset{\forall x \notin A}{\text{Max}}\, \pi(x)\end{aligned} \tag{4}$$

In Eq. (4), $\pi(x)$, is the possibility distribution representing the information about x. The above equations are explained using illustrations in Fig. 3. In Fig. 3(a), $\pi(x)$ is assumed to be rectangular, and in Fig. 3(b), it is assumed to have any shape. In both figures, the horizontal lines in $\pi(x)$ are meant to be the individual pieces of evidence pointing to the nested sets. In the figures, $C_A(x)$ represents the crisp set A through its characteristic function.

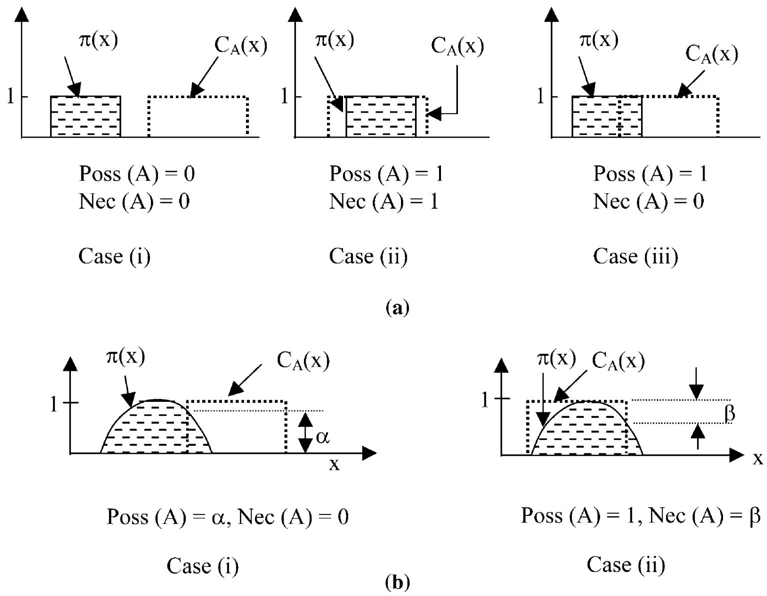


Fig. 3. Possibility and necessity measures when (a) $\pi(x)$ is rectangular and (b) $\pi(x)$ is any general shape.

In Fig. 3(a) Case (i), $\pi(x)$ and A have no overlap, or no evidence supports A, thus, Poss(A) and Nec(A) are both zero. In Case (ii), $\pi(x)$ is completely contained in set A, or all the evidence supports A; thus both Poss(A) and Nec(A) are 1. In Case (iii), $\pi(x)$ and A partially overlap, or evidence supports A as well as Not A; thus using Eq. (4), Poss(A) = 1 and Nec(A) = 0.

Fig. 3(b) illustrates the application of Eq. (4) to a general shape of $\pi(x)$. In Case (i), evidence as represented by the horizontal dashed lines, points to set A partially, thus Poss(A) = α, which is the maximum available evidence pointing to A. In this case, Nec(A) = 0, because the evidence supporting Not A is fully available. In Case (ii), the evidence supporting A is fully available, thus Poss(A) = 1, and the evidence that supports Not A is $1-\beta$; hence Nec(A) $= 1-(1-\beta) = \beta$.

4.2.2. When set A is a fuzzy set

This is a case of determining whether x satisfies a vaguely defined criterion, A. The principle of measuring the truth is the same as the previous case, except for introducing the membership function of fuzzy set A in Eq. (4). That is, if evidence that supports x (measured as possibility of x) is $\pi(x)$, and the membership of x in set A is $h_A(x)$, then the contribution of the evidence towards the truth of "x is A" is given as $\mathrm{Min}(\pi(x), h_A(x))$; hence considering Eq. (4), the expression becomes,

$$\begin{aligned}&\text{Truth of "}x\text{ is }A\text{" (optimistically)} = \mathrm{Poss}(A) = \underset{\forall x}{\mathrm{Max}}\,\mathrm{Min}(\pi(x), h_A(x))\\&\text{Truth of "}x\text{ is }A\text{" (conservatively)} = \mathrm{Nec}(A) = 1-\mathrm{Poss}(\mathrm{Not}A) = 1-\underset{\forall x}{\mathrm{Max}}\,\mathrm{Min}(\pi(x), 1-h_A(x))\end{aligned} \tag{5}$$

Note that Eq. (5) is a general version of Eq. (4) as a fuzzy set is a general version of a crisp set. For example, if A is a crisp set then $h_A(x)$ should be replaced by $C_A(x)$ (which is 1 when $x \in A$ and 0 otherwise) and then Eq. (5) becomes the same as Eq. (4). Eq. (5) is illustrated in Fig. 4.

The value α in Fig. 4(a) illustrates the meaning of Poss(A). The bold line is the locus of the point $\mathrm{Min}(\pi(x), h_A(x))$. Hence, according to Eq. (5), Poss(A) is the maximum value of the bold line in Fig. 4(a); this value is α. The value β in Fig. 4(b) illustrates the meaning of Nec(A). The dotted line represents the function, $1-h_A(x)$, or $h_{\mathrm{Not\,A}}(x)$. The point with a value of $\beta\{=\mathrm{Max}_{\forall x}\mathrm{Min}(\pi(x), 1-h_A(x))\}$ is Poss(Not A). Hence, Nec(A) is $1-\beta$, which indicates the elimination of the weight of evidence that points to Not A.

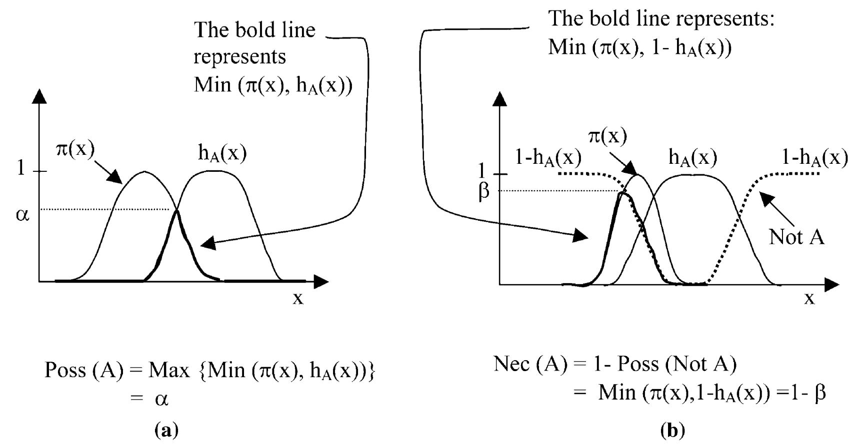


Fig. 4. Illustration of computations presented in Eq. (5).

5. Relevant properties of probabilistic and possibilistic measures

While specific properties of possibility theory and probability theory are different, these theories are subsumed in measure theory, which measures the degree of truth of a set. A measure function, $g(S)$, provides the degree of truth of a set S by the values between 0 and 1. An important axiomatic condition of $g(S)$ is monotonicity, that is

$$\text{when } A \subseteq B, \quad \text{then } g(A) \leqslant g(B); \quad g(A \cup B) \geqslant \text{Max}\{g(A), g(B)\} \quad \text{and} \quad g(A \cap B) \leqslant \text{Min}\{g(A), g(B)\}$$

This condition states when A is a subset of B, then $g(A)$ should be smaller than $g(B)$. This comes natural when observing the real world; the larger the item, the greater the measure of that item. The specific form of the monotonicity condition characterizes the differences between probability and possibility theory in terms of (1) additivity of measures, (2) expression of uncertainty, and (3) effects of improved information.

5.1. Additivity

Probability theory evaluates the degree of truth of a set by weighting all the pieces of evidence that point to the subsets of the set. Hence, the degree of truth of $A \cup B$, is the sum of the degrees of truth of A and B. This makes the probability measure additive.

$$\begin{aligned} &\text{Prob}(A \cup B) = \text{Prob}(A) + \text{Prob}(B), \quad \text{when } A \cap B = \phi; \quad \text{or} \\ &\text{Prob}(A \cup \text{Not } A) = \text{Prob}(A) + \text{Prob}(\text{Not } A) = 1 \end{aligned} \tag{6}$$

The attractiveness of probability theory is rooted in this additivity property, which lends the calculations with probability theory to the classical arithmetic operators.

If probability theory is applied to measure the truth of A, (Prob(A)), subjectively, then the truth of Not A, Prob(Not A), is made equal to (1 − Prob(A)), even though this may not conform to the analyst's subjective judgment. In other words, under probability theory, once Prob(A) is known, then its complement, Prob(Not A) is automatically known with or without the evidence for it.

In *possibility theory*, because the evidential pattern supports both A and Not A, the strict additivity principle is not satisfied. Monotonicity principle is however satisfied as follows:

$$\text{Poss}(A \cup B) = \text{Max}\{\text{Poss}(A), \text{Poss}(B)\} \quad \text{and} \quad \text{Nec}(A \cap B) = \text{Min}\{\text{Nec}(A), \text{Nec}(B)\}$$

Further,

$$\begin{aligned} &\text{Poss}(A) + \text{Poss}(\text{Not}A) \geqslant 1, \quad \text{Nec}(A) + \text{Nec}(\text{Not}A) \leqslant 1 \quad \text{and} \\ &\text{Poss}(A) + \text{Nec}(\text{Not}A) = 1, \quad \text{Poss}(\text{Not}A) + \text{Nec}(A) = 1 \end{aligned} \tag{7}$$

The interested reader may refer to Klir and Wierman (2000) for a more detailed understanding of the above and other properties of different types of measures. This non-strict additive property allows the analyst to assign his/her belief to set A and Not A independently. Compared with probability theory, this property is useful when quantifying perception or belief under uncertainty, without being restricted by the additivity condition.

5.2. Representation of total uncertainty (or ignorance)

How to present the notion of ignorance or "I do not know" in the face of incomplete information, is an important aspect of analysis in transportation. However, this has not been rigorously studied, partly because probability theory lacks the mechanism to express ignorance.

In *probability theory*, ignorance about the occurrence of any particular outcome (out of n possible events) is represented by assigning $1/n$, as the probability of its occurrence. However, $1/n$ can also indicate the probability of occurrence of equi-probable events, and not total uncertainty (or ignorance). If one is totally uncertain (or ignorant), logically, one cannot develop a function (like the uniform distribution) that represents information. In other words, under probability theory, total uncertainty (ignorance) cannot be distinguished from uniformly distributed evidence. Probability distribution actually represents the conflict among the evidence, thus entropy, often used for the measure of uncertainty, represents the lack of order in the evidence.

Possibility theory, on the other hand, allows the expression of total uncertainty distinctly as a combination of possibility and necessity measures.

$$\left.\begin{array}{lll} \text{Total uncertainty about } A & \text{Poss}(A)=1 \text{ and } \text{Nec}(A)=0, & \text{or } (\text{Poss}(\text{Not } A)=1) \\ \text{Total certainty of } A & \text{Poss}(A)=1 \text{ and } \text{Nec}(A)=1, & \text{or } (\text{Poss}(\text{Not } A)=0) \\ \text{Total certainty of Not } A & \text{Poss}(A)=0 \text{ and } \text{Nec}(A)=0, & \text{or } (\text{Nec}(\text{Not } A)=1) \end{array}\right\} \quad (8)$$

The first case refers to total uncertainty about the truth of A, in which everything is possible but nothing is necessarily true (or nothing is "not possible"). The second case refers to the situation, in which every evidence supports A and no evidence supports "Not A". Hence, in this case there is absolute certainty about A. The third case refers to the situation, in which every evidence supports "Not A", and no evidence supports A. It is seen from the above that the difference between Poss and Nec measures represents a degree of ignorance; such a representation is not possible by probability theory.

Perincherry (1994) proposed a measure of confidence, Con(A), as the difference between the measure of possibility and possibility of its complement: where $\text{Con}(A) = \text{Poss}(A) - \text{Poss}(\text{Not } A) = \text{Poss}(A) + [1 - \text{Nec}(A)] = \text{Poss}(A) + \text{Nec}(A) - 1$. This indicates the range of analyst's confidence, from the total affirmation of the proposition, to ignorance, and to the total non-affirmation. $\text{Con}(A) = -1$ for total non-affirmation of the proposition when $\text{Poss}(A) = 0$ and $\text{Nec}(A) = 0$. $\text{Con}(A) = 0$ for total uncertainty, when $\text{Poss}(A) = 1$ and $\text{Nec}(A) = 0$. $\text{Con}(A) = 1$, for total affirmation, when $\text{Poss}(A) = 1$ and $\text{Nec}(A) = 1$.

5.3. Effects of improved information

How much the additional information reduces uncertainty is a relevant subject when dealing with the problem of data collection and also when providing information to travelers. As information becomes more specific, or the range for the distribution (evidence) narrows, then the evidential support for (or against) a set A becomes clear; hence, the proposition's truth should become easier to determine. Eventually, under the perfect information, the degree of truth should be dictated by the nature of the predicate (A) of the proposition, not by the nature of the information about x.

Consider that perfect information is obtained and it is $x = c$, and also set A is a crisp set. Then, both in the probability framework and possibility framework, truth is obtained as either 0 or 1. That is,

$$\left.\begin{array}{l} \text{Prob}(A)=1, \text{ if } c\in A \quad \text{and} \quad \text{Prob}(A)=0, \text{ if } c\notin A; \quad \text{and} \\ \text{Poss}(A)=1 \quad \text{and} \quad \text{Nec}(A)=1, \text{ if } c\in A, \quad \text{and} \\ \text{Poss}(A)=0 \quad \text{and} \quad \text{Nec}(A)=0, \text{ if } c\notin A \end{array}\right\} \quad (9)$$

Consider, on the other hand, if A is a fuzzy set with membership $h_A(x)$, then using the earlier equation (Eqs. (2) and (5)), truth is computed for probability and possibility theory as follows. In the probabilistic framework: Truth of "x is A" $= \sum_{\forall x} h_A(x) \cdot \mathrm{Prob}(x) = h_A(c)$. This is because, under perfect information, Prob(x) becomes 1 when $x = c$, and 0 when $x \neq c$. In the possibilistic framework, under perfect information, $\pi(x) = 1$ when $x = c$ and 0 when $x \neq c$.

$$\text{Truth of "}x\text{ is }A\text{" (in possibility measure and optimistically)}$$
$$= \mathrm{Poss}(A) = \underset{\forall x}{\mathrm{Max}}\,\mathrm{Min}\{\pi(x), h_A(x)\} = \mathrm{Min}\{1, h_A(c)\} = h_A(c) \quad \text{and}$$

$$\text{Truth of "}x\text{ is }A\text{" (in necessity measure or conservatively)} = \mathrm{Nec}(A) = 1 - \underset{\forall x}{\mathrm{Max}}\,\mathrm{Min}\{\pi(x), 1 - h_A(x)\}$$
$$= 1 - \mathrm{Min}\{1, 1 - h_A(c)\} = 1 - (1 - h_A(c)) = h_A(c)$$

The above shows that as information become more specific, or "sharpened", probability and possibility yield the same result. When A is a crisp set, then the truth is known as yes or no (0 or 1), i.e., clearly. When A is a fuzzy set, then the fuzziness of set A dictates the truth. In other words, even if the information is perfect, if set A is not defined well, then uncertainty still remains.

This situation is often encountered in the planning process when the objective or target is not clearly defined; no matter how precise the information is, uncertainty still exists as to whether the target is met or not. Therefore, improvement in the quality of information or data must be performed relative to the precision of one's objective.

6. Place of possibility theory in transportation analysis

We now present the nature of uncertainty in transportation problems that are amenable to treatment by possibility theory, examples of application, and how to develop a possibility distribution for the real world applications.

6.1. Nature of uncertainty in transportation

The study of transportation entails prediction, diagnosis, control, and regulation (optimization) of a system. The important elements of analysis are data (or input), knowledge base (model), interpretation of the model outcome, and the objective(s) or goal(s). Some of the properties of these elements, as described in the following, make possibility theory a useful analytical framework.

Data and information: The truly statistical information, which lends itself to probabilistic analysis, is rather hard to come by. Even though the original data may be based on the statistics, it is often filtered through perception, and information is presented as a range of values or an approximate (possible) value.

Knowledge base or the model: The knowledge about transportation phenomena is largely empirical. The physical principles that dictate the phenomena, such as the conservation of flow, are few. Causality is mostly represented through empirical approximate language-based rules. In addition, inferences are often made using the logic of similarity; for example, justifying improvement of transit service by referring to the success in other cities.

Interpretation of the result: The results of analysis are interpreted as approximate, even though the procedure leading to them may have been deterministic. For example, the forecast travel demand is understood as an approximate (or possible) value, even though the procedure for deriving it is deterministic, another example is the capacity of roadway.

Objectives and goals: What we want to achieve is often not too specific; usually there is a window of tolerance. The measure of performance variables, like travel time, delay, cost, utility, and the level of service, are open-ended, e.g., "lesser the better", or "greater the better". In other words, what is aimed in the analysis is satisfaction, referring to meeting the target or desire. As a result, the statements that the analyst makes regarding the conclusion of the study are in the category of, "it is possible that…", or "it is acceptable that…", or "it is consistent with the available data that…"

6.2. Typical problems suited to possibility theory application

In view of the above, the problems typically suited to possibility theory arise in the following situations.

First, *the information* about the character of the proposition (x) or predicate (A) is approximate due to the measurement imprecision and perception. Such information includes,

(a) Notion of desire, e.g.,
 Desired departure time and desired arrival time.
(b) Notion of satisfaction and acceptability (vague threshold values), e.g.,
 Satisfactory cost, acceptable cost, willing to pay.
 Acceptable level of error, acceptable delay, acceptable air pollution level.
(c) Perception and quantities based on memory, e.g.,
 Travel time, distance, appearance and condition.
(d) Descriptive condition, e.g.,
 Traffic congestion—bad traffic and good traffic condition.
 Comfort, safety, level of service.
(e) Imprecise values—hard to measure or hard to summarize, e.g.,
 Sight distance, reaction time, value of time.

Among the above examples, one's estimated travel time by automobile between two points is a typical case of possibility. Although the statistically based information about past travel times may exist, because one is able to control the travel time somewhat, by driving fast or slowly, the travel time *estimated by the traveler before travel* is a set of values which are "possible" to achieve by the traveler and differ among the individuals. Another case is the capacity of a roadway, whose precise value is perhaps impossible to determine; it is understood as an approximate value, which the roadway can "possibly" handle. To some extent, the forecast value of travel demand is in this category.

Second, *the analysis situations* suited for the possibility framework are (a) classifying a situation into one of the predetermined classes whose boundaries are not precise, (b) comparing two approximate quantities, or a value with a reference value, and (c) determining preference (or ranking) between alternatives.

More specifically,

(a) Classification, e.g., assigning the current (or future) traffic condition, whose measurement is approximate, to a level of service category.
(b) Justification of investment, e.g., comparing the estimated transit ridership with a threshold ridership value that justifies the investment.
(c) Feasibility, e.g., comparing the estimated arrival time with the desired arrival time.
(d) Preference and ordering, e.g., comparing utilities of different alternatives, comparing the predicted performance and the desired performance.

In dealing with these problems, possibility theory is useful for (a) incorporating the analyst's (or decision maker's) attitude—optimistic and conservative views, (b) analyzing the anxiety of the analyst (or decision maker) that accompanies uncertainty, and (c) analyzing what remains unknown and the impact of additional information. These subjects naturally arise whenever uncertainty exists, but particularly when the information is approximate (not as rigid as the probability distribution). In the following we present four examples of the use of possibility theory.

6.3. Example 1: Comparing estimated transit demand with the threshold value for investment justification

Suppose that in an early stage of planning a light rail transit (LRT) line, the "estimated ridership" during the peak hour is known to be "approximately 30,000 (set B)" persons per hour. Also, suppose that the conventional reference that justifies the LRT investment is "ridership around 40,000 persons per hour or more (set

A)''. To determine whether the LRT investment is justified (only from the standpoint of peak hour demand), two approximate numbers, approximately 30,000 (B), and greater than approximately 40,000 (A), are compared. The proposition for which the truth is investigated is whether ''B is greater than A'', ($B \geqslant A$).

With the absence of any other information, the analyst may make up the possibility distribution of ''approximately 30,000 (B)'' and a fuzzy set for ''greater than approximately 40,000 (A)'' with examples being shown in Fig. 5. In the figure, $\pi(x)$ and $h_A(x)$ denote the distribution and the membership function, respectively (how to develop the shapes of $\pi(x)$ and $h_A(x)$ are discussed in the latter part of this paper).

Poss($B \geqslant A$) and Nec($B \geqslant A$) are obtained using Eq. (5): $\text{Poss}(B \geqslant A) = \text{Max}\,\text{Min}(\pi(x), h_A(x)) = \alpha$, and $\text{Nec}(B \geqslant A) = 1 - \text{Poss}(\text{Not } B \geqslant A) = 1 - \beta$. This result can be interpreted as, the LRT investment is *possibly justified* with the value of α, but there is not much evidence that it is *necessarily justified*. These two measures provide the feeling of the analyst's uncertainty more faithfully under incomplete information, by reflecting both the optimistic and conservative views, respectively.

In this example, if the probabilistic approach were used, the approximate value, ≈30,000, needs to be translated into a probability distribution, $p(x)$, which must satisfy the additivity axiom. The result will be *the probability that investment is justified*. Further, *the probability that the investment is not justified* is also obtained automatically. This *probabilistic* interpretation (which implies randomness of the event), however, is not natural and awkward given the way that the ridership estimate was derived (assuming it does not have probabilistic derivation).

6.4. Example 2: Comparing estimated arrival time and desired arrival time—effect of attitude and anxiety associated with departure time decisions

Suppose that one wishes to reach a destination ''before 5 p.m. (A)'' by automobile, and the estimated travel time is ''*approximately* 120 min (x)''. This travel time is interpreted as what the traveler, who uses the automobile, considers as ''possible'' and forms a possibility distribution. Possibility and necessity of ''on-time arrival'' (and also ''late arrival''), if one departs at a departure time, t, are examined. The situation is illustrated in Fig. 6.

6.4.1. Possibility and necessity of on-time arrival and late-arrival

Possibility and necessity measures of on-time arrival are obtained for a given departure time (t). Let us denote the arrival time as T ($=t + x$), where x is the travel time. If ''before A'' is the desired arrival time, then Poss($T \leqslant A$) and Nec($T \leqslant A$), respectively, represent the possibility and necessity measure of on-time arrival.

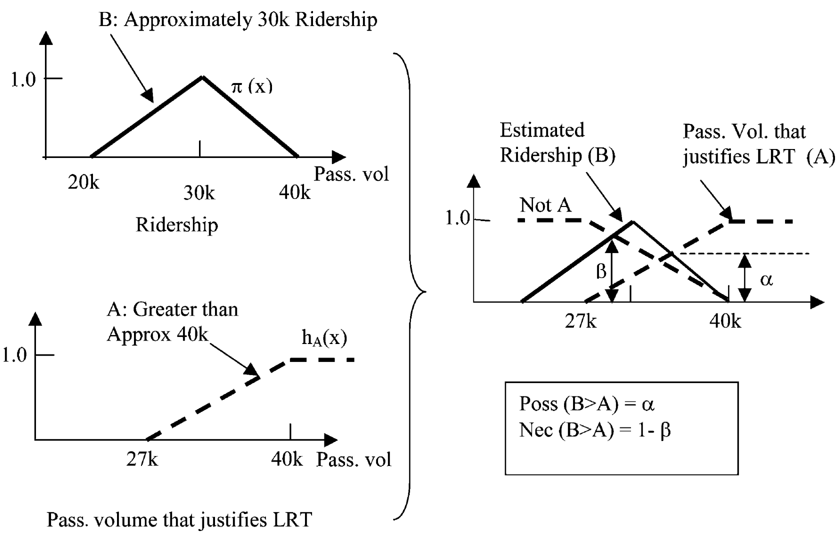


Fig. 5. Illustration of Poss(·) and Nec(·) for Example 1.

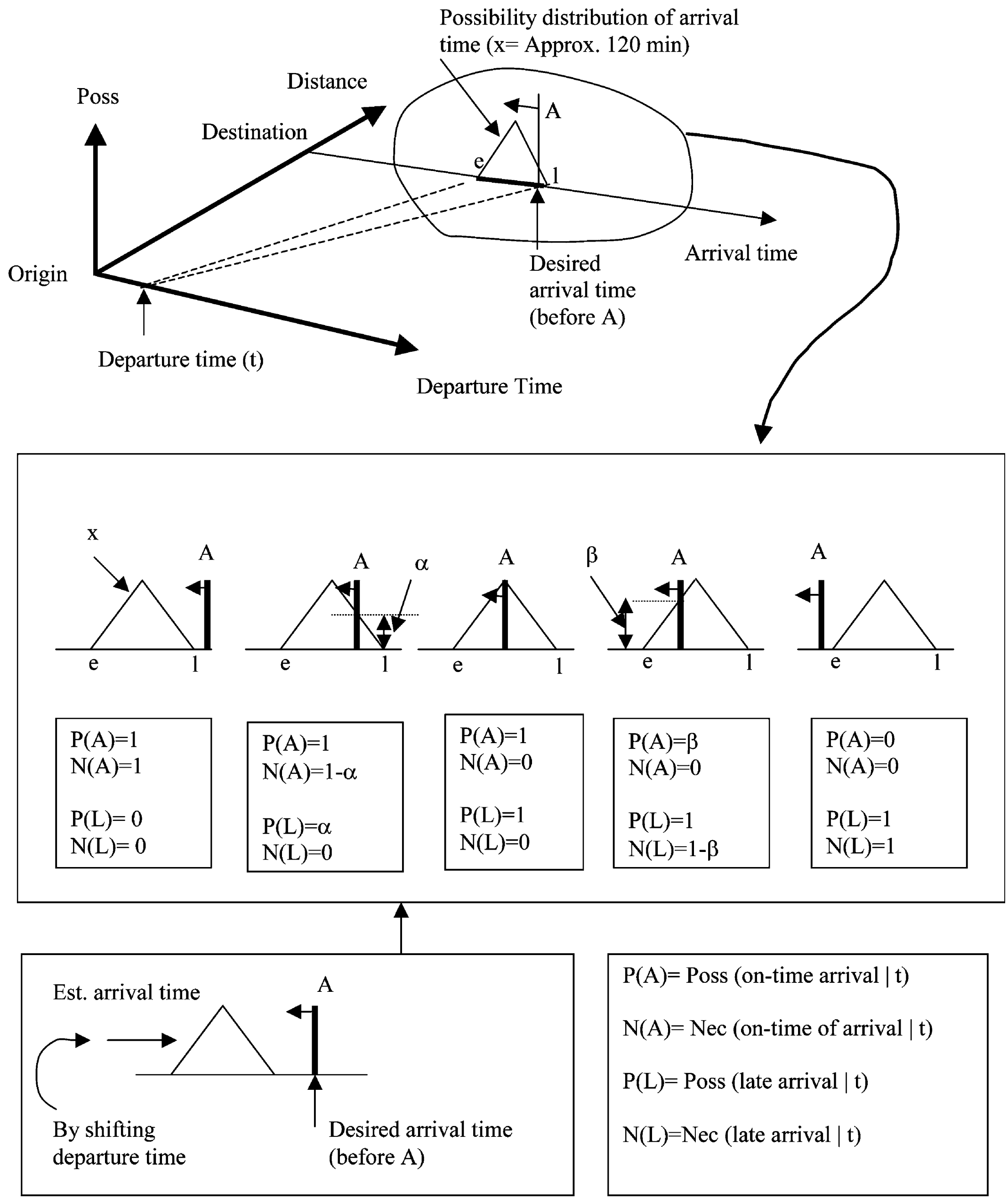


Fig. 6. Poss(·) and Nec(·) measures of on-time arrival and late arrival as a function of departure time, t.

The computation process is illustrated in Fig. 6, as a function of departure time, t, and under the assumption that "before A" is precise. As we shift the departure time, t, to the right (delayed departure), then the possibility distribution of arrival time slides to the right also. Fig. 6 shows Poss(·) and Nec(·) of on-time arrival ($T \leqslant A$) and late arrival ($T > A$) graphically, as the departure time moves to the right on the x-axis. The expressions are consistent with Eq. (5) (see also Table 1).

6.4.2. *Traveler's anxiety about deciding on the departure time*

The use of possibility theory allows us to study the anxiety faced by travelers about selecting a particular departure time. In this paper, anxiety about the departure time is defined as the discord between "to leave now'', and ''not to leave now, but later'' in the mind of the decision maker (traveler). The forces (momentum or push) supporting each of these alternatives are represented by the measures of possibility and necessity of on-time arrival or late arrival.

At a given time point, one's motivation to leave is given by either Poss(·) or Nec(·) of ''late arrival'', because of the prospect of being late if one does not leave now. Also at the same time one may be motivated not to leave. This motivation is supported by Poss(·) or Nec(·) of ''arrive on or before the desired arrival time'' because there is no threat of arriving late if one leaves now (thus one may decide not to leave now, rather leave later). In this analysis we assume that the decision-maker (regardless of personality) wants to delay the departure time as long as he is not late at the destination.

The question then is to evaluate the anxiety a traveler faces while choosing a departure time, given that any choice will have conflicting motivations. Let us consider two types of traveler, one optimistic and another conservative. We model anxiety that each of the two types of traveler feels, with respect to the time of departure.

The optimistic traveler considers the *possibility* measure, Poss(arrive on time | leave at t), as the indication of how possible it is to arrive on-time if he leaves at time t; thus this measure supports the momentum for ''not to leave now'' (because ''It is still possible to arrive by leaving later''). At the same time, this traveler considers the *necessity* measure of late-arrival, Nec(late arrival | leave at t) as the momentum to ''leave now'', (because, ''otherwise, I will necessarily arrive late''). Hence, the discord between, ''leave at time t'', and ''not to leave at time t'', are represented by Nec(late arrival | leave at time t) and Poss(on-time arrival | leave at time t), respectively.

The conservative traveler, on the other hand, considers that *necessity* of on-time arrival, Nec(arrive on time | leave at time t), as the force to ''not to leave (yet), because I will necessarily arrive, if I leave now (time t)''. But, he considers the *possibility* of late arrival, Poss(arrive late | leave time t), as the momentum to ''leave now (time t), because, otherwise, there is a possibility I may not arrive on time''. The decision criteria for the two travelers are summarized in Table 2.

Yager (1982) introduced a measure of anxiety when one has to choose one alternative from many. It is expressed as a function of the strength of the forces that support different alternatives, more specifically, as cardinality (or the size) of the decision set. In the case of two alternatives (i.e., binary choice) anxiety, Anx, is modeled as

$$\text{Anx} = 1 - \text{Max}[a_1, a_2] + 0.5\text{Min}[a_1, a_2]$$

where, Anx is the measure of anxiety when choosing one from two alternatives (1 and 2), and a_1 and a_2 are the strength of support for alternatives 1 and 2, respectively. The derivation of this equation is found in Kikuchi and van Zuylen (2003).

Using this expression, we compute traveler's anxiety at time t; with regard to ''leave'' or ''not leave''. In this equation Anx is maximum (=1) when no support is available for either alternative (when $a_1 = a_2 = 0$), and it is minimum (=0) when support for one of the alternative is 1 and the other is 0, i.e., ($a_1 = 1$, $a_2 = 0$) or ($a_1 = 0$, $a_2 = 1$).

For the optimistic traveler, a_1 and a_2 are Poss(on-time arrival | leave time t) for ''not to leave now (Alternative 1)'' and Nec(late arrival | leave time t) for ''to leave now (Alternative 2)'', respectively. Thus, introducing these into the expression above (also refer to Fig. 6), one finds the measure of anxiety associated with departure time t, Anx(t).

$$\begin{aligned}\text{Anx}(t) = 1 &- \text{Max}[\text{Poss(on-time arrival)}, \text{Nec(late arrival)}] \\ &+ 0.5\text{Min}[\text{Poss(on-time arrival)}, \text{Nec(late arrival)}]\end{aligned}$$

Table 2
Decision criteria for optimistic and pessimistic drivers (see Example 2)

Type of person	Decision to leave now	Decision not to leave now
Optimistic person	Nec(late arrival | leave now)	Poss(on-time arrival | leave now)
Conservative person	Poss(late arrival | leave now)	Nec(on-time arrival | leave now)

That is,

$$\begin{aligned}\text{Anx}(t) &= 1 - \text{Max}[P(A), N(L)] + 0.5\text{Min}[P(A), N(L)] = 1 - \text{Max}[\beta, (1-\beta)] + 0.5\text{Min}[\beta, (1-\beta)]\\ &= 1.5(1-\beta), \text{ when } \beta > 0.5; \quad \text{and} \quad 1.5\beta, \text{ when } \beta < 0.5\end{aligned}$$

For the conservative traveler, a_1 and a_2 are Nec(on-time arrival | leave time t) for "not to leave now" (Alternative 1) and Poss(late arrival | leave time t) for "to leave now" (Alternative 2), respectively.

$$\begin{aligned}\text{Anx}(t) = 1 &- \text{Max}[\text{Poss(late arrival)}, \text{Nec(on − time arrival)}]\\ &+ 0.5\text{Min}[\text{Poss(late arrival)}, \text{Nec(on − time arrival)}]\end{aligned}$$

That is,

$$\begin{aligned}\text{Anx}(t) &= 1 - \text{Max}[P(L), N(A)] + 0.5\text{Min}[P(L), N(A)] = 1 - \text{Max}[\alpha, (1-\alpha)] + 0.5\text{Min}[\alpha, (1-\alpha)]\\ &= 1.5(1-\alpha), \text{ when } \alpha > 0.5; \quad \text{and} \quad 1.5\alpha, \text{ when } \alpha < 0.5.\end{aligned}$$

Anx(t) becomes maximum at time t^* when the value of α and β are near 0.5. At this time, the support for both alternatives, "leave now" and "not leave now" are equal. This condition occurs at different times for the optimistic and pessimistic traveler.

Fig. 7 shows an example of the values of Poss(·) and Nec(·) associated with each potential departure time (t), and the bottom graph shows how the corresponding anxiety develops and subsides along the time axis for

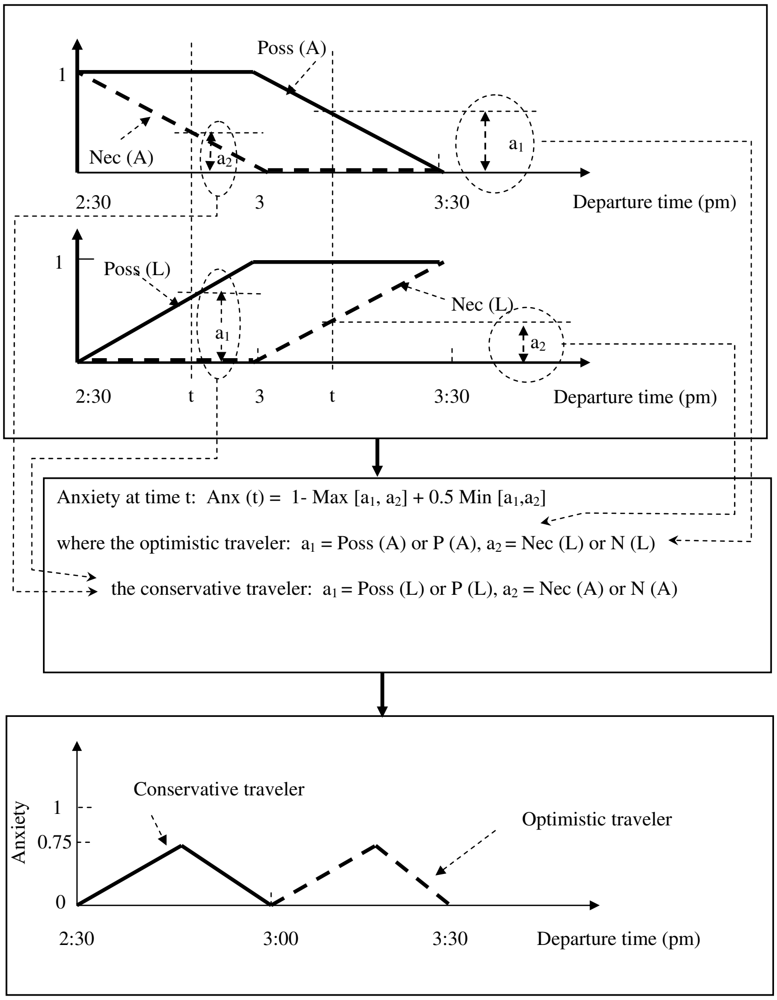


Fig. 7. Illustration of calculation of anxiety measure.

the two types of travelers. The figure has been developed for a symmetric possibility distribution whose base ($1 - e$ in Fig. 6) is 60 min. It is seen that, as expected, the conservative traveler faces anxiety before the optimistic traveler. A detailed discussion of this analysis is found in Kikuchi and van Zuylen (2003) and Yager and Kikuchi (2004).

The fact that people depart at different times in the real world (even under the same information for the same travel circumstances) can be explained by the difference in attitude. Most persons' attitude should fall between two extremes, optimistic and conservative, and possibly explains the range of departure time. An analysis similar to this discussion is found in Kikuchi et al. (1993), in which the issue of the driver's anxiety either to stop or go in the face of appearance of yellow signal phase is modeled.

6.5. Example 3: Effects of improved information

The analysis shown above can be extended to examine the effects of the quality of information on traveler's decision. It is expected that the more precise the information on travel time becomes, the narrower becomes the range of time of anxiety. This can be understood intuitively, as well as by examining the values of Poss(on-time arrival) and Poss(late arrival) and Nec(on-time arrival) and Nec(late arrive) when the base of the possibility distribution of the estimated travel time (i.e., $1 - e$ in Fig. 6) is narrowed. Eventually, when the information becomes very precise, to a single value, then possibility and necessity of "on-time arrival" or "late arrival" becomes either 0 or 1 (this result is consistent with Eq. (8)); thus, anxiety vanishes. Further, the decision pattern of optimistic and conservative person becomes the same. This process is shown in Fig. 8. This is what is expected; when there is absolute certainty about the outcome, there is no scope for differences in perceptions (as what exists between an optimistic and conservative view). Actually, this is close to the situation of traveling by a reliable public transportation (e.g., rail transit), for which the traveler has no control over the travel time.

It must be noted that how the desired arrival time is perceived is also an important factor. As more precise information about the travel time is given, Poss(·) and Nec(·) of on-time arrival and late arrival begin to be controlled by the vagueness of the desired arrival time. In other words, the imprecision of the "desired" arrival time, not the information about the travel time, dictates the traveler's uncertainty about the departure time. This underscores, as also indicated earlier, the importance of examining both the information about x and also the specificity of A, when analyzing the truth of "x is A". For example, when designing the quality of information under Intelligent Transportation System, ITS, one must be cognizant that too precise information may not be useful beyond a certain point as long as the decision maker has some uncertainty about the decision objective.

6.6. Example 4: Degree of uncertainty

Consider the situation where one needs to compare an approximate number (A) with a crisp value (B), in which A may be an estimated cost and B may be the limit of available budget; say one wants to examine whether a project is feasible from the budget standpoint. Let us examine one's confidence in asserting that "the estimated cost (A) is within the budget (B)", when the estimated cost (A) is a fuzzy value and the available budget (B) is a crisp value.

Assuming a triangular distribution for A, Fig. 9(a) shows five cases of relative positions of A and B. In each case, the values of $\{\text{Poss}(A > B), \text{Nec}(A > B)\}$, and $\{\text{Poss}(A < B)$ and $\text{Nec}(A < B)\}$ are presented. It is seen (see Fig. 9(b)) that Case (3) corresponds to maximum uncertainty. This is because $\text{Poss}(A > B) = 1$ and $\text{Nec}(A > B) = 0$, and also $\text{Poss}(A < B) = 1$ and $\text{Nec}(A < B) = 0$. (This is the situation in which optimistic and conservative views are maximally separated, see Eq. (8).) These conditions correspond to, as explained earlier, the situation in which Confidence, $\text{Con}(x \text{ is } A) = 0$.

It is interesting to consider the probabilistic counterpart of the above analysis. If A was represented by a probability distribution, the situation similar to Case 3 in Fig. 9(a), would imply that the truth that A is greater than B is expressed by $\text{Prob}(A > B)$, which here, is equal to 0.5. This would simply mean that, at the current location of B, it is equally *likely* that A is greater than B and A is less than B. This situation cannot be interpreted any further in probability theory; it will remain to be just a case of equally likely events as opposed to a situation in which the analyst is most uncertain about his or her assertion.

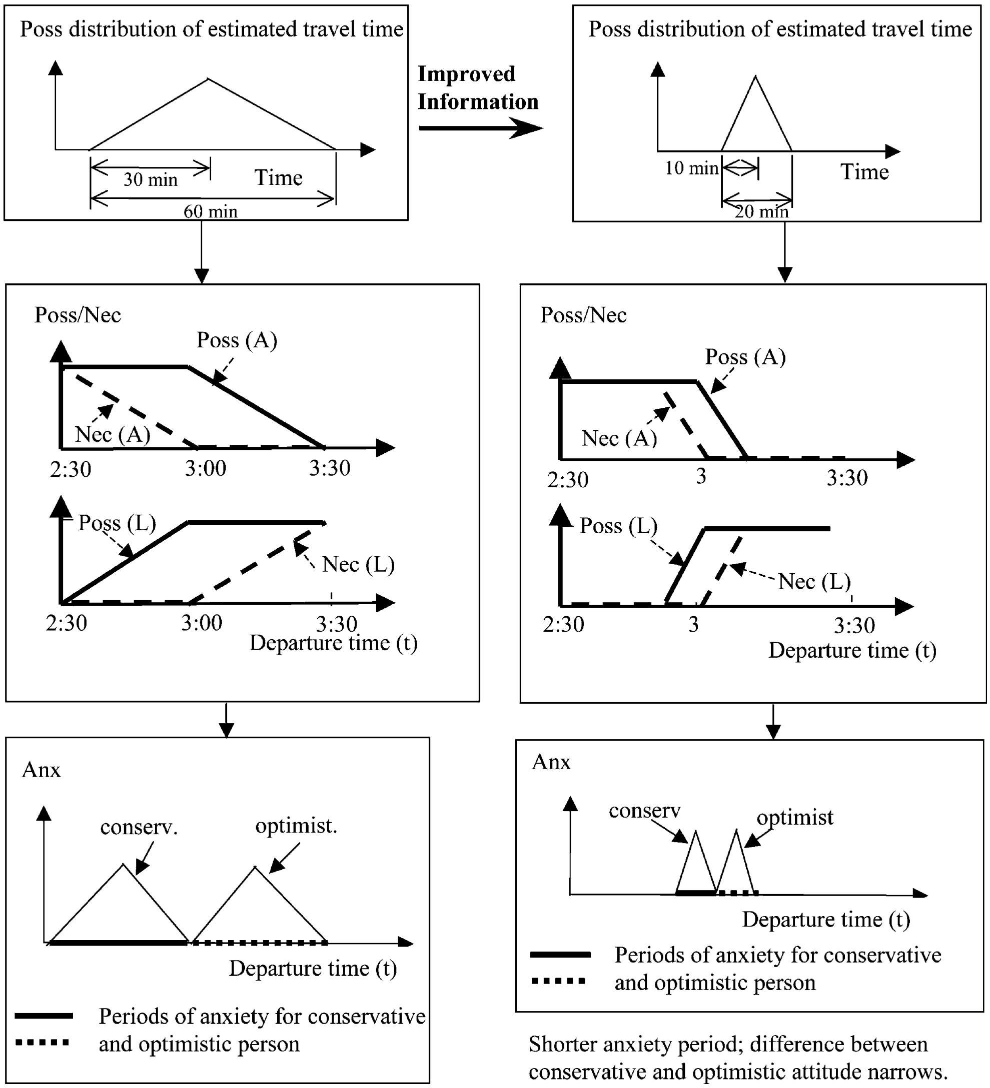


Fig. 8. Effect of improved information on anxiety.

6.7. Development of possibility distribution

One may ask how to develop the shape of a possibility distribution. Since possibility distribution is a formalization of perceived information under a large degree of uncertainty, the axiomatic requirements for the shape are not rigorous, except for the convexity and normality requirements. Convexity implies that the possibility of a point, say x, between two separate points, say b and c, should be between the possibilities of b and c, in other words, $\text{Min}(\pi(b), \pi(c)) \leqslant \pi(x) \leqslant \text{Max}(\pi(b), \pi(c))$. Normality suggests that the maximum value of the possibility distribution must be one. Additivity is not required. This makes it easier to draw the shape of a possibility distribution (compared with the case of the probability distribution), basically by following one's judgment based on the available knowledge and information, and the context of the problem.

The possibility distribution can be viewed as a representation of a set of opinions that are in general agreement; for example, an "ideal" target (which everyone generally agrees) exists, and each opinion represents the acceptable (or tolerable) level of closeness to the target, with the difference among the opinions being the boundaries of acceptability. The shape can be constructed through the consensus of a group of people, or it can also be constructed from the analyst's subjective judgment.

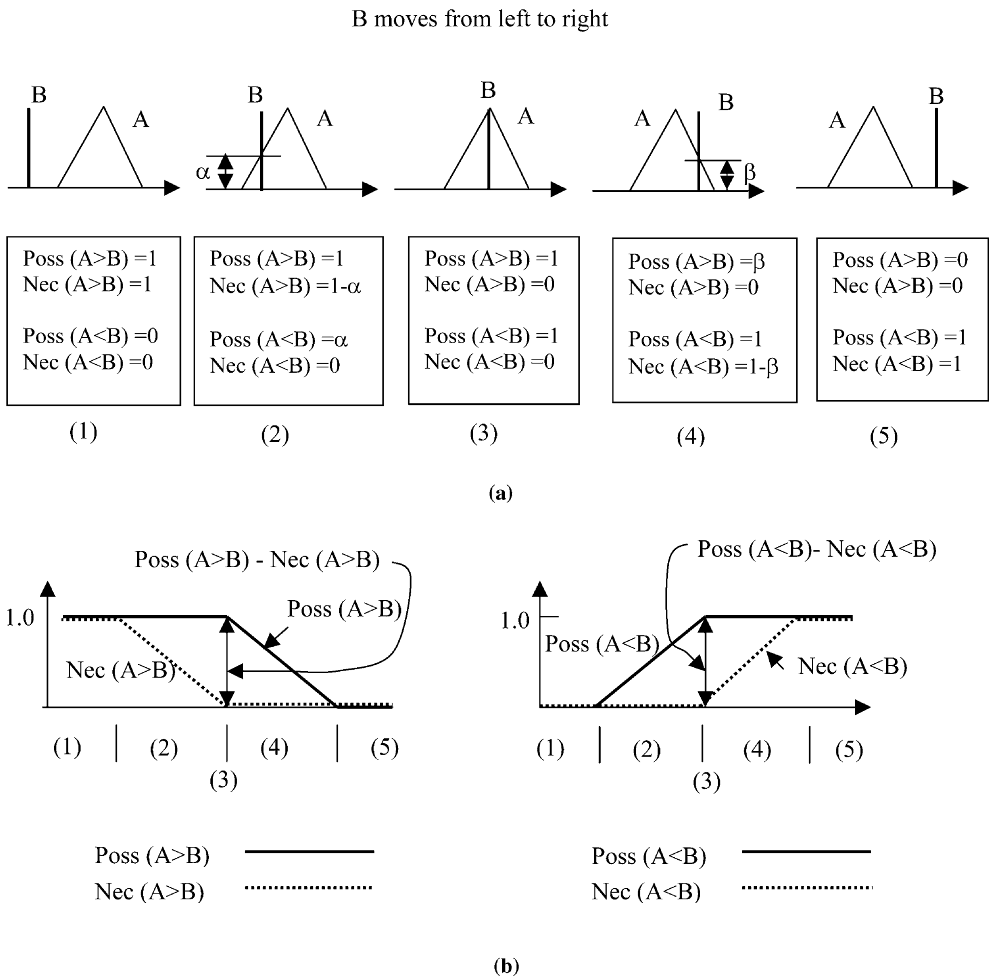


Fig. 9. Different degree of knowledge—total certainty to total uncertainty.

One approach to develop a possibility distribution is a graphical expression of the opinions. For example, a group of subjects are asked about the "acceptable" delay at an intersection. Each subject draws a line representing his/her feeling of acceptability on a x–y graph, y-axis being the acceptability level between 0 and 1, and x-axis being the delay time. Then, as shown in Fig. 10(a), the line will start from $y = 1$ at $x = 0$, and eventually end at the point where the time is totally unacceptable ($y = 0$ at this point). Many similar lines by different subjects are drawn, and by combining them (either taking the maximum, the minimum, or the middle) a representative possibility distribution can be obtained (as seen in Fig. 10(b)).

Another approach (for the same example) is to ask each to indicate the limit of the acceptable delay, and the analyst draws a horizontal line corresponding to the range of acceptable delay, and stacks all the responses as shown in Fig. 10(c). Since all subjects agree that zero delay is the "ideal" state and acceptable to all, the height of the stacked lines is 1 at $x = 0$. Joining the rightmost ends of the lines (as seen in Fig. 10(d)), the analyst obtains the possibility distribution. This is also consistent with the definition of possibility distribution in Fig. 1.

When no information other than a single value is available, for example the case of a forecast travel volume in Example 1, the analyst may choose a triangular or trapezoidal shaped distribution. The most possible value is obviously the available single value and the upper (like 1 in Fig. 6) and lower (like e in Fig. 6) values of the range are subjectively determined. When one provides a single value as *an approximate value*, then one is expected to know a vague range associated with the approximate value. Or, the analyst may want to make a rectangular shaped function in which all values in the range are equally possible with the possibility one.

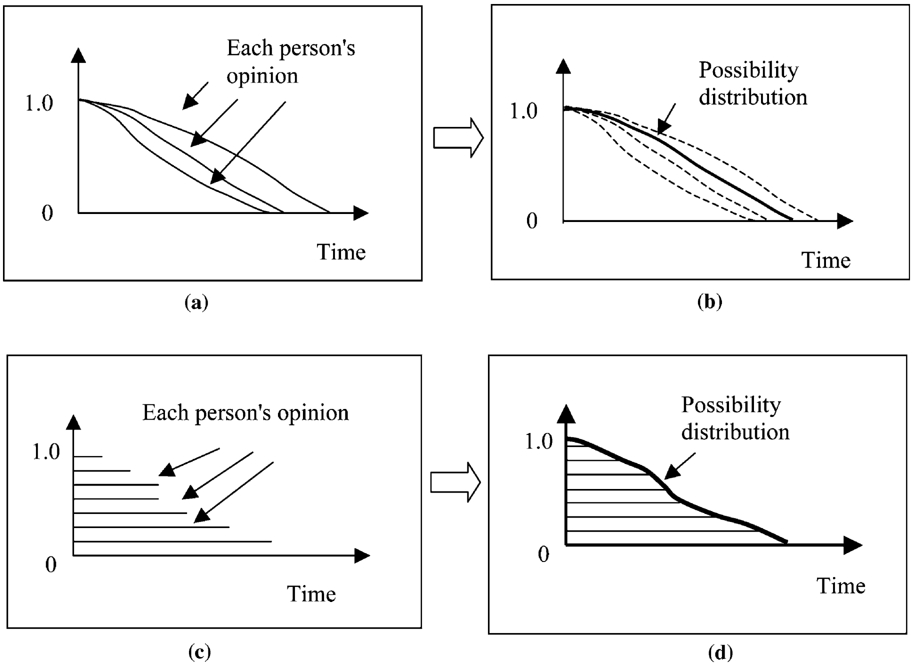


Fig. 10. Examples of constructing possibility distribution.

Zadeh (1978) states that the membership function of fuzzy set induces the possibility distribution and numerically the values of the membership function of a fuzzy set and the corresponding possibility distribution are the same. Hence, it is also possible to develop a possibility distribution by the same approaches as used for developing a membership function of a fuzzy set (see Li and Yen, 1995).

7. Probability vs. possibility

In summary, the distinction between the two theories (possibility and probability) is rooted in the type of information they handle, and how it is formalized in a functional form, the distribution. The probability distribution represents a much more specific (rigid) information than the possibility distribution. It is characterized by the concept of propensity, or actual occurrence of events. The additivity property of the probability distribution clearly suggests consistency in the evidential support.

The possibility distribution, on the other hand, is founded on the concept of disposition, which implies "judgment" in the feeling of "possibility", "achievability", "acceptability", and "capacity of the events to occur". The possibility distribution covers a set of "possible ranges", less precise information than the probability distribution. Hence, it is natural that how to express ignorance and uncertainty is an important part of the possibility theory framework.

The possibility and necessity measures of possibility theory constitute the upper and lower bounds of probability measure. Conceptually, this is because only the possible events can be probable (Smets, 1998). With a better quality of information, the difference between possibility and necessity measures narrows and each converges to the probability measure as shown earlier.

The value of probability is interpreted as propensity of occurrence of an event in an objective sense; and, hence, it clearly has application to engineering design and risk, such as the allowable probability of queue overflow at a queuing facility (like the length of a turn-lane at an intersection). The value of possibility and necessity, on the other hand, is associated with the sense of force or momentum to *support* a particular decision alternative. Its uses are suited to comparing (ordering) two situations, or understanding the degree of uncertainty or degree of support for an alternative. Hence, applications of possibility theory are found in the problems of ranking, ordering, and evaluation; such as "this is more possible than that".

8. Conclusion

In the analysis of transportation planning and engineering, uncertainty resides in many places in different forms. The integrity of analysis, hence, is largely subject to how well uncertainty is addressed and treated. What is important is how to represent uncertainty in a manner consistent with the information in the context of analysis. This paper introduces the framework of possibility theory, and characterizes its place of application in transportation. The paper also compares the possibility framework with that of probability theory, which has been the traditional approach.

The contribution of this paper is twofold. One, the frameworks of possibility and probability theories (which allow us to account for two basically different types of information—perceptive and statistical information) are presented side by side so that transportation researchers will be able to apply them appropriately. Two, the paper illustrates how the use of possibility theory allows analysis that is not available in the framework of probability theory, such as the expression of ignorance, measure of anxiety, incorporation of the decision maker's attitudes, and comparison and ranking of approximate values.

Information has become the central player in the arena of transportation research. The roles of information and its effects on decision-making by the travelers and also that by the supplier of transportation services have become the major research agenda, as seen in the study of ITS. Hence, understanding the types of information and proper mathematical tools for their treatment are very important, and that will lead to a better design of ITS and overall transportation systems.

Our future study will take two directions: one, the development of a comprehensive framework of uncertainty treatment and identification of issues relevant to transportation. Of particular interest is Demspter–Shafer theory, which was briefly discussed as Type 3 information in this paper. Type 3 information is characterized by the combination of probabilistic (Type 1) and possibilistic (Type 2) information. Such framework is particularly suited for diagnostic analysis such as accident investigation or cause-finding, where both types of information exist and need to be examined. Another direction of the future study is to explore application of tools developed under possibility theory. Among them are (1) multi-objective optimization under possibilistic information on the objectives and constraints, and (2) multi-criteria evaluation in which the weights are not completely additive due to interaction among the attributes.

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