**System Characterization (L - 4)**

RWPAs pointed out earlier, real world associated may be very complex and so modelling incorporating all the features of the real world may in itself become very complicated and unmanageable. This requires a simplification of the real world problem (RWP). It may be noted that not all the features of the RWP may be important from the point of view of the goal of the study. Also one may have to make some idealizations/ assumptions to obtain a real world model (RWM). The partial description of RWP w.r.t. the goal of the study gives us a ‘system’ and most of the time it is sufficient to study problem through system. This process of simplification and idealization is called ‘System Characterization’. It is an important step in modelling.

1. **Objects**: Objects are physical entities of the real world with specific characteristics.
2. **System**: It is a collection of one or more related objects of the RW. They have some interaction with each other. In general, it is a subset of a bigger set which may have more interacting/ non interacting objects. The system consists of those objects which the modeller finds to play an important role in the study of the problem w.r.t. the goal of the problem. Thus ‘system’ may differ from one modeller to the other.
3. **Closed System**: In case the objects of a system do not interact with the objects outside the system, we say it a ‘closed system’
4. **Open System**: Sometimes while simplifying the system, we leave out certain objects from the system but they still interact with the system objects. Such systems are called ‘open system’ and the objects lying outside the system but interacting with it provide an ‘Environment’ to the system.

**Why Environment?** - It is just a matter of convenience to keep the system manageable. Also, note that closed system has no Environment. It is only the open systems, which have Environment. Of course, one can simply move the objects in the environment to the system itself, thus making it a closed system. However, this may make the system more complex. Thus, we see that the boundary of a system is flexible. When to keep an object in the system or in environment depends on the intuition of the modeller.
the modeller. It is a subjective decision. One rule could be to keep only the essential objects in the system and others can go to environment. It is to simplify the RWM. The objects in the system and those in environment interact through the variables common to both.

5. **Parameters and Variables of the system**: The attributes or characteristics of the objects are described in terms of variables and parameters. Parameters are those attributes which are intrinsic to the objects. The variables are those characteristics which describe interactions between objects.

6. **Relations**: Once we have a system, we should try to define the relationship (interactions) between the objects.

Some examples:

1. Weight lifting Problem: One can consider various objects, e.g. weight lifters, weight to be lifted, location of competition, climate etc. However, it is the first two which are important and others hardly influence the outcome (i.e. the weight lifted). Thus one can consider it to be closed system with objects as weight lifters and weight to be lifted.
   In this case the parameters are (i) body weight of the lifter, (ii) weights to be lifted; and the variable is ‘weight lifted’ by the lifters.

2. Supermarket Problem of Checkout counters: Here the system consists of (i) customers, (ii) items for sale, and (iii) counters. Anything else? If not, then system is a closed system.
   However, if there is a supermarket in the neighbourhood, then it may influence the customers by providing better service. In that case we can consider the system to be open with an environment (other supermarket).
   The variables of interest is the queue length and the items purchased by a customer, what else? Parameters – price of various items, number of counters, ...

3. Fall of a rain drop: The system consists of rain drop, cloud and ground. We can take it to be open system with air in the environment.
   The variables here are time, distance travelled, velocity, force etc. and the parameters distance between cloud and ground, etc.

4. Amul Factory: The system consists of milk procured and the various items, like chee, processed milk, Ghee, butter, yoghurt. What are the variables and parameters of the problem?

5. Demand of soft drink: If we predict the future demand based on the previous sales only, then it is a closed system. However demand may also depend on change in population, advertisement, and climate. Then these can be considered as objects in the environment and system as an open system. The relationship between ‘Sales’ ‘Profit’ and ‘Advertisement’ can be defined as follows:

```
Advertiseent  ----->  Sales
               Profits
```

So, it can noted that the same system can be closed or open depending upon the modeller.
**Degree of Detail:** A system is a simplified version of RW, question arises how much simplification? For example, look at the examples 2 and 5, we have considered two models in each case. The second model is more detailed in each case. In the case of World Population problem one can simply consider the whole population as one object or one can consider them three categories – Young and juvenile, Mature and Child bearing and Old. This relates to degree of detail in system characterization. If we simplify it too much some important features may be lost and the results may either may not be adequate or of limited use (e.g. rain drop model – the first model was too simple and the second one did not consider the air drag hence we got results which were not useful). On the other hand if we include everything, the model may become unmanageable and again the results may be of limited use. Hence one has to make a sensible compromise. This is again a very subjective decision and corresponds to art aspect of modelling.

Sometimes, we do not have much information about the real world or we want to obtain some information about it, then it is helpful to describe the system in terms of one single variable through which it interacts with its environment. Such an approach is called “Black Box” approach. Here we simply ignore the inner structure of the RWM. We use input to get an output, e.g. in Soft Drink Problem if we just work with the ‘Sale records of previous months’ to estimate the demand in the next year, then it is a “Black Box” approach.

On the other hand if we consider the system in detail, i.e. by considering all possible objects and the relationships in detail, we get a White Box. This may be very close to the RW, but may become unmanageable and so of limited use.

In general one should enough and relevant details so as to get a tractable model. It always does not help to include all the details of the RW in the model (Recall Weight Lifting Problem!) but one should also not leave significant details of the RW. One often starts with a simple model (can even be a black box model) and then improve it by adding more complications or one can start with a white box and then keep simplifying it till a tractable model is obtained.

**Static or Dynamic Model:** In case time does not play any role in the various interactions and all the variables are independent of time, we say model is a *Static model*. However when one or more variables dependent on time we say it is a *Dynamic Model*. For example, weight lifting system provides a Static Characterization while Rain drop or Rocket launch problem distance, velocity etc. depend on time, so they are dynamic models. How about the ‘Soft drink demand’ and ‘Alloy Selection’ Problems’?

In case of Dynamic Model we should see if the dependence on time is continuous or is it discrete (note that the data is available only at discrete time intervals). If the variables are changing continuously with time then we have to describe the system for all times, so making it a detailed description. If it is chosen discrete, then the system has to be defined only at some discrete times only. In that case the question is ‘what should be time of discretization’? If time of discretization is small model may become detailed or if it is very large then we may get only average outputs only as fluctuations with time be lost – this will lead to a coarse model.
Time scales: It indicates the duration of significant variations in a variable. For example if we consider populations of tiger and rabbits, then the population of tigers change very slowly as compared to that in rabbit population.

If the duration of study is $T$ and the time scale of a variable is $T^*$ such that $T^* \gg T$ then this implies that there is hardly any change in the variable during the duration of the study and hence that parameter can be taken as ‘static’ – thus reducing the number of dynamic variables. Similarly we can compare time scales of two variables. This will be a process of simplification.

If $x(t)$ and $y(t)$ are two dynamical variables such that changes in $x$ cause changes in cause changes in $y$ over time then only past values of $x$ can influence the present and future of $y$ not the reverse, i.e., $y(t)$ is influenced by $x(\tau)$ for $\tau < t$ and not for $\tau > t$. This is called the principle of causality.

**Deterministic and Stochastic characterization:** Depending on the uncertainty in the system we characterize it as deterministic/stochastic. As such the real world is always unpredictable – however depending on the degree of uncertainty, we can make characterizations. If degree of uncertainty is small (as in the case of weight lifting problem! Or Rocket Launch Problem) we say the system is Deterministic otherwise it is Stochastic. We shall see some examples in the next lecture.

### Some System Characterizations (L-5)

Now we shall consider some case studies and give a systems characterization for them.

1. **Weight Lifting Problem:** Here the goal of the study is to find a handicapping rule for a weight lifting competition, where no individual categories and the body weights of the lifters vary significantly.
   
   As we have seen earlier, the weight lifted by an individual mostly depends on the body weight of the lifter and other factors like location of the completion, climate conditions, and spectators do not significantly affect the performances. Thus, we can consider the system to consist of two objects – weight to be lifted and the competitors, with no environment (why!). So the system is closed. Here time does not play any role on the performance, thus it is a static system. Further, the performance a lifter can be predicted with a fair degree, so it is taken as deterministic system.

2. **Alloy Selection:** The Company is interested to manufacture an alloy with the least thermal coefficient. For this, ores can be obtained from 4 mines and there two different processes available to produce the alloy. Company has to choose a proper combination of the method and mine to get alloy of desired characteristic.
   
   Note that if the samples from each combination do not differ from each other, then any combination can be selected, or if there is a significant difference in various combinations, again the selection is easy (why?). The problem is that various samples
for each combination show variations – then what to do? This could be due the
geographical conditions at the mine or due to uncontrollable factors in the process.
Here the system consists of objects – ‘Method’, ‘Mine’ (from where ore is taken) and
‘Alloy’. It is stochastic as randomness is involved due to unknown factors. The system
is static. Is it open or closed?
What are the variables of the system? Also give the relationship between different
objects.

3. **Rocket Launch Problem**: The problem is to find the thrust so as to launch the rocket
in an orbit at a given height. The motion of the rocket is influenced by the thrust
generated and the forces acting on it.
The system consists of Rocket and earth. The effect of other celestial bodies on the
motion is neglected. Air (which resists the motion) is taken to be in environment. So
it is Open system. The variables are - distance travelled by Rocket, its velocity and
acceleration, forces acting on it. The parameters are: mass of the rocket, gravitational
constant etc. It is dynamic as the velocity and forces vary with time. It is deterministic
as the motion can be predicted by the laws of motion.

4. **World Population**: One can consider a simple model where the object is ‘Population’
which is changing with time. Thus it is a dynamical model, where Population at any
given time depends on the birth rate and death rate.
To make it more detailed, one can consider the system to consist of populations in
three different age groups – (i) Young and juvenile, (ii) Mature and child bearing, (iii)
Old, who do not reproduce. Here we have to give relations between different groups.
Their birth rates and death rates may also be different. Further, population can be
taken to be continuously changing with time and age. One can also incorporate
‘gestation period’ – these factors will make the model more detailed. Also, time can
be taken as continuous or discrete.

5. **Component Reliability**: Here a control element is essential for the working of a
machine. The machine stops operation as soon as the element (component) fails. The
problem is how many spare parts should be there so as to keep the machine working
for certain time period (T, say). Here the system can be just the control element with
machine in the environment. This can be taken as a Black Box approach. As the life
time of the element is random, it becomes a stochastic problem. It is dynamic as time
is involved. There are two time scales, one is of course life time of the element and
the other is time required to change the element. If this time is much smaller than the
mean life time of control element, we can take it to be zero, i.e. it can then be assumed
that ‘the element is replaced instantaneously’. Here the mechanism of failure is
ignored. If one has to account that also, e.g., if the failure is due to crack propagation,
then the model becomes more detailed.

6. **River Pollution**: Goal is to keep the pollutant concentration level below a prescribed
limit. For this we need to obtain changes in the level of pollutant concentration. Give
a system characterization here. Note the concentration may change due to (i)
diffusion, (ii) transportation, (iii) chemical reaction, if any – hence there will be three
time scales involved. Justify the statement that ‘system is dynamic and deterministic’.
7. **Demand of Soft Drink**: Demand varies with advertising, population changes, climate, price, competition, if any. These can be kept in environment and past sale records can be taken to form the system. The monthly sales record are taken as daily or weekly records will be very detailed and annual records will remove any seasonal fluctuations. So here we consider the time to be discrete and time of discretization as a month. So it is open dynamic and stochastic system – stochastic because of fluctuations due to uncertain factors (!). However, if other factors are ignored and only sale records are considered, we can have it as a closed system with demand as one variable.