α_s Measurements

- Introduction
- Scaling Violation
- Inclusive Jet Production
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- Z Lineshape
- τ Decays
- Event Shapes
- Summary

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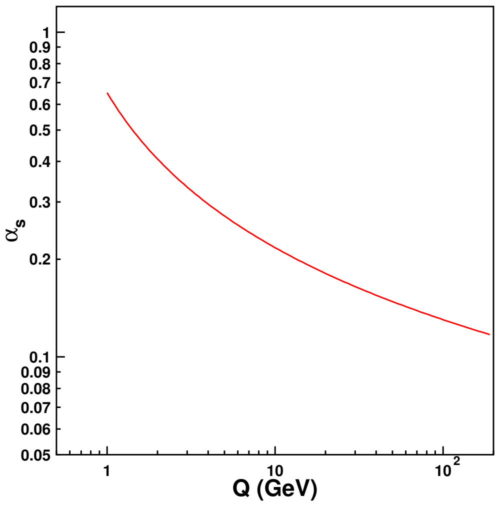
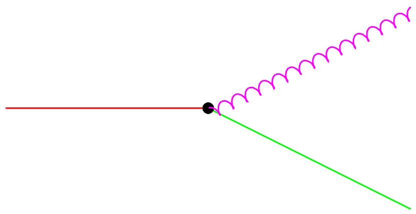
Energy dependence of α_s, the only free parameter in QCD (+ # of flavours N_F and quark masses), has a well defined prediction from theory

Confinement at large distance

Asymptotic freedom at small distances

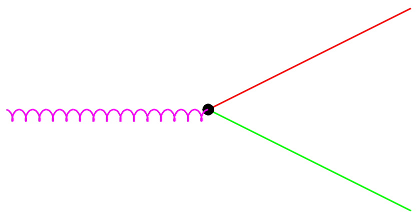


α_s appears with colour factors in all basic couplings:



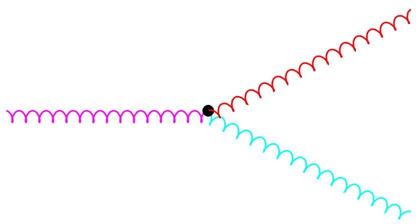
Quark Bremsstrahlung

$\sim C_F \cdot \alpha_s$ $\qquad C_F = \frac{{N_C}^2-1}{2N_C} = \frac{4}{3}$



Gluon Splitting

$\sim T_R \cdot \alpha_s$ $\qquad T_R = \frac{1}{2}$



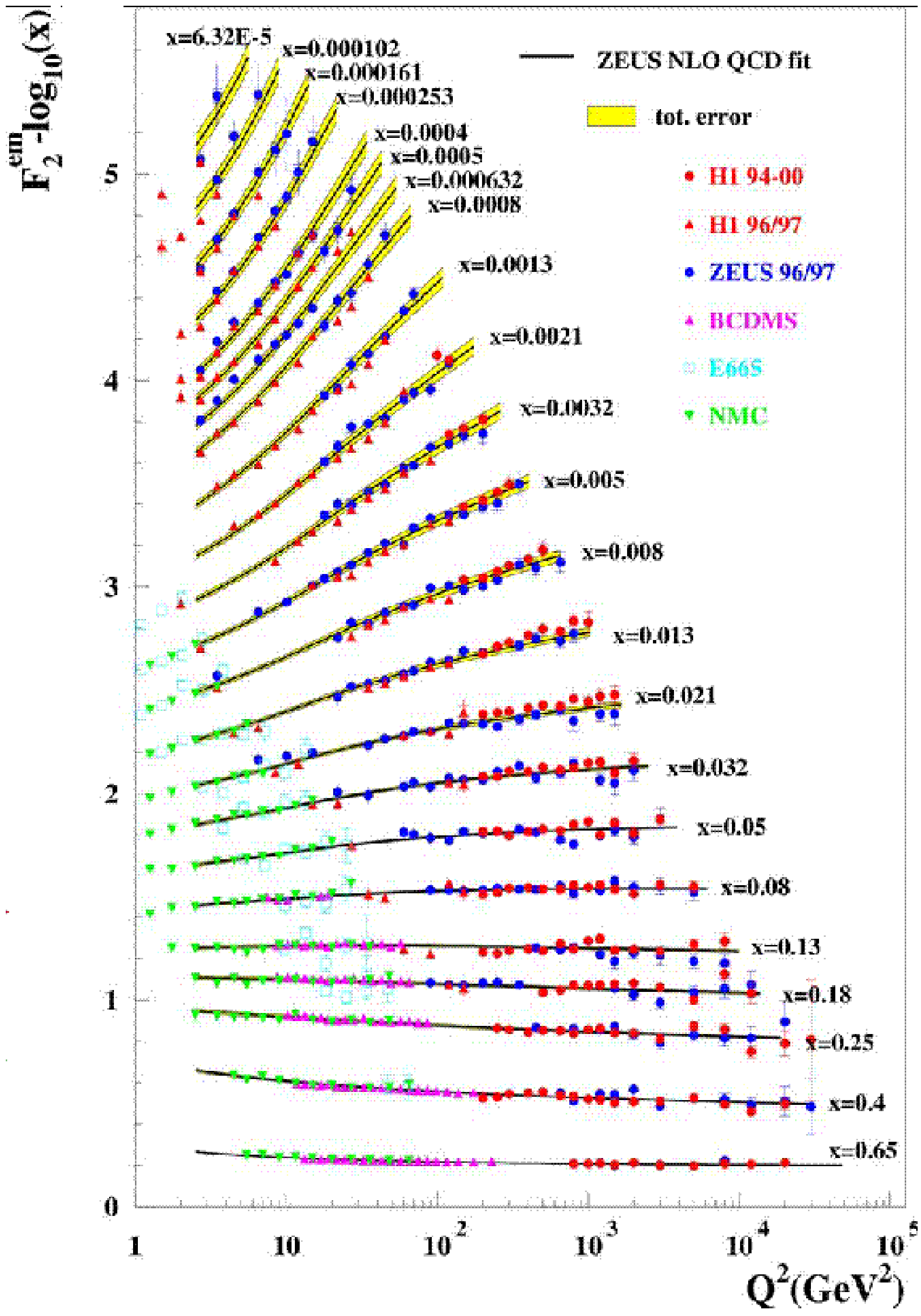
Triple Gluon Vertex

$\sim C_A \cdot \alpha_s$ $\qquad C_A = N_C = 3$

α_s is measured from a variety of pure inclusive and semi-inclusive measurements:

- Total hadronic cross section in Z decays $\mathcal{O}\left(\alpha_s^3\right)$
- Hadronic decays of τ - branching ratio, spectra $\mathcal{O}\left(\alpha_s^3\right)$
- Kinematic distributions of hadronic final states in e^+e^- interactions:
  - Jet Rates $\mathcal{O}\left(\alpha_s^2\right)$
  - All event shape variables $\mathcal{O}\left(\alpha_s^2\right)$
  - Selective event shape variables (1−T, ρ_H, $\cdots$) $\mathcal{O}\left(\alpha_s^2\right)$ + Resummed
- Scaling violation in deep-inelastic scattering and in fragmentation functions $\mathcal{O}\left(\alpha_s^2\right)$
- Jet rates in high energy hadron-hadron or lepton-hadron scattering $\mathcal{O}\left(\alpha_s^2\right)$
- Hadronic decays of heavy quarkonia $\mathcal{O}\left(\alpha_s^2\right)$

The original quantitative test of perturbative QCD is the breaking of Bjorken scaling in deep inelastic scattering

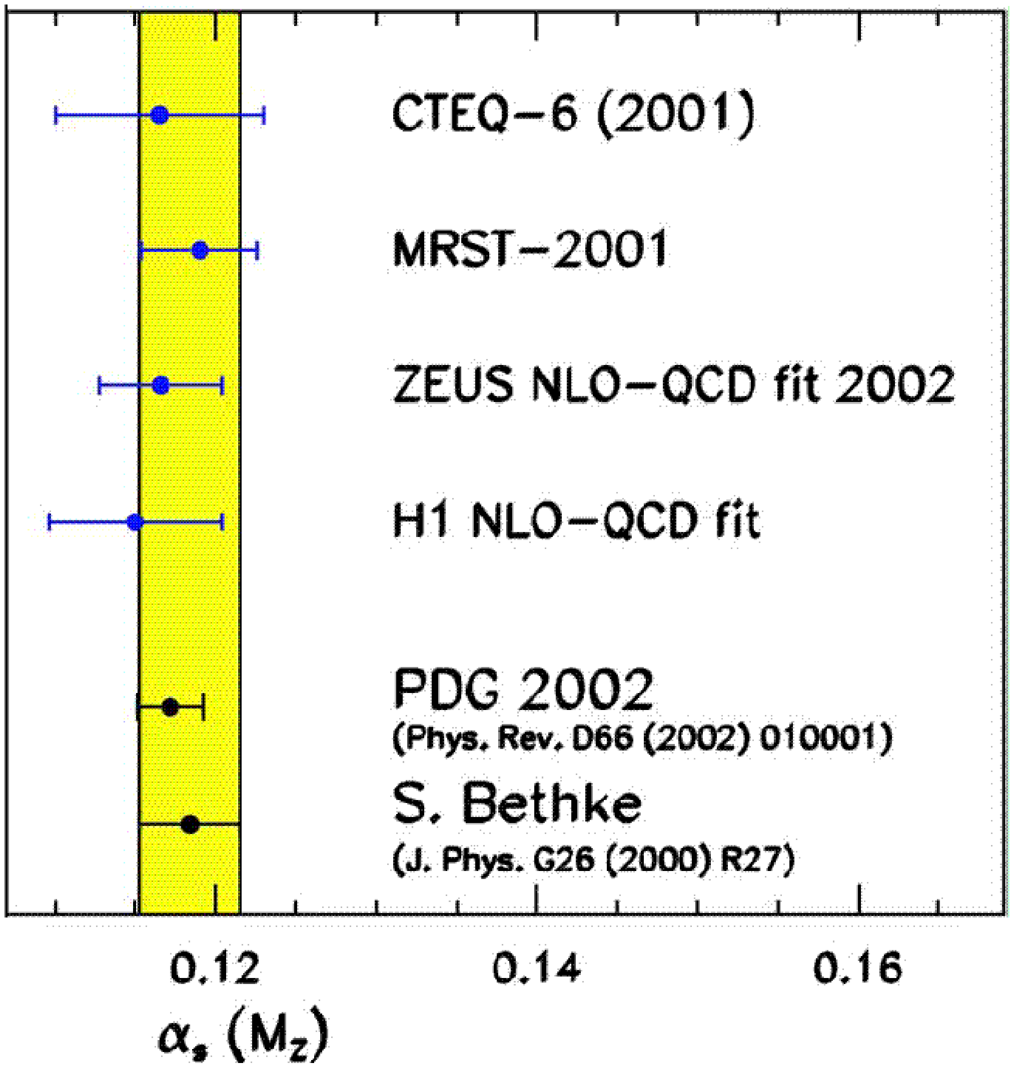


Large amount of data exist on structure function from ep/μ p/$\cdots$ experiments

$$\frac{dF_2^{em}}{d\ln Q^2} = \frac{\alpha_s}{2\pi}\{[P_{qq} \otimes F_2^{em}] + [P_{qG} \otimes xG]\}$$

- Fit to determine Parton Density Function and α_s
- Use Tevatron jet data to constrain gluon density at large x
- Treat correlated errors properly

$$\alpha_s(m_Z) = 0.119 \pm 0.002\,(\text{expt}) \pm 0.003\,(\text{theory})$$



- QCD evolution of non-singlet structure function known to $\mathcal{O}\,(\alpha_s^3)$:
  $\int_0^1 dx \left(F_3^{\bar{\nu}p}(x,Q^2) + F_3^{\nu p}(x,Q^2)\right) = 3\Big[1 - \left(\frac{\alpha_s}{\pi}\right) - 3.58\left(\frac{\alpha_s}{\pi}\right)^2 - 19.0\left(\frac{\alpha_s}{\pi}\right)^3 - \Delta HT\Big]$
  The higher twist term has been estimated and a fit to the existing ν data gives

$$\alpha_s(m_Z) = 0.118 \pm 0.011$$

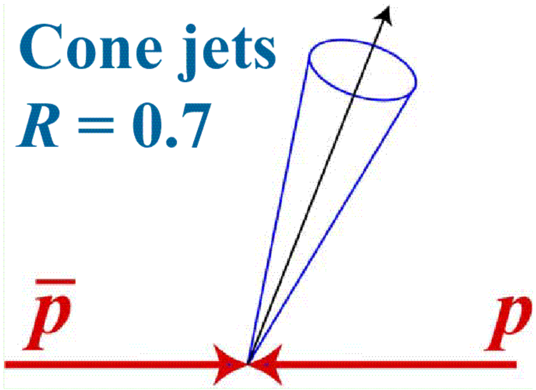
- The spin dependent structure functions measured in polarised lepton-hadron scattering are used to determine α_s:

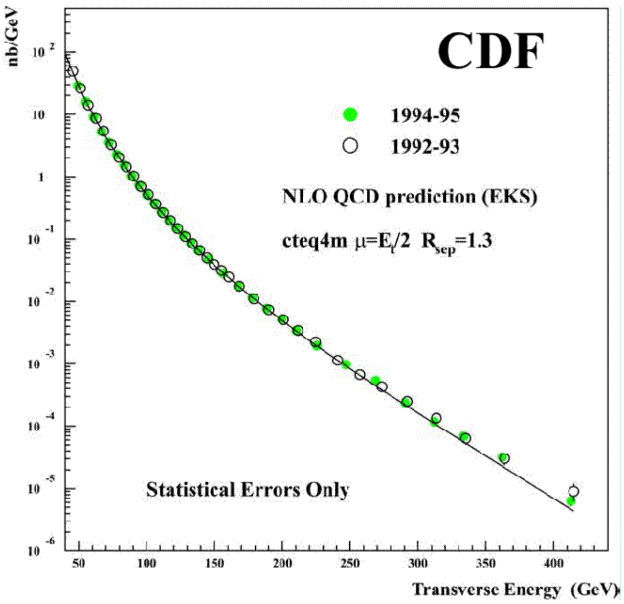
$$\alpha_s(m_Z) = 0.114\,^{+0.004}_{-0.005}\,(\mathrm{expt}) \pm 0.009\,(\mathrm{theory})$$

- Fragmentation functions $(d_i(z,E))$ measured in e^+e^- collisions for a variety of hadrons (i) with energy fraction (z) of the initial parton. Flavour tag and 3-jet analysis are used to dis-entangle quark and gluon fragmentation. Global analysis of the energy evolution measures α_s:

$$\alpha_s(m_Z) = 0.117\,^{+0.006}_{-0.007}\,(\mathrm{expt})\,^{+0.002}_{-0.003}\,(\mathrm{theory})$$

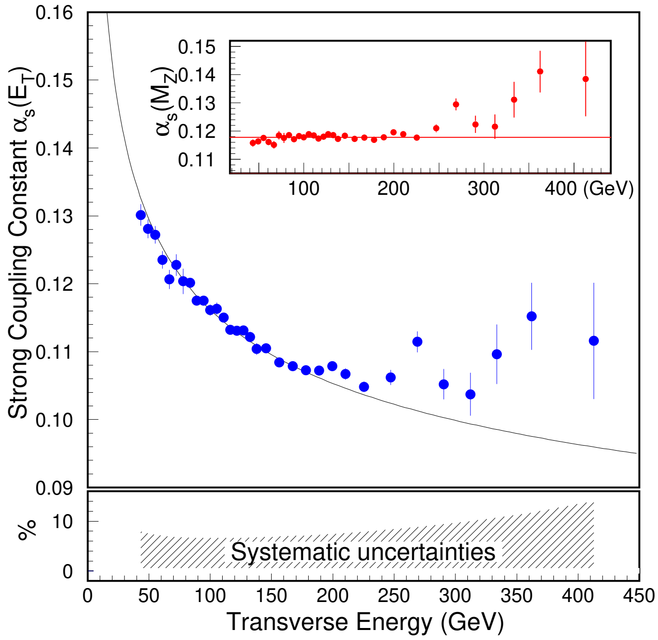
Jets are reconstructed in CDF using cone algorithm



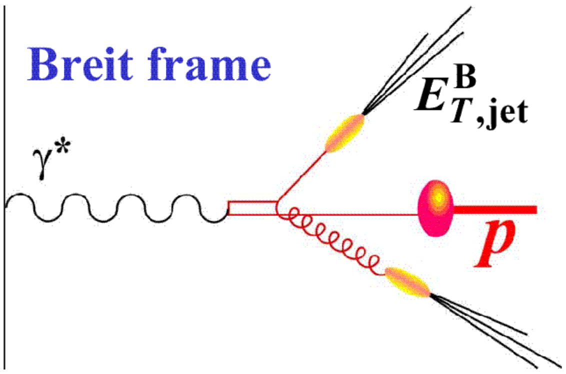


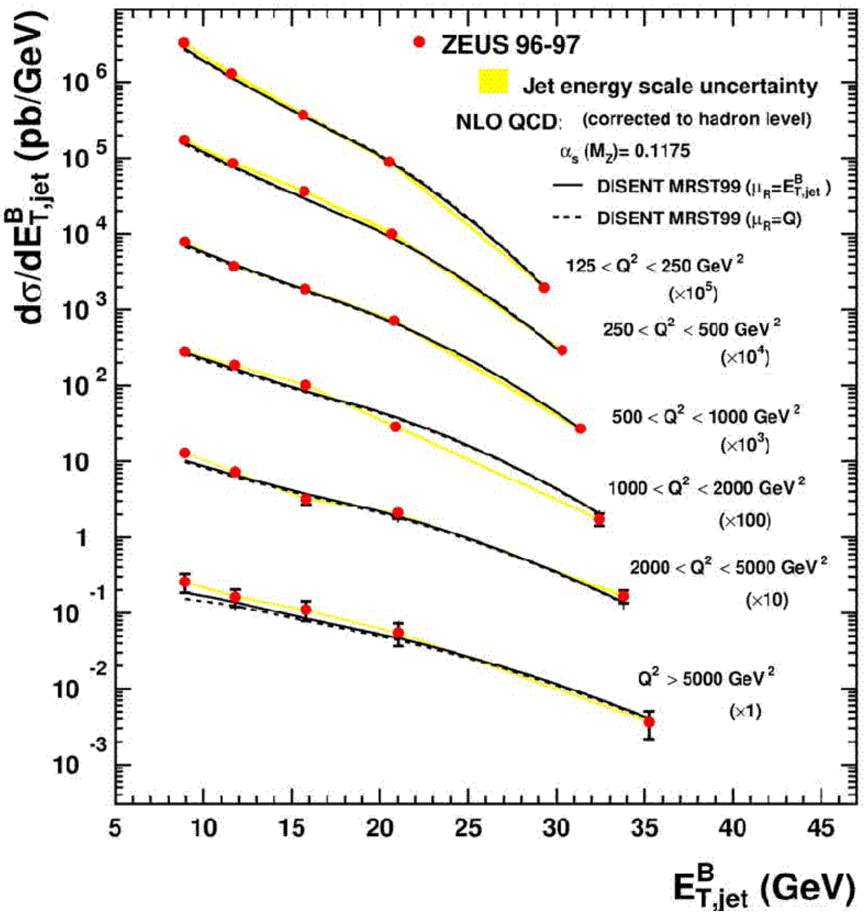
$$\frac{d\sigma}{dE_T} = \alpha_s^2 \times F_{LO} + \alpha_s^3 \times F_{NLO}$$

Evolution of α_s over a large energy range has been tested



Jets reconstructed in ep collision using $k_\perp$ algorithm in the Breit frame





NLO $(\mathcal{O}(\alpha_s^2))$ gives reasonable description of data over a wide range of Q^2 and E_T

$$\alpha_s(m_Z) = 0.1212 \pm 0.0017\,(\mathrm{stat})\,{}^{+\,0.0023}_{-\,0.0031}\,(\mathrm{syst})\,{}^{+\,0.0028}_{-\,0.0027}\,(\mathrm{theory})$$

Assume hadronic and leptonic decay widths of $Q\bar{Q}$ resonances factorise into a perturbative and a non-perturbative part:

- Measure the ratio of partial widths to hadrons and to muon pair

$$R_\mu(\Upsilon) = \frac{\Gamma(\Upsilon \to \mathrm{hadrons})}{\Gamma(\Upsilon \to \mu^+\mu^-)}$$

  Correct for relativistic nature of $Q\bar{Q}$ system and for non-perturbative correction due to colour octet contribution

- Measure the ratio of partial widths:

$$R_\gamma(\Upsilon) = \frac{\Gamma(\Upsilon \to \gamma gg)}{\Gamma(\Upsilon \to ggg)}$$

  Use higher order calculation for photon energy distribution

- Measure the moments of the ratio of cross section:

$$M_n = \int_0^\infty ds \frac{R_b(s)}{s^{n+1}}$$

$$R_b(s) = \frac{\sigma(e^+e^- \to b\bar{b})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

Use all measurements done at Υ

$$\alpha_s(m_Z) = 0.109 \pm 0.004$$

Define

$$R_Z = \frac{\Gamma(Z \to q\bar{q})}{\Gamma(Z \to e^+e^-)} = \frac{\Gamma_q}{\Gamma_\ell}$$

- Determine at LEP and SLC from total cross section measurements
- Compare with theory
  - Electroweak theory in improved Born approximation
  - Assume QCD correction to factorise out

$$R_Z = {R_Z}^0 \cdot (1 + \delta_{QCD})$$

  $\delta_{QCD} \simeq 4\%$ and is used to measure α_s

- No assumption on the hadronisation mechanism
- Ratio helps partial cancellation of Electroweak radiative corrections

$\Rightarrow$ α_s measurement with small theoretical uncertainty

- Measure cross section as a function of centre of mass energy at $\sqrt{\mathrm{s}} \approx \mathrm{m_Z}$
  $\Rightarrow$ peak cross section $\sigma(\mathrm{e^+e^-} \to \mathrm{f\bar{f}}) \sim \Gamma_\mathrm{e}\Gamma_\mathrm{f}$

- Fit simultaneously cross sections for the processes
  $\mathrm{e^+e^-} \to$ hadrons and $\mathrm{e^+e^-} \to \ell^+\ell^-$
  $\Rightarrow \mathrm{R_Z} = \frac{\Gamma_\mathrm{q}}{\Gamma_\ell} \simeq \frac{\sigma_0^\mathrm{q}}{\sigma_0^\ell}$

- Take care of mass effect of lepton:

$$\frac{\Gamma_{ell}}{\Gamma_\mathrm{e}} = \frac{\Gamma(\mathrm{Z} \to \ell^+\ell^-)}{\Gamma(\mathrm{Z} \to \mathrm{e^+e^-})} = 1$$
$$\frac{\Gamma_\ell}{\Gamma_\mu} = \frac{\Gamma(\mathrm{Z} \to \ell^+\ell^-)}{\Gamma(\mathrm{Z} \to \mu^+\mu^-)} = 1.00001$$
$$\frac{\Gamma_\ell}{\Gamma_\tau} = \frac{\Gamma(\mathrm{Z} \to \ell^+\ell^-)}{\Gamma(\mathrm{Z} \to \tau^+\tau^-)} = 1.0022$$

If assume lepton universality but do not make mass correction

$$\mathrm{R}' = \frac{\Gamma_q}{\Gamma_\ell} = \frac{3 \cdot \Gamma_q}{\Gamma_\mathrm{e} + \Gamma_\mu + \Gamma_\tau} = 1.0007 \cdot R$$

$\approx$ 2% effect on α_s value

$$\delta_{QCD} = a_1 \left(\frac{\alpha_s}{\pi}\right) + a_2 \left(\frac{\alpha_s}{\pi}\right)^2 + a_3 \left(\frac{\alpha_s}{\pi}\right)^3 + \cdots$$

But

- QCD corrections affect ${R_Z}^0$ in Z self energy and also in $Zb\bar{b}$ vertex
- QCD corrections to the vector and the axial vector components are different

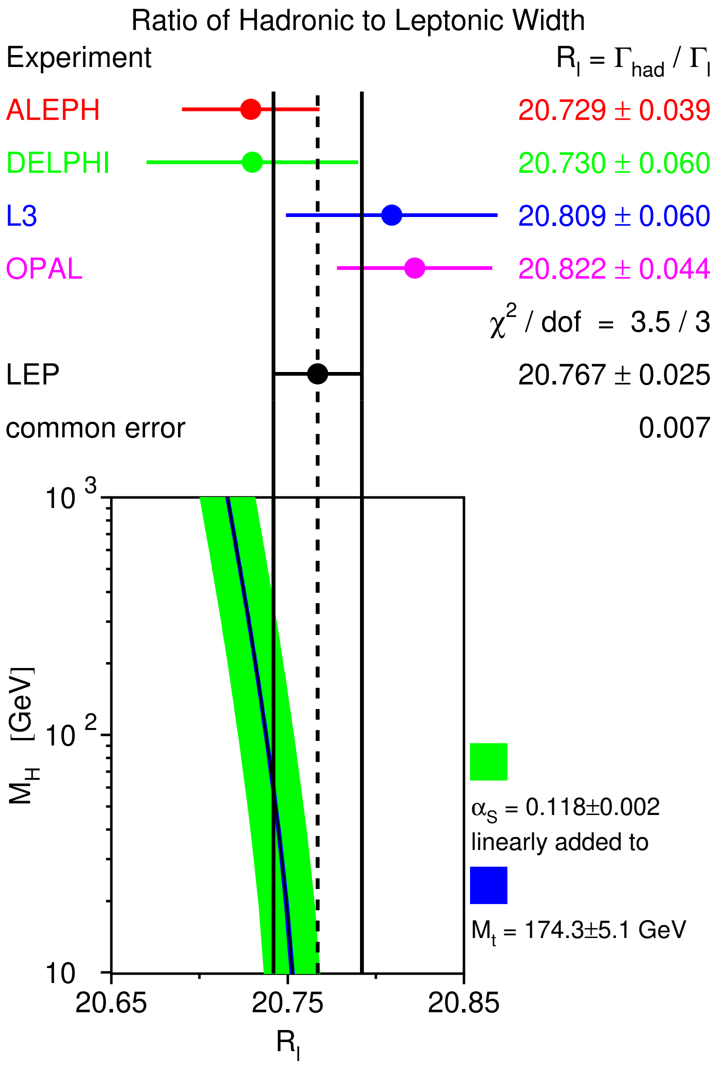
So factorisation is not exact

Some higher order calculations exist

- $\mathcal{O}\left({G_F}^2{m_t}^4\right)$ correction for ρ and $Zb\bar{b}$ vertex
- Effect of heavy top on Z self energies and $Zb\bar{b}$ vertex in $\mathcal{O}\left(\alpha_s G_F {m_t}^2\right)$
- $\mathcal{O}\left(\alpha\alpha_s\right)$ correction for Z self energy
- $\mathcal{O}\left(\alpha_s^3\right)$ correction to Z decay rates to hadrons for both vector and axial vector components
- $\mathcal{O}\left(\alpha_s^2\right)$ m_t dependent corrections
- Complete mass corrections of $\mathcal{O}\left(\alpha_s^2 \frac{{m_b}^2}{{m_Z}^2}\right)$ to the axial coupling of Z boson

Combining results from 4 LEP experiments:

$$R_Z = 20.767 \pm 0.025$$



$$\alpha_s(m_Z) = 0.1210 \pm 0.0037$$

Theoretical uncertainties include:

- Uncertainties in the electroweak corrections and their implementations

  Studied using different electroweak libraries
- Uncertainty due to light quark contribution in radiative corrections for α_{QED}
- Uncertainty due to error in b quark mass
- Uncertainty due to unknown mass corrections
- Missing higher orders in QCD calculations
- Non-perturbative corrections
- Unknown Higgs mass

$\Rightarrow$

$$\alpha_{\mathrm{s}}(\mathrm{m_Z}) \ = \ 0.121 \ \pm \ 0.004 \ (\mathrm{Exp}) \ \pm \ 0.004 \ (\mathrm{Theory})$$

LEP EW Working Group performed a Standard Model fit to all EW measurements from LEP:

$$\alpha_{\mathrm{s}}(\mathrm{m_Z}) \ = \ 0.1199 \ \pm \ 0.0030$$

Define

$$R_\tau = \frac{\Gamma(\tau \to \nu_\tau + \mathrm{hadron})}{\Gamma(\tau \to \ell\bar{\nu}_\ell \nu_{tau})} = \frac{1 - B_e - B_\mu}{B_\ell}$$

B is the branching ratio of τ

(mass of the hadronic system varies between m_π to m_τ)

- R_τ is again a pure inclusive measurement
- R_τ can be measured in e^+e^- colliders from leptonic branching ratio and lifetime measurements
- Compare with theory

$$R_\tau = 12\pi \int_0^{{m_\tau}^2} \frac{ds}{{m_\tau}^2}\left(1 - \frac{s}{{m_\tau}^2}\right)^2 \left[\left(1 + \frac{2s}{{m_\tau}^2}\right) \mathrm{Im}\Pi^{(1)}(s) + \mathrm{Im}\Pi^{(0)}(s)\right]$$

$$\mathrm{Im}\Pi^{(J)} = \text{Hadronic spectral function}$$

- Use analyticity $\Rightarrow$ contour integration in complex s-plane
- Use Operator product expansion for Π
- Use QCD sum rule

$$R_\tau = 3\left[|V_{ud}|^2 + |V_{us}|^2\right] S_{EW}\left[1 + \delta_{EW} + \delta_{QCD}\right]$$

Naive Parton model $\Rightarrow R_\tau = 3$

Overall correction $\sim$ 20%

- Measure directly $\mathrm{B_e}$ and B_μ
  - Identify τ pair events in $\mathrm{e^+e^-}$ collision
  - Tag τ decays to one charged particle and identify the charged particle as e or μ
  - Take world average of all existing measurements:

$$\mathrm{B_e} = 0.1784 \pm 0.0006; \quad \mathrm{B}_\mu = 0.1737 \pm 0.0006$$

  - Correct for mass effect:

$$\mathrm{B}_\ell = \frac{\mathrm{B_e}}{\mathrm{f_e}} = \frac{\mathrm{B}_\mu}{\mathrm{f}_\mu}$$

Phase space calculation $\Rightarrow$

$$\mathrm{f_e} \simeq 1; \quad \mathrm{f}_\mu = 0.9726.$$

$\mathrm{R}_\tau = 3.629 \pm 0.015$

- Extract B_ℓ from τ lifetime measurement

$$\mathrm{B}_\ell = \frac{\tau_\tau}{\tau_\mu}\left(\frac{\mathrm{m}_\tau}{\mathrm{m}_\mu}\right)^5$$

Use world average values for m_τ and τ_τ

$$\mathrm{m}_\tau = (1.7770 \pm 0.0003)\ \mathrm{GeV}; \quad \tau_\tau = (0.2906 \pm 0.0011)\ \mathrm{ps}$$

$\mathrm{R}_\tau = 3.645 \pm 0.020$

$$\mathrm{R}_\tau = 3.635 \pm 0.012$$

- S_{EW} is the sum of leading logs and estimated using RGE $(= 1.0194)$
- δ_{EW} NLL EW correction term $(= 0.0010)$
- δ^{Pert}_{QCD} has been calculated to $\mathcal{O}\left(\alpha_s^3\right)$:

$$\delta^{Pert}_{QCD} = \left(\frac{\alpha_s(m_\tau)}{\pi}\right) + 5.2023\left(\frac{\alpha_s(m_\tau)}{\pi}\right)^2 + 26.366\left(\frac{\alpha_s(m_\tau)}{\pi}\right)^3 + \cdots$$

  Estimate the effect of missing terms from the convergence of the series

- δ^{NP}_{QCD} has been estimated using operator product expansion and QCD sum-rule

  Effect of quark mass effect has been estimated

$$\alpha_s(m_\tau) = (0.35 \pm 0.03)$$

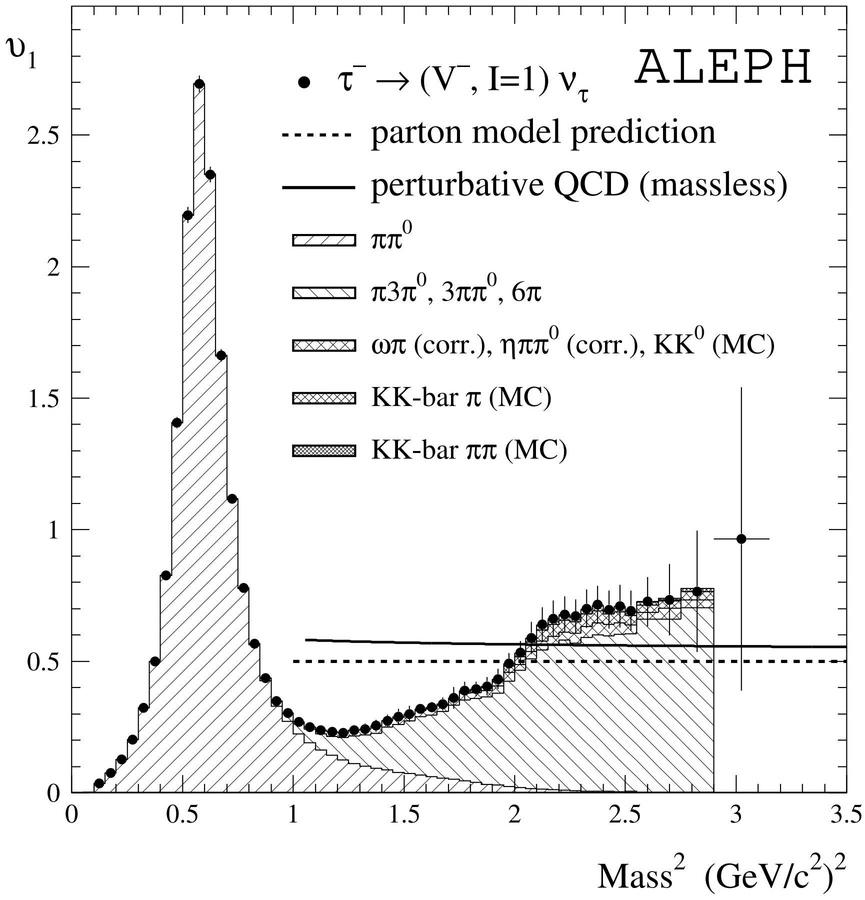
Propagate to a scale m_Z with 5 quark-flavours

$$\alpha_s(m_Z) = (0.121 \pm 0.003)$$

ALEPH has measured the spectral function for the spin 1 and 0 state of the hadronic system for the vector and axial-vector component separately:

$$v_1(s)/a_1(s) = \frac{m_\tau^2}{6|V_{ud}|^2 S_{EW}} \cdot \frac{B(\tau \to V/A\nu_\tau)}{B(\tau \to e\bar{\nu}_e\nu_\tau)} \frac{1}{N_{V/A}} \cdot \frac{dN_{V/A}}{ds} \left[\left(1 - \frac{s}{m_\tau^2}\right)^2 \left(1 + \frac{2s}{m_\tau^2}\right)\right]^{-1}$$

$$a_0(s) = \frac{m_\tau^2}{6|V_{ud}|^2 S_{EW}} \cdot \frac{B(\tau \to \pi\nu_\tau)}{B(\tau \to e\bar{\nu}_e\nu_\tau)} \cdot \frac{1}{N_A} \cdot \frac{dN_A}{ds} \left(1 - \frac{s}{m_\tau^2}\right)^{-2}$$



- Select τ-pair final state
- Identify the final state from the number of reconstructed charged and neutral pion
- Correct the invariant mass spectrum using a regularised inversion matrix
- Use constraints from isospin symmetry to extract the branching ratios

Extract the spectral moments from the spectral distribution:

$$\mathrm{R}^{kl}_{\tau,\mathrm{V/A}} = \int_o^{m_\tau{}^2} ds \left(1 - \frac{s}{m_\tau{}^2}\right)^k \left(\frac{s}{m_\tau{}^2}\right)^l \frac{d\mathrm{R}_{\tau,\mathrm{V/A}}}{ds}$$

Fit simultaneously $\alpha_s(m_\tau)$ and phenomenological operators from the non-perturbative components

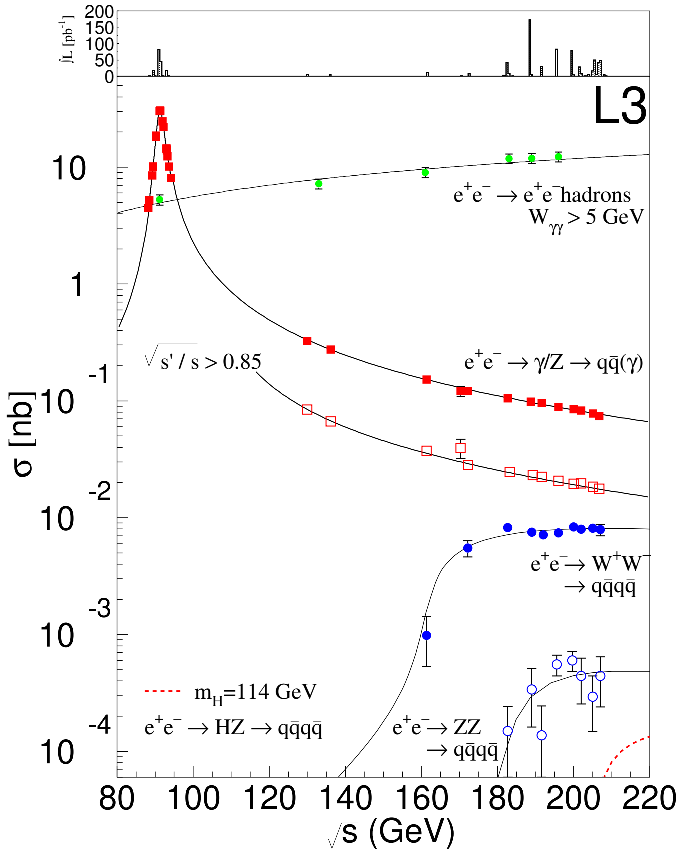
$$\alpha_s(m_\tau) = (0.334 \pm 0.022)$$

Propagate to a scale m_Z with 5 quark-flavours

$$\alpha_s(m_Z) = (0.120 \pm 0.003)$$

Event Shapes

- Select events of the type $e^+e^- \longrightarrow$ hadrons
- Issues:
  - ISR, 4 fermion background (WW, ZZ)
  - Several energy points at high energies, combine some of them (e.g. 204—209 $\Rightarrow$ 206 GeV)



Typical number per expt.

$\sqrt{s}$ (GeV)	$\int$L dt (pb^{-1})	# events	Background
91.2	100	$> 3 \times 10^6$	Negligible
133	12	800	2 %
161	11	300	5 %
172	10	250	10 %
183	60	1300	12 %
189	170	3500	13 %
200	200	3500	14 %
206	210	3500	15 %

Six event shape variables for which improved analytical calculations are available:

Thrust:

$$T = \left(\sum_a |\vec{p}_a \cdot \hat{n}_T| / \sum_a |\vec{p}_a| \right)_{\max}$$

Heavy jet mass:

$$\rho_{\mathrm{H}} = \left(\sum_{a \in S_\pm} p_a / \sqrt{\mathrm{s}} \right)^2_{\max}$$

Jet broadenings: Compute in each hemisphere the quantity

$$\mathrm{B}_\pm = \frac{\sum_{i \in \mathrm{S}_\pm} |\vec{\mathrm{p}}_\mathrm{i} \times \vec{\mathrm{n}}_\mathrm{T}|}{2 \sum_i |\vec{\mathrm{p}}_\mathrm{i}|}$$

Then

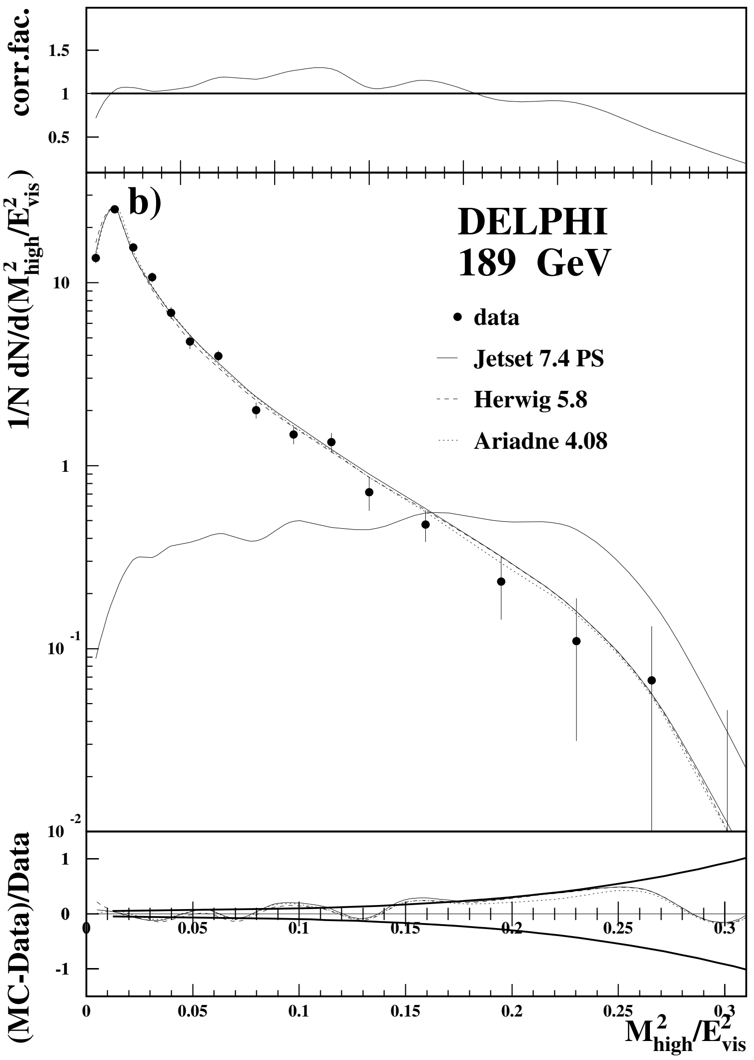
$$\mathrm{B}_\mathrm{T} = \mathrm{B}_+ + \mathrm{B}_- \quad \text{and} \quad \mathrm{B}_\mathrm{W} = \max(\mathrm{B}_+, \mathrm{B}_-)$$

C- and D-Parameters: from the eigenvalues of spherocity tensor

$$\theta^{ij} = \frac{\sum_a \mathrm{p}_a^i \mathrm{p}_a^j / \mid \vec{\mathrm{p}}_a \mid}{\sum_a \mid \vec{\mathrm{p}}_a \mid} \quad i, j = 1, 2, 3 \; ;$$

$$\mathrm{C} = 3(\lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_3\lambda_1) \quad \text{and} \quad \mathrm{D} = 27\lambda_1\lambda_2\lambda_3$$

- Measure the event shape distributions from charged and neutral particles
- Subtract known backgrounds (2-photon, 4-fermion, ISR)
- Correct for residual contamination, detector resolution and acceptance



MC provides good description of data

Soft gluon emission, responsible for hadronisation, is controlled by an effective coupling constant ($\alpha_{\rm eff}$) differing from $\alpha_{\rm s}$ only in the infrared region

$\Rightarrow$ Shift in the distribution for linear event shape variables (T, $\rho_{\rm H}$, C) proportional to Q^{-1}:

$$\frac{d\sigma}{df} = \left.\frac{d\sigma}{df}\right|^{\rm Pert}_{f=f-c_f\mathcal{P}}$$

An additional logarithmic skew for jet broadening variables

Thus for a variable f

$$\langle f\rangle = \langle f_{\rm pert}\rangle + \langle f_{\rm pow}\rangle$$

Perturbative contribution:

$$\langle f_{\rm pert}\rangle = A\frac{\alpha_{\rm s}(Q)}{2\pi} + (B + A\cdot(\beta_0 \ln\mu - 2))\left(\frac{\alpha_{\rm s}(Q)}{2\pi}\right)^2 ,$$

and the power correction term:

$$\langle f_{\rm pow}\rangle = c_f \cdot \mathcal{P}$$

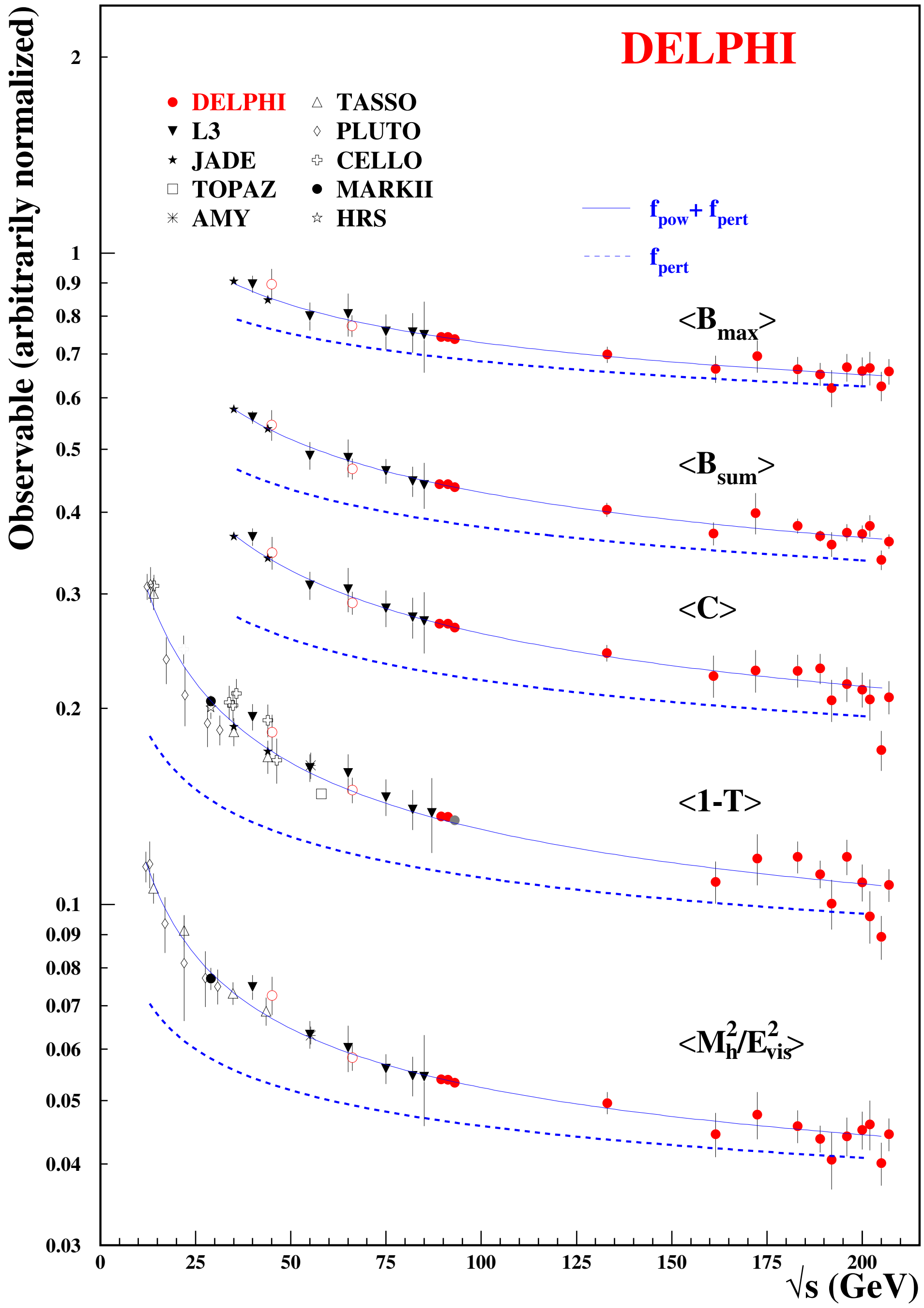
	1−T	$\rho_{\rm H}$	$\rm B_T$	$\rm B_W$	C	D
c_f	2	2	1	$\frac{1}{2}$	9.425	$195\cdot\frac{\alpha_{\rm s}}{2\pi}$

$\mathcal{P}$ is supposed to have a universal form:

$$\mathcal{P} = \frac{4C_F}{\pi^2}\mathcal{M}\frac{\mu_I}{\sqrt{\rm s}}\left[\alpha_0(\mu_I) - \alpha_{\rm s}(\sqrt{\rm s}) - \beta_0\frac{\alpha_{\rm s}^2(\sqrt{\rm s})}{2\pi}\left(\ln\frac{\sqrt{\rm s}}{\mu_I} + \frac{K}{\beta_0} + 1\right)\right]$$

One additional free parameter: $\alpha_0(\mu_I)$. Extra factor for $\rm B_T$/ $\rm B_W$:

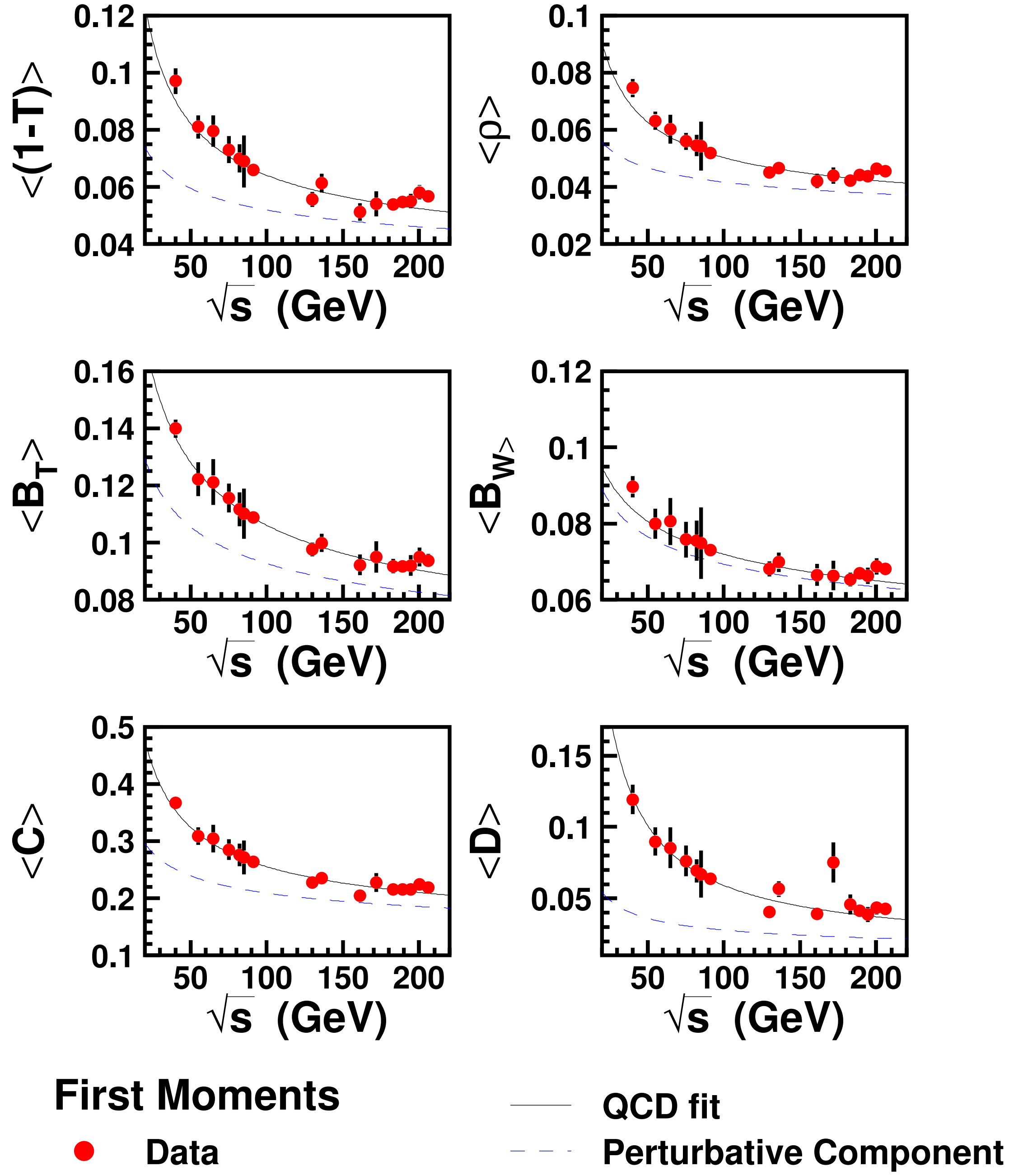
$$F = \left(\frac{\pi}{2\sqrt{a\cdot C_F\alpha_{\rm CMW}}} + \frac{3}{4} - \frac{\beta_0}{6a\ C_F} + \eta_0 + \mathcal{O}(\sqrt{\alpha_{\rm s}})\right)$$



	α_0 (2 GeV)	$\alpha_s(\mathrm{m_Z})$
$< 1 - T >$	0.491± 0.018	.1241 ± .0034
$< \rho_H >$	0.601± 0.059	.1177 ± .0036
$< B_T >$	0.463± 0.033	.1174 ± .0029
$< B_W >$	0.438± 0.049	.1167 ± .0019
$< C >$	0.444± 0.021	.1222 ± .0036

$\alpha_s(\mathrm{m_Z}) = 0.1217$
$\pm\ 0.0046$ (exp)
$\pm\ 0.0030$ (thy)

Event Shapes



	α_0 (2 GeV)	$\alpha_s(m_Z)$
$< 1 - T >$	0.514± 0.054	.1162 ± .0049
$< \rho_H >$	0.399± 0.030	.1068 ± .0036
$< B_T >$	0.443± 0.039	.1163 ± .0034
$< B_W >$	0.323± 0.067	.1172 ± .0034
$< C >$	0.453± 0.027	.1161 ± .0030
$< D >$	0.639± 0.044	.1371 ± .0092

$$\alpha_s(m_Z) = 0.1183 \pm 0.0046\ (\text{exp}) \pm 0.0044\ (\text{thy})$$

Fixed order calculation exists up to $\mathcal{O}(\alpha_s^2)$ for all event shape variables

Cumulative cross section $R(y)$ for event shape variable y:

$$\begin{aligned} R(y,\alpha_s) &\equiv \int_0^y \frac{1}{\sigma}\frac{d\sigma}{dy} \\ &= \overline{\alpha}_s A(y) + \overline{\alpha}_s^2 \left[B(y) + 2\pi\beta_0 \ln\left(\frac{Q^2}{s}\right) A(y) \right] \end{aligned}$$

with

$$\begin{aligned} \overline{\alpha}_s &= \frac{\alpha_s(Q)}{2\pi} \\ \beta_0 &= \frac{33-2\mathrm{N_F}}{12\pi} \end{aligned}$$

$A(y)$, $B(y)$ computed by integrating ERT matrix elements $[\mathcal{O}(\alpha_s^2)]$ using the program EVENT (Kunzst and Nason) or EVENT2 (Catani and Seymour)

Describes data well in the multi-jet region, but fails in the two jet region (small y)

Resummation of Large Logarithms: For certain variables ($y \equiv 1-T$, $\rho = \frac{m_H^2}{s}$, B_T, B_W, C, y_{23}) leading and next-to-leading log terms are summed to all orders in $L \equiv \ln(1/y)$

	LL	NLL	Sub-leading			
$\mathcal{O}(\alpha_s)$	$\overline{\alpha}_s L^2$	$\overline{\alpha}_s L$	$\overline{\alpha}_s$	$\overline{\alpha}_s \mathcal{O}\left(\frac{1}{L}\right)$		
$\mathcal{O}(\alpha_s^2)$	$\overline{\alpha}_s^2 L^3$	$\overline{\alpha}_s^2 L^2$	$\overline{\alpha}_s^2 L$	$\overline{\alpha}_s^2$	$\overline{\alpha}_s^2 \mathcal{O}\left(\frac{1}{L}\right)$	
$\mathcal{O}(\alpha_s^3)$	$\overline{\alpha}_s^3 L^4$	$\overline{\alpha}_s^3 L^3$	$\overline{\alpha}_s^3 L^2$	$\overline{\alpha}_s^3 L$	$\overline{\alpha}_s^3$	$\overline{\alpha}_s^3 \mathcal{O}\left(\frac{1}{L}\right)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

- ❒ Resummed LL and NLL terms dominate at small y
- ❒ Sub-leading terms important at large y

$\Rightarrow$ Combine fixed order calculations with resummed calculations

Avoid double counting of terms present in $\mathcal{O}\left(\alpha_s^2\right)$ fixed order and resummed calculations

$\Rightarrow$ Many matching Schemes

Log R Matching Take log of fixed order; expand in power series and match in $\ln R(y)$

Modified Log R Replace L in the resummed terms by $L' = \ln\left((\frac{1}{y})^p - (\frac{1}{y_{\max}})^p + 1\right)$ with modification degree p $=$ 1; carry out Log R matching

R Matching Remove $\mathcal{O}\left(\alpha_s^2\right)$ terms from resummed $R(y)$; take fixed order calculation in toto

Modified R One sub-leading term included in the argument of exponent and subtracted after exponentiation

- ❒ All matching algorithms exact up to $\mathcal{O}\left(\alpha_s^2\right)$
- ❒ Sub-leading terms determined using Monte Carlo

Also use kinematic constraints :Cross sections must vanish beyond kinematic limit

$$\begin{aligned} R\big|_{y=y_{\max}} &= 1 \\ \left.\frac{dR}{dy}\right|_{y=y_{\max}} &= 0 \end{aligned}$$

has to be added because they are strictly obeyed in the fixed order calculations but not in the resummed expansion

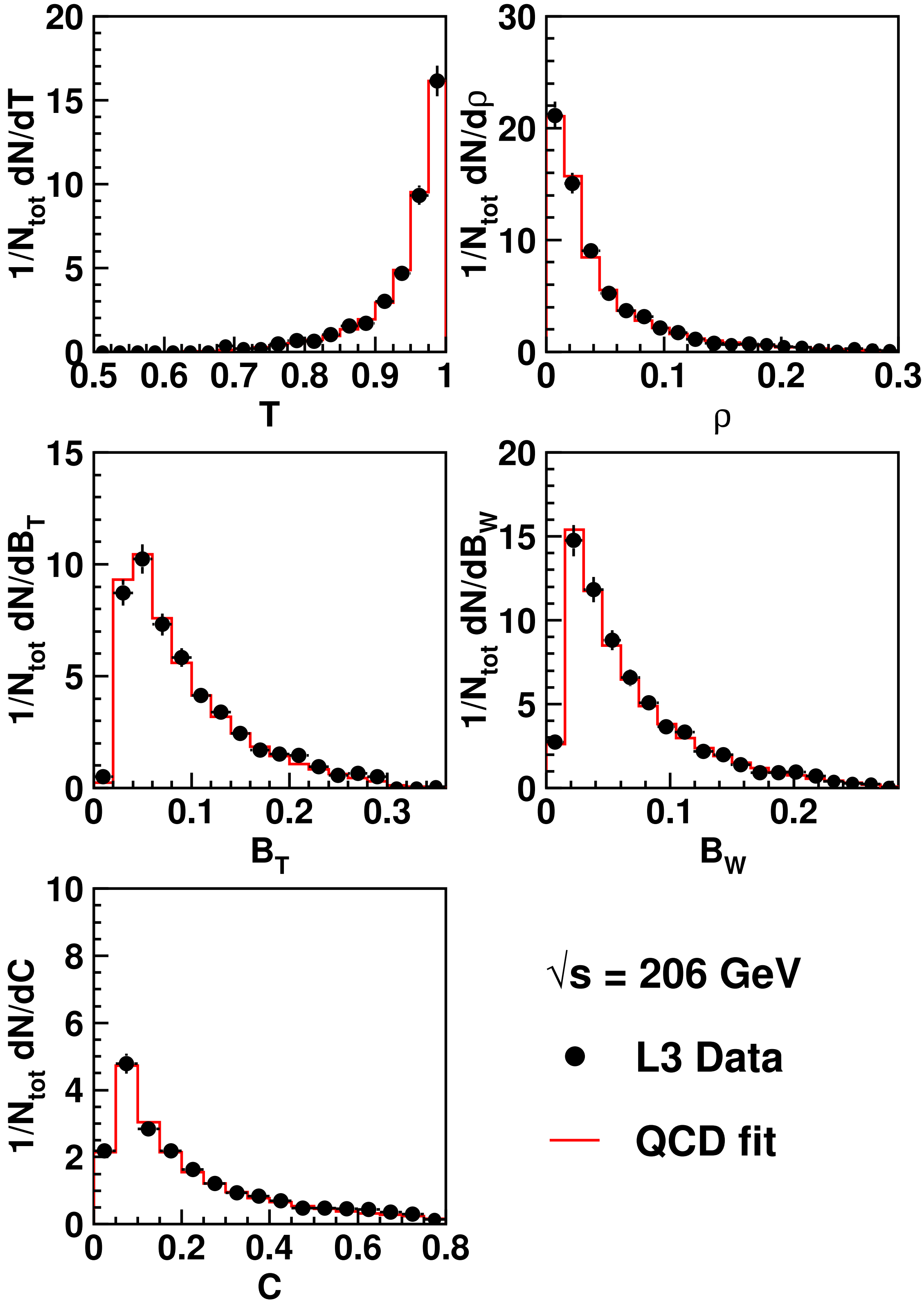
Advantage of this approach

- ❏ Can explain small y (high statistics) region
- ❏ Get good fit at a reasonable scale ($Q \approx \sqrt{s}$)
- ❏ Since more theoretical terms known, uncertainty due to uncalculated terms should be reduced

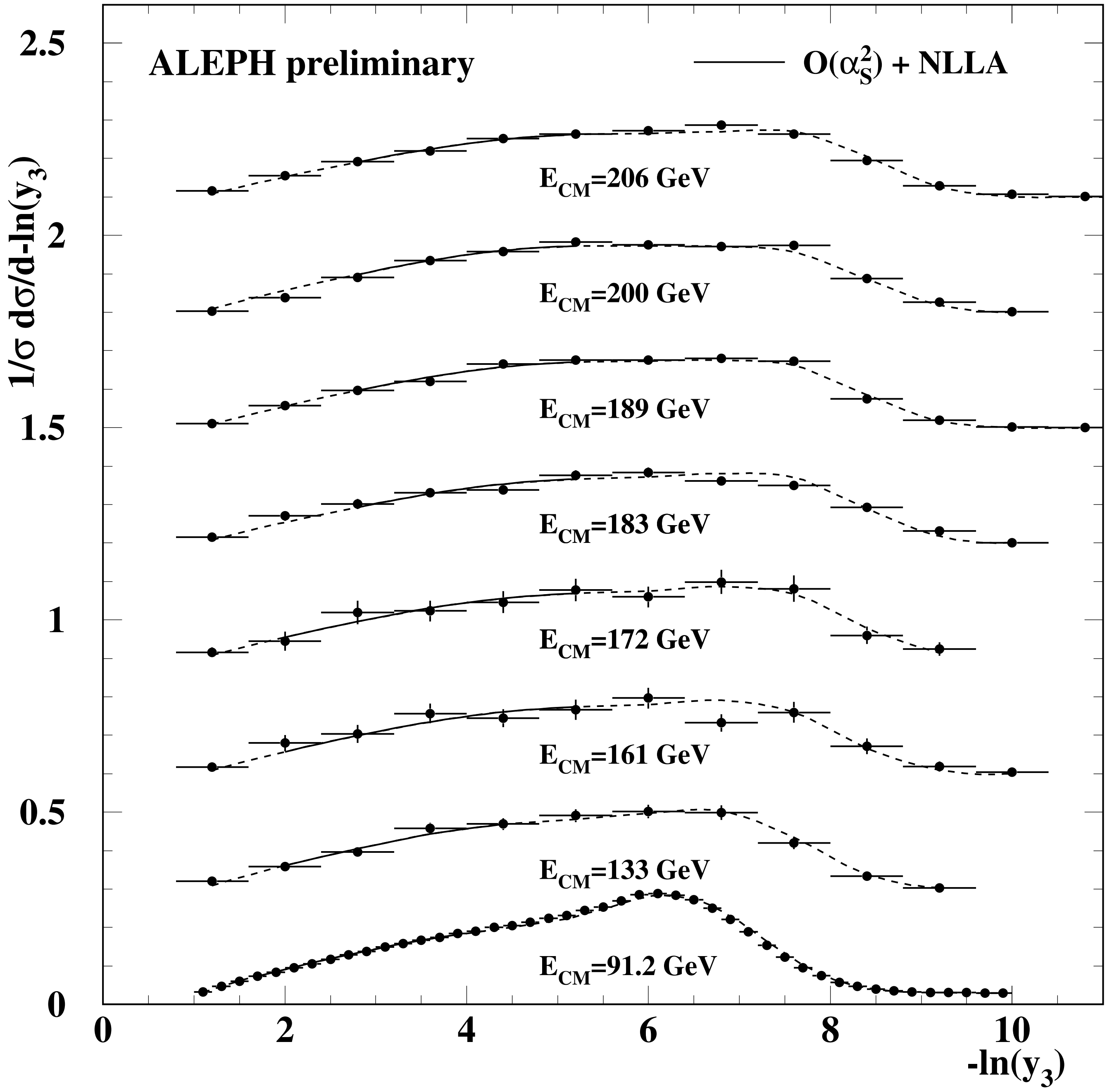
To include hadronisation effects convolute:

$$f(y) = \int f^{\text{pert}}(y') \cdot p^{\text{non-pert}}(y', y)\, dy'$$

where $p^{\text{non-pert}}(y', y)$ is the probability to find a value y after fragmentation and decays for a parton level value y' evaluated using Parton Shower Monte Carlo programs



Obtain good fits using pQCD predictions corrected for hadronisation effects using MC



Differential 2-jet rate (Durham) has now got an improved theoretical prediction

But the four LEP experiments

- ➠ use different set of variables (y)
- ➠ differ in fitting range
- ➠ differ in matching scheme
- ➠ differ in estimation (method) of systematic errors

LEP QCD Working Group:

- ☛ Choose two matching schemes for all 4 experiments (Modified log R and R schemes)
- ☛ Classify the error on α_s in four categories:
  - ➢ Statistical
  - ➢ Experimental systematic (dominated by BG)
  - ➢ Hadronisation (dominated by JETSET vs HERWIG)
  - ➢ Theoretical uncertainty

Average α_s (91.2 GeV) (as in 2001)

	α_s	$\Delta\alpha_s^{Stat}$	$\Delta\alpha_s^{Exp}$	$\Delta\alpha_s^{Had}$	$\Delta\alpha_s^{Scale}$
ALEPH	0.1195	0.0001	0.0008	0.0017	0.0029
DELPHI	0.1246	0.0002	0.0024	0.0032	0.0063
L3	0.1221	0.0008	0.0018	0.0039	0.0053
OPAL	0.1130	0.0004	0.0022	0.0075	0.0070

Theoretical errors differ quite a lot!!

Is this effect present in one energy?

Average α_s (183 GeV)

	α_s	$\Delta\alpha_s^{Stat}$	$\Delta\alpha_s^{Exp}$	$\Delta\alpha_s^{Had}$	$\Delta\alpha_s^{Scale}$
ALEPH	0.1066	0.0032	0.0018	0.0011	0.0026
DELPHI	0.1132	0.0038	0.0013	0.0016	0.0049
L3	0.1088	0.0021	0.0010	0.0030	0.0044
OPAL	0.1056	0.0027	0.0019	0.0010	0.0039

or due to choice of variables?

α_s (183 GeV) from Thrust

	α_s	$\Delta\alpha_s^{Stat}$	$\Delta\alpha_s^{Exp}$	$\Delta\alpha_s^{Had}$	$\Delta\alpha_s^{Scale}$
ALEPH	0.1111	0.0038	0.0025	0.0016	0.0018
DELPHI	0.1132	0.0038	0.0012	0.0015	0.0053
L3	0.1132	0.0023	0.0012	0.0028	0.0046
OPAL	0.1070	0.0032	0.0033	0.0015	0.0048

α_s (183 GeV) from ρ_H

	α_s	$\Delta\alpha_s^{Stat}$	$\Delta\alpha_s^{Exp}$	$\Delta\alpha_s^{Had}$	$\Delta\alpha_s^{Scale}$
ALEPH	0.0990	0.0084	0.0039	0.0016	0.0013
DELPHI	0.1133	0.0039	0.0012	0.0017	0.0045
L3	0.1075	0.0022	0.0011	0.0017	0.0034
OPAL	0.1074	0.0036	0.0032	0.0009	0.0040

Hadronisation error::

- ✧ L3 determines from a comparison of models and also within a model by changing model parameters
- ✧ ADO determine from comparison of models
  Choice of models may not always be the same

⇒ L3 is more conservative

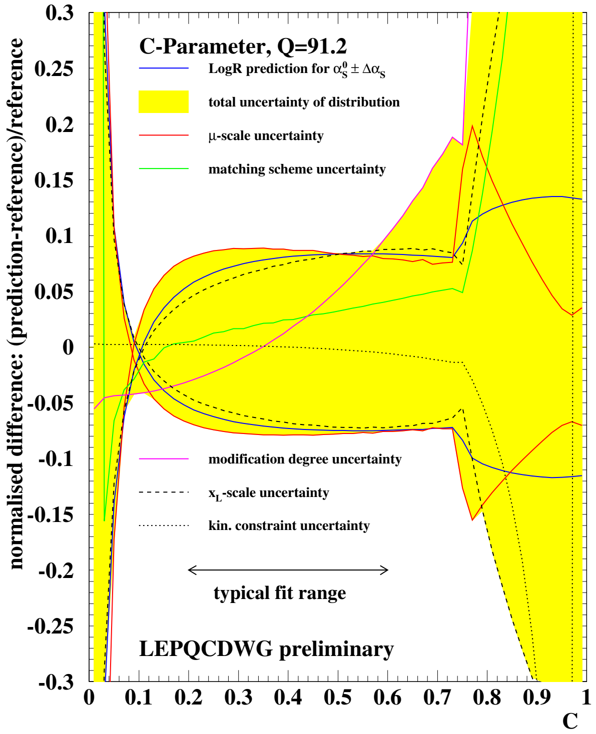
LEP QCD WG decides to estimate hadronisation uncertainty from difference between various models: PYTHIA, HERWIG and ARIADNE

In order to determine theoretical uncertainty, LEPQCD has developed a new prescription:

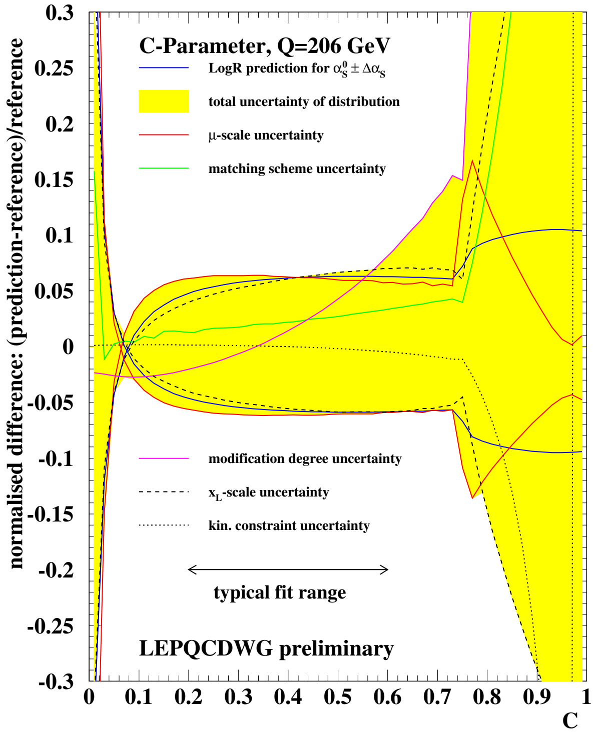
Uncertainty band method

- ❑ Obtain uncertainty band for a fixed α_s by varying
  - ✧ renormalisation scale (between $\frac{\sqrt{s}}{2}$ and $2\sqrt{s}$)
  - ✧ rescaling factor x_L $\left(L' = \ln\left(\frac{1}{x_L y}\right)\right)$
  - ✧ kinematic constraint (y_{max})
  - ✧ matching scheme
  - ✧ modification degree (p in L')
- ❑ For a fixed reference prediction (Log R) find α_s variation which covers the band within the fit range

Event Shapes



Typically 5% at LEP1



Goes down to 3.5% at the highest energy

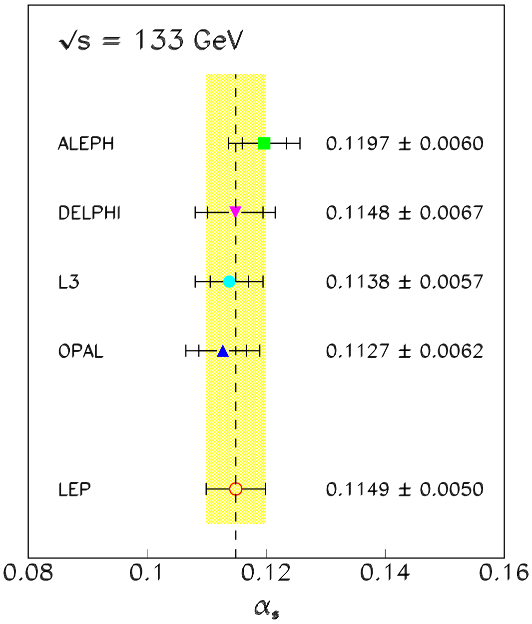
For combining different α_s values, one can treat correlated errors as:

- ❑ simple average of the errors — Conservative
- ❑ fully correlated — $W_{ij} = \sigma_i \cdot \sigma_j$
- ❑ minimum overlap — $W_{ij} = \min(\sigma_i^2, \sigma_j^2)$
- ❑ uncorrelated for weight calculation but repeat for fits with different hadronisation corrections and scale values — mixed

Final prescription:

Error Type	Between Experiments	Between Energies
Statistical	Uncorrelated	Uncorrelated
Exp. Systematic	Uncorrelated	Minimum overlap
Hadronisation	Mixed	Mixed
Scale	Mixed	Mixed

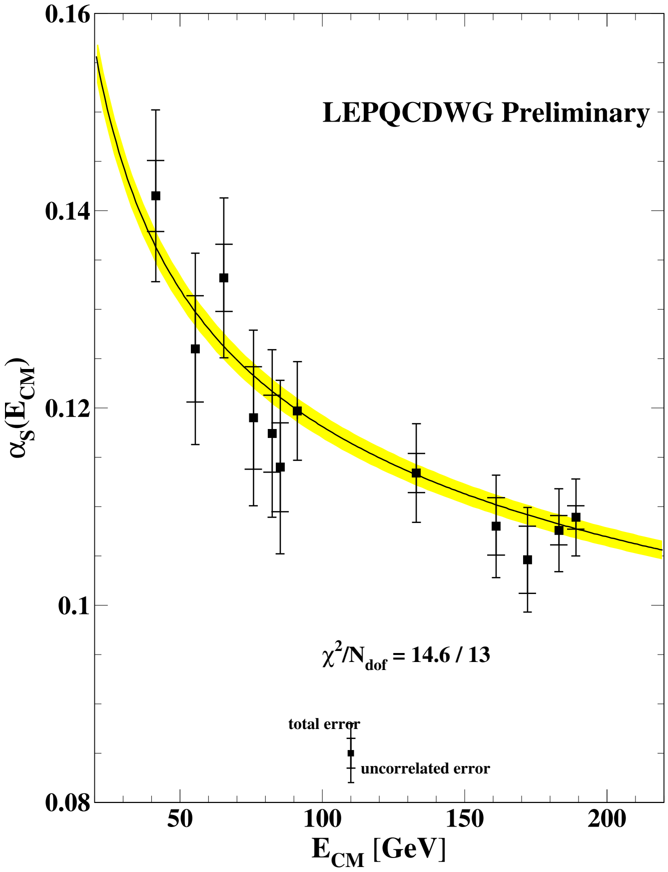
Average α_s (133 GeV)



$\sqrt{s}$ (GeV)	α_s	$\Delta\alpha_s^{Stat}$	$\Delta\alpha_s^{Exp}$	$\Delta\alpha_s^{Had}$	$\Delta\alpha_s^{Scale}$
41.4	0.1415	0.0024	0.0027	0.0018	0.0077
55.3	0.1260	0.0023	0.0049	0.0045	0.0067
65.4	0.1332	0.0015	0.0031	0.0041	0.0061
75.7	0.1190	0.0012	0.0051	0.0045	0.0056
82.3	0.1174	0.0013	0.0037	0.0051	0.0055
85.1	0.1140	0.0018	0.0041	0.0051	0.0056
91.2	0.1197	0.0002	0.0008	0.0010	0.0048
133.0	0.1149	0.0016	0.0012	0.0010	0.0045
161.0	0.1080	0.0025	0.0014	0.0003	0.0043
172.0	0.1046	0.0029	0.0017	0.0006	0.0040
183.0	0.1076	0.0013	0.0008	0.0007	0.0038
189.0	0.1089	0.0008	0.0009	0.0006	0.0037
200.0	0.1074	0.0009	0.0010	0.0006	0.0036
206.0	0.1073	0.0009	0.0008	0.0005	0.0034

Fit measured α_s values to QCD evolution equation in NNLO

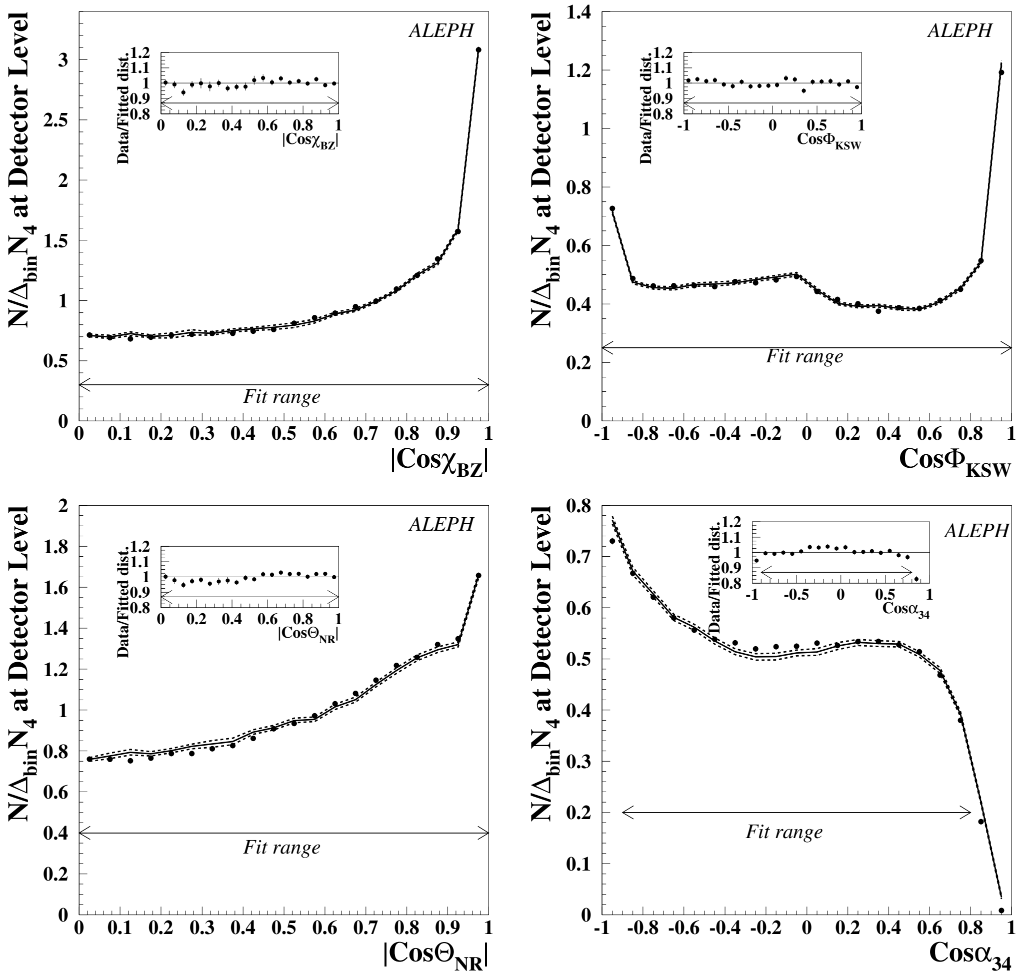
$$\alpha_s(\mu) = \frac{4\pi}{\beta_0 \ln\left(\frac{\mu^2}{\Lambda^2}\right)}\left[1 - \frac{2\beta_1}{\beta_0^2}\frac{\ln\left[\ln\left(\frac{\mu^2}{\Lambda^2}\right)\right]}{\ln\left(\frac{\mu^2}{\Lambda^2}\right)} + \frac{4\beta_1^2}{\beta_0^4 \ln^2\left(\frac{\mu^2}{\Lambda^2}\right)} \times \left(\left(\ln\left[\ln\left(\frac{\mu^2}{\Lambda^2}\right)\right] - \frac{1}{2}\right)^2 + \frac{\beta_2\beta_0}{8\beta_1^2} - \frac{5}{4}\right)\right]$$



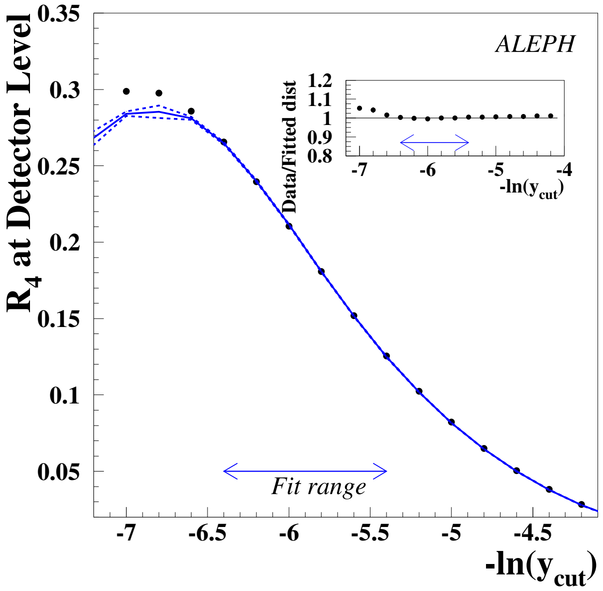
	All LEP	LEP I	LEP II
$\alpha_s(m_Z)$	0.1198	0.1197	0.1197
Δ stat	0.0002	0.0002	0.0006
Δ syst	0.0009	0.0008	0.0010
Δ had	0.0008	0.0010	0.0007
Δ theo	0.0045	0.0048	0.0047
Δ tot	0.0047	0.0050	0.0047

ALEPH has analysed 4-jet events in e^+e^- interactions at $\sqrt{s} \simeq m_Z$. From the measured energies of the 4-jets with $k_\perp$ algorithm at $y_{cut} = 0.008$, the different angular correlations are measured:

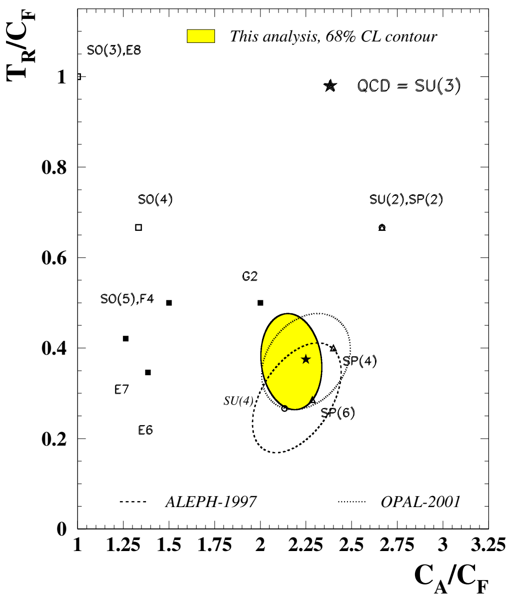
- The Bengton-Zerwas angle (angle between planes containing jets 1-2 and 3-4)
- The Körner-Shierholz angle (mean of the angles between planes containing jets 1-4 and 2-3 and between planes containing jets 1-3 and 2-4)
- The Nachtman-Reiter angle (angle between $(\vec{p}_1 - \vec{p}_2)$ and $(\vec{p}_3 - \vec{p}_4)$)
- The angle between the two lowest energetic jets



This makes simultaneous measurement of α_s and QCD colour factors:



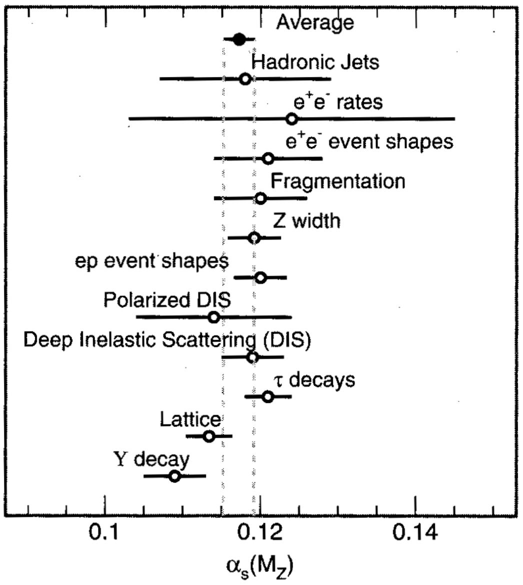
$$\alpha_s(m_Z) = 0.119 \pm 0.006\,(\mathrm{stat}) \pm 0.026\,(\mathrm{syst})$$



$$C_A = 2.93 \pm 0.14\,(\mathrm{stat}) \pm 0.58\,(\mathrm{syst})$$
$$C_F = 1.35 \pm 0.07\,(\mathrm{stat}) \pm 0.26\,(\mathrm{syst})$$

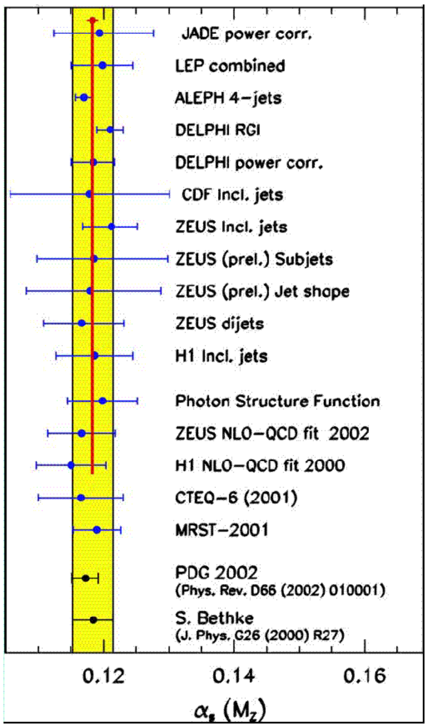
- A large number of measurements exist on α_s from a variety of experiments
- PDG averages 11 measurements with a χ^2 of 9

$$\alpha_s(m_Z) = 0.1171 \pm 0.0014$$



- At ICHEP 2002, 14 measurements were combined:

$$\alpha_s(m_Z) = 0.1183 \pm 0.0009$$



- More realistic error estimate $\pm$ 0.003