Chiral Quark Model

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<u>Outline</u>:

I) Introduction

- ⋆ QCD in the large N_C limit
- ⋆ chiral symmetry of QCD

II) NJL model for quark flavor dynamics

- ⋆ dynamical chiral symmetry breaking
- ⋆ "constituent quarks"

III) Mesons from bosonization

- ⋆ Bethe–Salpeter eqn in NJL model
- ⋆ axial vector coupling and f_π
- ⋆ hadronic tensor and structure functions

IV) Baryons as chiral solitons

- ⋆ effective action for static backgrounds
- ⋆ constructing the soliton
- ⋆ connection to Skyrme model
- ⋆ quantizing the soliton
  static baryon properties
- ⋆ nucleon structure functions

V) Conclusions

I) Introduction

⋆ Theory for the interaction of hadron constituents: QCD

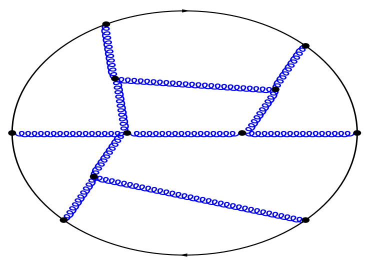
⋆ Solution to QCD at low energy scales unknown
(except perhaps lattice)

⋆ No rigorous prediction for hadron properties

⋆ Hidden expansion parameter: N_C
(number of color degrees of freedom
't Hooft '72, Witten '79)

⋆ Leading diagrams:

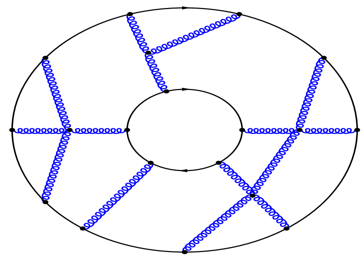
planar diagrams with no
internal quark loops



cut $\longrightarrow$ single color trace: $\bar{q} A_\mu A_\nu \ldots A_\sigma q$
meson quantum numbers

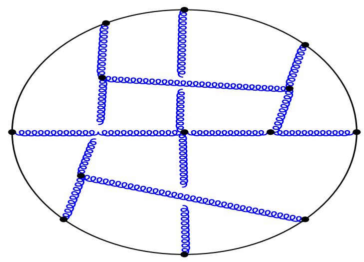
⋆ Suppressed diagrams:

planar diagrams with internal quark loops



cut $\longrightarrow$ no single color trace: $\bar{q} A_\mu \ldots A_\nu q \; \bar{q} A_\sigma \ldots A_\tau q$
quantum numbers of multiple meson states

non–planar diagrams



cut $\longrightarrow$ no single color trace: $\bar{q}\, \mathrm{tr}(A_\mu \ldots A_\nu)\; \mathrm{tr}(A_\sigma \ldots A_\tau) q$
quantum numbers of multiple meson and glueball states

⋆ In the large N_C limit QCD becomes equivalent to an effective meson theory:

– meson loops are suppressed
– mesons are stable and interact weakly: $g_{\rm eff}^{(n)} \sim \left(1/\sqrt{N_C}\right)^n$
– soliton solutions of the effective meson theory $\longrightarrow$ baryons
(scaling of soliton properties with $g_{\rm eff}^{(n)}$ and thus N_C)

⋆ Model the effective meson theory:
Light flavors should be essential for hadron structure

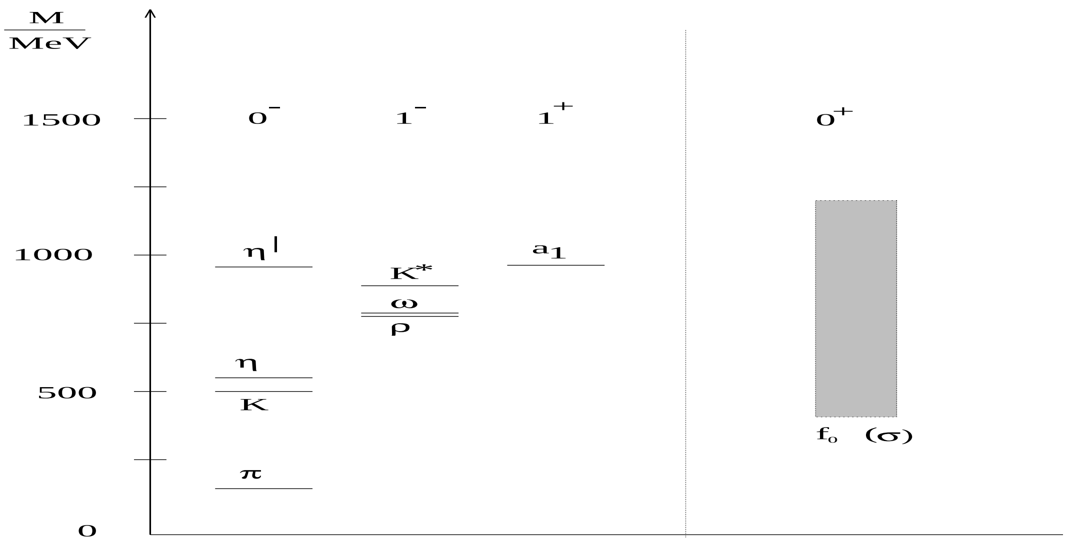
⋆ Basic building block: Chiral symmetry

$$\vec{\sigma}\cdot\left(\vec{p}+\vec{A}\right) q_{\mathrm{R}} + m q_{\mathrm{L}} \; = \; (E + A_0)\, q_{\mathrm{R}}$$
$$\vec{\sigma}\cdot\left(\vec{p}+\vec{A}\right) q_{\mathrm{L}} - m q_{\mathrm{R}} \; = \; (E + A_0)\, q_{\mathrm{L}}$$

$m_{\text{current}} \ll \Lambda_{\text{QCD}}$ $\qquad q_{\mathrm{R/L}} = \frac{1}{2}\,(1 \pm \gamma_5)\, q$ $\qquad$ decouple

– approximate Lagrangian symmetry: $SU_L(3)\times SU_R(3)$

– Lagrangian symmetry not reflected in the spectrum:



– symmetry spontaneously (dynamically) broken to:

$$\underline{SU_{L+R}(3)}$$

– pions as Goldstone bosons

II) NJL model for quark flavor dynamics

(Nambu and Jona–Lasinio '61)

- ⋆ simplified starting point to obtain an effective meson theory from quark interactions
- ⋆ based on chiral symmetry
- ⋆ exemplifies dynamical chiral symmetry breaking
- ⋆ contains soliton solutions

$$\mathcal{L}_{\rm NJL} = \bar{q}\,(i\partial\!\!\!/ - m_c)\,q + G\left[(\bar{q}q)^2 - (\bar{q}\vec{\tau}\gamma_5 q)^2\right]$$

$$= \bar{q}\,(i\partial\!\!\!/ - m_c + \sigma + i\vec{\pi}\cdot\vec{\tau}\,\gamma_5)\,q - \tfrac{1}{4G}\left(\sigma^2 + \vec{\pi}^{\,2}\right)$$

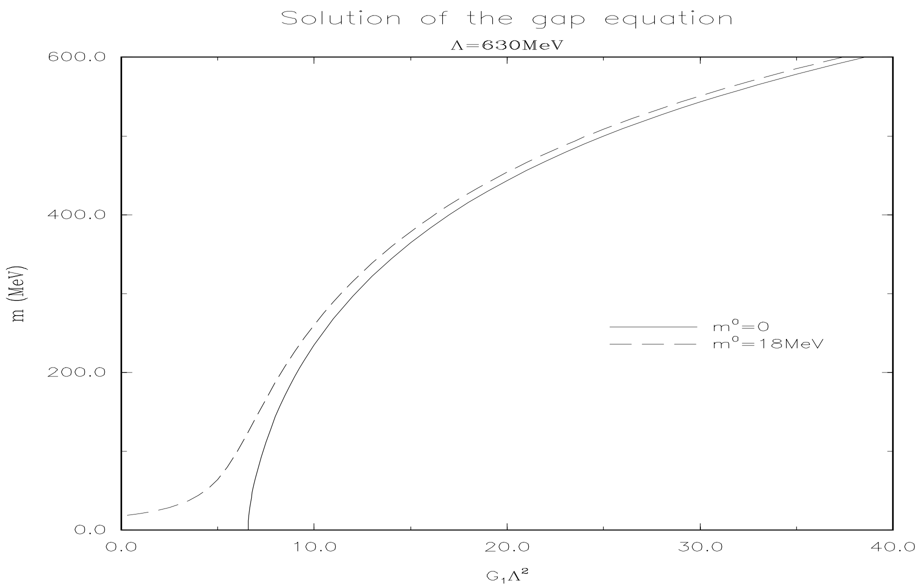
- ⋆ one–loop effective action:

$$\boxed{\mathcal{A}_{\rm NJL} = i\,{\rm Tr}_{\Lambda}\log\,(i\partial\!\!\!/ - m_c + \sigma + i\vec{\pi}\cdot\vec{\tau}\,\gamma_5) - \frac{1}{4G}\int d^{\,4}x\,\left(\sigma^2 + \vec{\pi}^{\,2}\right)}$$

$\star$ gap–equation $(\sigma \longrightarrow \sigma + m_c)$:

$$\langle x \left| \frac{i}{i\partial\!\!\!/ - \sigma} \right| x \rangle_\Lambda = \frac{m_c - \sigma(x)}{2G}$$

$\star$ ansatz $\sigma = m = \text{const.}$: $\quad m = m_c + 2G \int_\Lambda \frac{d^4 k_E}{k_E^2 + m^2}$



$\star$ $G < G_{\text{crit.}}$: $\qquad m = 0 \qquad \Leftrightarrow \qquad \langle \bar{q}q \rangle = 0$

uninteresting for hadron physics .....

$\star$ $G > G_{\text{crit.}}$: $\qquad m \neq 0 \qquad \Leftrightarrow \qquad \langle \bar{q}q \rangle \neq 0$

chiral symmetry dynamically broken

fermions acquire constituent mass $\quad m \sim 300...400\text{MeV}$!

fermions are constituent quarks

III) Mesons from bosonization

⋆ Dirac operators in Minkowski space:

$$i\mathbf{D} \; = \; i\partial\!\!\!/ - (S + i\gamma_5 P) + v\!\!\!/ + a\!\!\!/\gamma_5 =: i\mathbf{D}^{(\pi)} + v\!\!\!/ + a\!\!\!/\gamma_5$$

$$i\mathbf{D}_5 \; = \; -i\partial\!\!\!/ - (S - i\gamma_5 P) - v\!\!\!/ + a\!\!\!/\gamma_5 =: i\mathbf{D}_5^{(\pi)} - v\!\!\!/ + a\!\!\!/\gamma_5$$

⋆ S, P : scalar and pseudoscalar fields
(matrices in flavor space)
generated as quark–antiquark bound states

⋆ v_μ, a_μ : vector and axialvector sources
(photons, Z^0 or $W^\pm$)

⋆ Minkowski $\longrightarrow$ Euclidean : $D \longrightarrow D_E$, $\quad D_5 \longrightarrow D_E^\dagger$

Bosonized action

$$\boxed{\mathcal{A}_{\rm NJL} = \mathcal{A}_{\rm R} + \mathcal{A}_{\rm I} + \frac{1}{4G}\int d^4x \, {\rm tr}\left[S^2 + P^2 + 2m_c S\right]}$$

⋆ local piece: $(\bar{q}q)^2\,,\quad (\bar{q}\gamma_5 q)^2 \quad \longrightarrow \quad S^2\,,\quad P^2$

⋆ real part of fermion loop (in E–space):

$$\mathcal{A}_{\rm R} = -i\frac{N_C}{2}\sum_{i=0}^{2} c_i {\rm Tr}\log\left[-\mathbf{D}\mathbf{D}_5 + \Lambda_i^2 - i\epsilon\right]$$

- Pauli–Villars regularization for UV divergence:

$$c_0 = 1\,,\quad \Lambda_0 = 0\,,\quad \sum_{i=0}^{2} c_i = 0\,,\quad \sum_{i=0}^{2} c_i \Lambda_i^2 = 0$$

- Regulators acquire physical meaning
- Λ_i additive to constituent quark mass

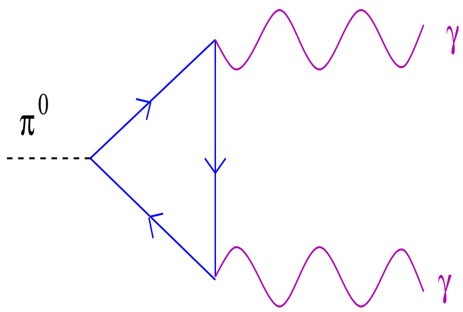
⋆ imaginary part (in E–space):

$$\mathcal{A}_{\rm I} = -i\frac{N_C}{2}{\rm Tr}\log\left[-\mathbf{D}\left(\mathbf{D}_5\right)^{-1} - i\epsilon\right]$$

- Conditionally UV finite (principal value)

- Correct triangle anomaly

(unregularized quark loop)



⋆ non–linear realization of the pion field

$$S + iP \;=\; \langle S\rangle\, U = m\, U = m \exp\left(i\frac{g}{m}\,\vec{\pi}\cdot\vec{\tau}\right)$$

$$U \;:\; \text{chiral field}$$

$$g \;:\; \text{pion} - \text{quark Yukawa coupling}$$

⋆ bound states from quark loop $(v_\mu = 0\,, \quad a_\mu = 0)$

$$\mathcal{A}_{\rm NJL} \;=\; g^2 \int \frac{d^4q}{(2\pi)^4}\, \vec{\tilde{\pi}}(q)\cdot\vec{\tilde{\pi}}(-q) \underbrace{\left[2N_C q^2 \Pi(q^2) - \frac{m_c}{2Gm}\right]}_{\text{inverse pion propagator}} + \mathcal{O}\left(\vec{\pi}^4\right)$$

⋆ polarization function reflects quark loop

$$\Pi(q^2,x) = \sum_{i=0}^{2} \frac{d^4k}{(2\pi)^4 i} \frac{c_i}{\left[-k^2 - x(1-x)q^2 + m^2 + \Lambda_i^2 - i\epsilon\right]^2}$$

$$\Pi(q^2) \;=\; \int_0^1 dx\, \Pi(q^2,x)$$

⋆ Physics identification

- pion mass = pole of propagator

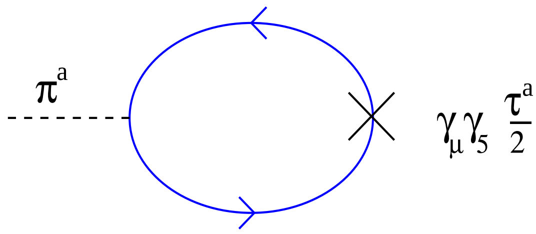
$$m_\pi^2 = \frac{1}{2G}\frac{m_c}{m}\frac{1}{2N_C\Pi(m_\pi^2)}$$

pion is massless when $m_c = 0$
(would-be) Goldstone boson of
dynamical symmetry breaking

- wavefunction normalization = unit residuum

$$\frac{1}{g^2} = 4N_C\frac{d}{dm_\pi^2}\left[m_\pi^2\Pi(m_\pi^2)\right]$$

- coupling to axial source a_μ = pion decay constant



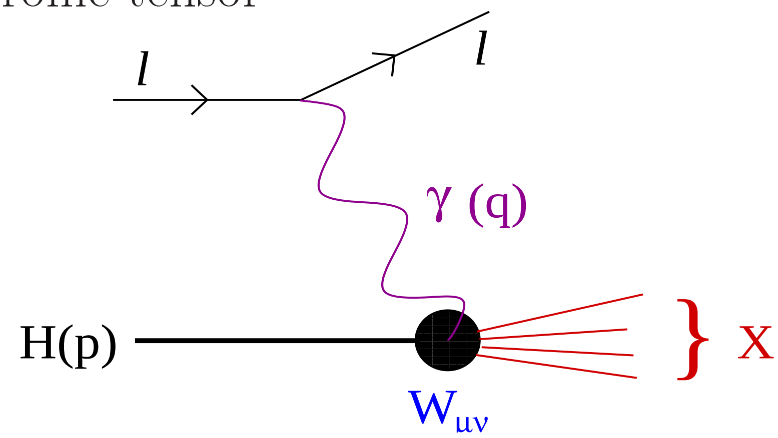
$$f_\pi = 4N_C mg\Pi(m_\pi^2)$$

⋆ Parameter fixing

G, m_c, Λ from $f_\pi = 93$MeV, $m_\pi = 139$MeV and $m \approx 400$MeV
constituent quark mass treated as adjustable parameter

Structure functions

⋆ DIS and hadronic tensor



$$W^{ab}_{\mu\nu}(p,q;s) = \frac{1}{4\pi}\int d^4x\, e^{iq\cdot\xi}\left\langle p,s\left|[J^a_\mu(\xi),J^{b\dagger}_\nu(0)]\right|p,s\right\rangle$$

- p: hadron momentum
- $a\,b$: flavor indices, s: spin projection (eventual)
- J^a_μ: symmetry current coupled to exchanged boson

⋆ Relation to Compton amplitude

$$W^{ab}_{\mu\nu}(p,q;s) = \frac{1}{2\pi}\,\mathrm{Im}\,T^{ab}_{\mu\nu}(p,q;s)$$

$$T^{ab}_{\mu\nu}(p,q;s) = i\int d^4x\, e^{iq\cdot\xi}\left\langle p,s\left|T\left\{J^a_\mu(\xi)J^{b\dagger}_\nu(0)\right\}\right|p,s\right\rangle$$

$$\boxed{T\left\{J^a_\mu(\xi)J^{b\dagger}_\nu(0)\right\} = \left.\frac{\mathrm{Tr}_\Lambda \log\left(i\partial\!\!\!/ - mU^{\gamma_5} + v\!\!\!/\right)}{\delta v^{\mu a}(\xi)\,\delta v^{\nu b}(0)}\right|_{v_\mu=0}}$$

WELL DEFINED IN BOSONIZATION !

⋆ Cutkosky's rules $\longrightarrow$ absorptive part $\mathrm{Im}T$

⋆ Bjorken limit $\longrightarrow$ hadron structure functions

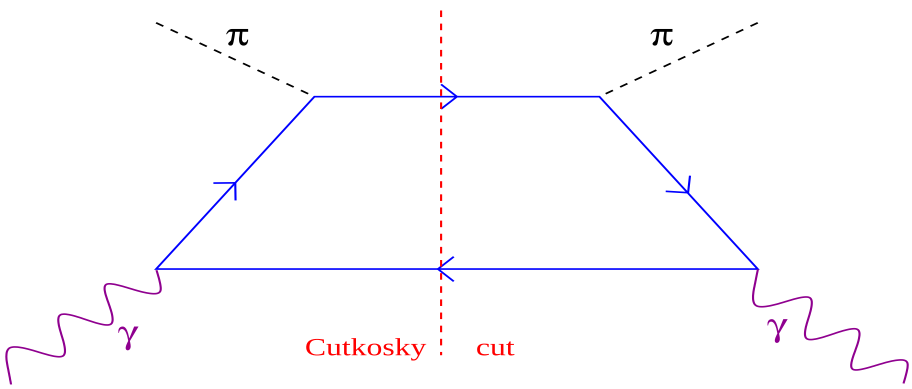
Bj : $q^2 \to -\infty$ with $x = -q^2/2p\cdot q$ fixed

⋆ Only single pion structure function:

$$\frac{1}{2\pi}\mathrm{Im}\, T_{\mu\nu}(p,q) \xrightarrow{\mathrm{Bj}} F(x)\left[-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} - \frac{1}{q^2}\left(p_\mu - \frac{q_\mu}{2x}\right)\left(p_\nu - \frac{q_\nu}{2x}\right)\right]$$

⋆ Usually: only handbag diagram

(expanding A_{NJL} to quadratic order in both π^a and v_μ)

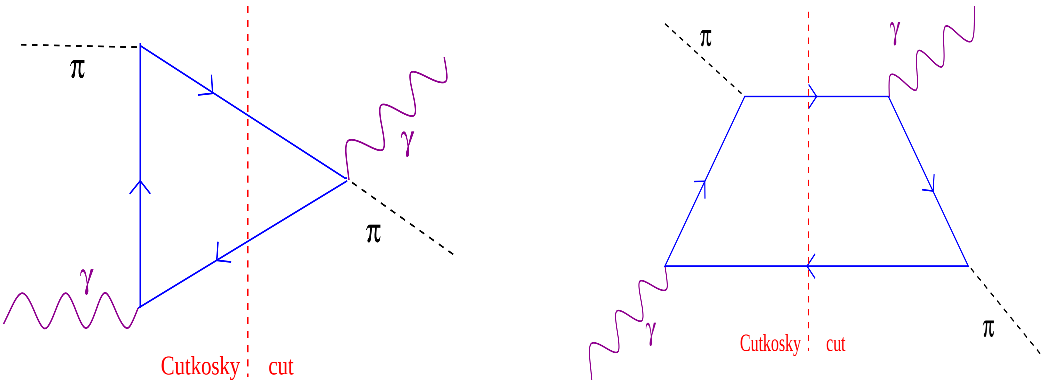


regularization induces
dimension–five operators

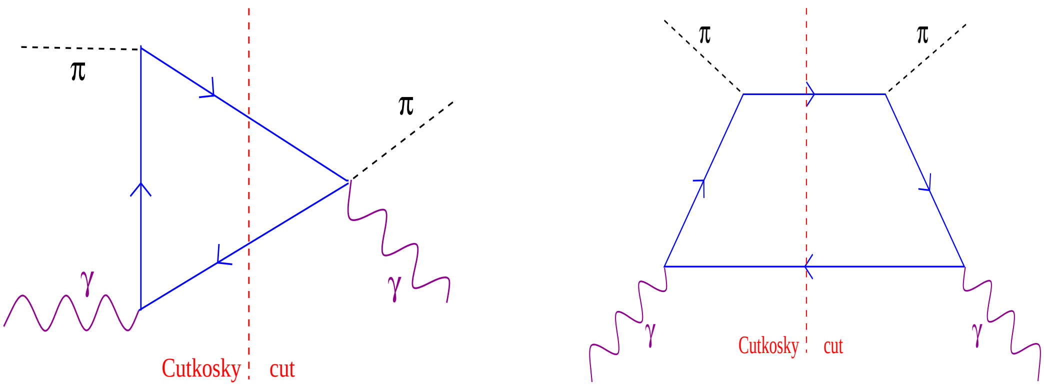
$$-\mathbf{DD}_5 = \partial^2 + m^2 + g\gamma_5\left[\not{\partial}, \vec{\pi}\cdot\tau\right] - i\left(\not{\partial}\not{v}\mathcal{Q} + \not{v}\mathcal{Q}\not{\partial}\right)\ldots$$

charge matrix $\mathcal{Q} = \mathrm{diag}\left(\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3}\right)$

⋆ Diagrams to compute:



• isospin violating: $\mathrm{tr}\left(\tau^a Q \tau^b Q\right)$



• isospin conserving: $\mathrm{tr}\left(\tau^a \tau^b Q Q\right)$

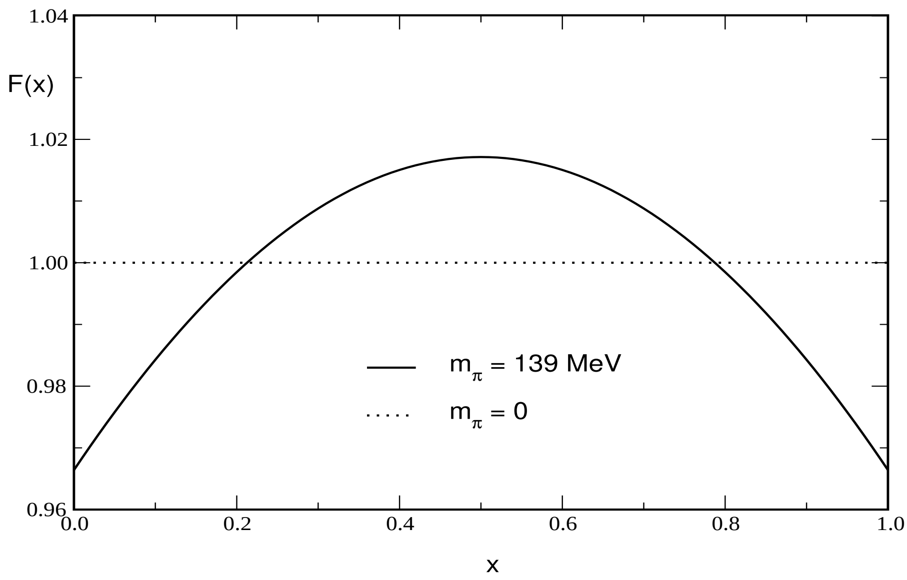
⋆ isospin violating pieces cancel in the Bjorken limit $\surd$

$$\boxed{F(x) = \frac{5}{9}(4N_C g^2)\frac{d}{dm_\pi^2}\left[m_\pi^2 \Pi(m_\pi^2, x)\right]}$$

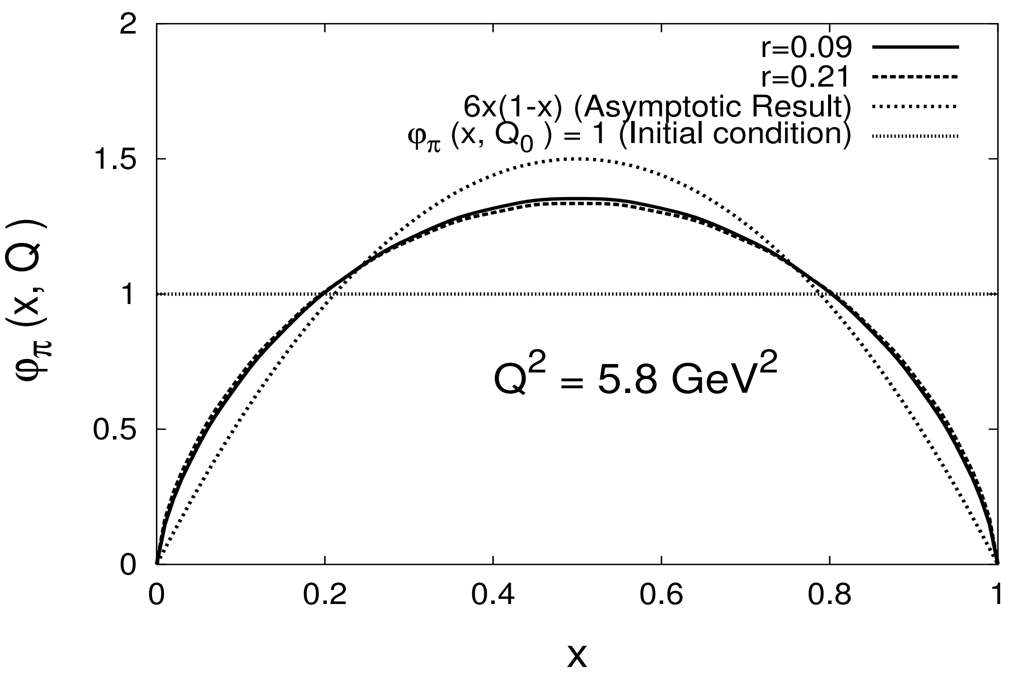
– correctly normalized

– $F(x) = \text{const.}$ for $m_\pi \to 0$ (chiral limit)

Numerical Results for F(x)

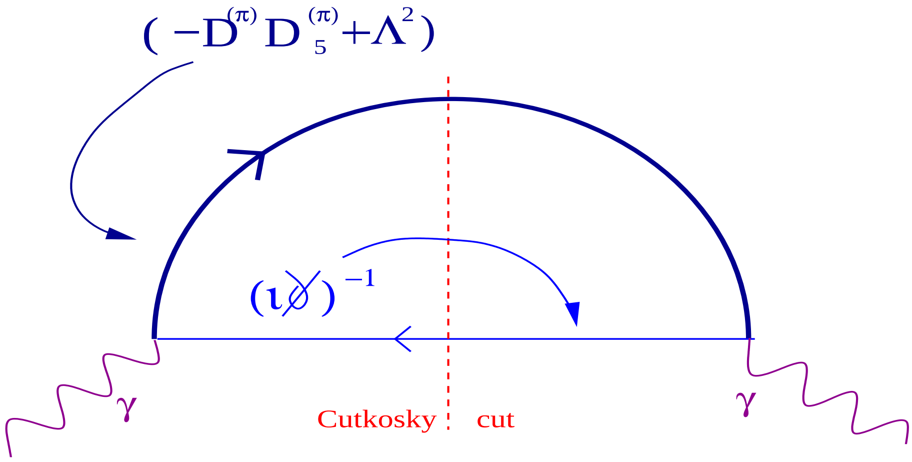


- DGLAP–evolution $(log(|q|/|q_0|)^2$ corrections)
  E. Ruiz Arriola & W. Broniowski, hep–ph/0207266



- evolution ratio: $r = \alpha_{\rm QCD}(|q|)/\alpha_{\rm QCD}(|q_0|)$

- ⋆ take Bjorken limit before expanding $\mathcal{A}_{\rm NJL}$ in pion fields
  - – dominating diagrams: large photon momentum ($|q^2| \to \infty$) goes through only one quark propagator
  - – this propagator is well approximated by the free, massless Dirac propagator
    no dependence on cut–off Λ $\longrightarrow$ scaling
  - – other propagator contains pion fields to all orders



- ⋆ Subsequent expansion of $(-\mathbf{D}^{(\pi)}\mathbf{D}_5^{(\pi)} - \Lambda_i^2 - i\epsilon)^{-1}$ to quadratic order in pion field $\longrightarrow$ previous result

- ⋆ Simplified computation also applicable to non–perturbative pion configurations: solitons

IV) Baryons as chiral solitons

⋆ static backgrounds (general framework)

$$i\not{D} = \gamma_0 \left(i\partial_t - h \right)$$

- eigenvalue problem: $\boxed{h\Psi_\nu = \epsilon_\nu \Psi_\nu}$
- ϵ_ν reflect the interaction with the meson fields that are contained in the Dirac–Hamiltonian h $\longrightarrow$ functionals of background fields

$$\begin{aligned} \mathrm{Det}(i\not{D}) &= \Pi_n \Pi_\nu \left(\frac{2n+1}{T} - \epsilon_\nu \right) \\ &= C(T)\, \mathrm{e}^{i\mathcal{A}_0} \sum_{\{\eta_\nu\}} \exp\left[i\mathcal{A}_V{}^{\{\eta_\nu\}} \right] \end{aligned}$$

- T: time interval to discretize eigenvalues of ∂_t
- occupation numbers: $\eta_\nu = 0, 1$
- valence quark part for a set $\{\eta_\nu\}$:

$$\mathcal{A}_V{}^{\{\eta_\nu\}} = -\tfrac{T}{2} \textstyle\sum_\nu \eta_\nu |\epsilon_\nu| = -T E_V$$

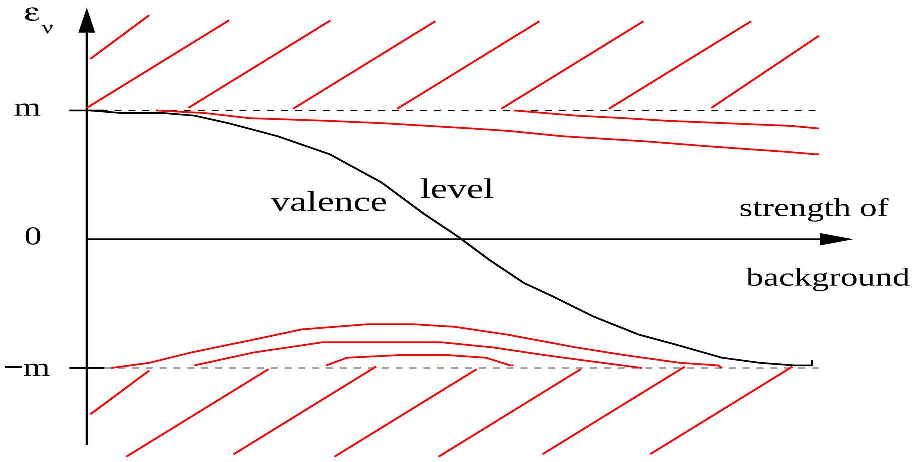
- UV divergent vacuum part:

$$\mathcal{A}_0 = \tfrac{T}{2} \textstyle\sum_\nu |\epsilon_\nu| = -T E_0$$

- baryon number for given set $\{\eta_\nu\}$:

$$B = \textstyle\sum_\nu \left(\eta_\nu - \tfrac{1}{2} \right) \mathrm{sign}(\epsilon_\nu)$$

⋆ typical Dirac spectrum:



⋆ energy functional in PV regularization:

$$
\begin{aligned}
E_{\rm tot} &= \frac{N_C}{2}\left(1-{\rm sign}(\epsilon_{\rm val})\right)\epsilon_{\rm val} \\
&\quad -\frac{N_C}{2}\sum_{i=0}^{2} c_i \sum_{\alpha}\left\{\sqrt{\epsilon_\alpha^2+\Lambda_i^2}-\sqrt{\epsilon_\alpha^{(0)2}+\Lambda_i^2}\right\}+E_{\rm local}
\end{aligned}
$$

⋆ adjust $\eta_{\rm val}=0,1$ such that $\underline{B=1.}$

⋆ $\epsilon_\alpha^{(0)}$ Dirac eigenvalues without background

⋆ $E_{\rm tot}$ is functional of background field.

⋆ Soliton = Minimum of $E_{\rm tot}$

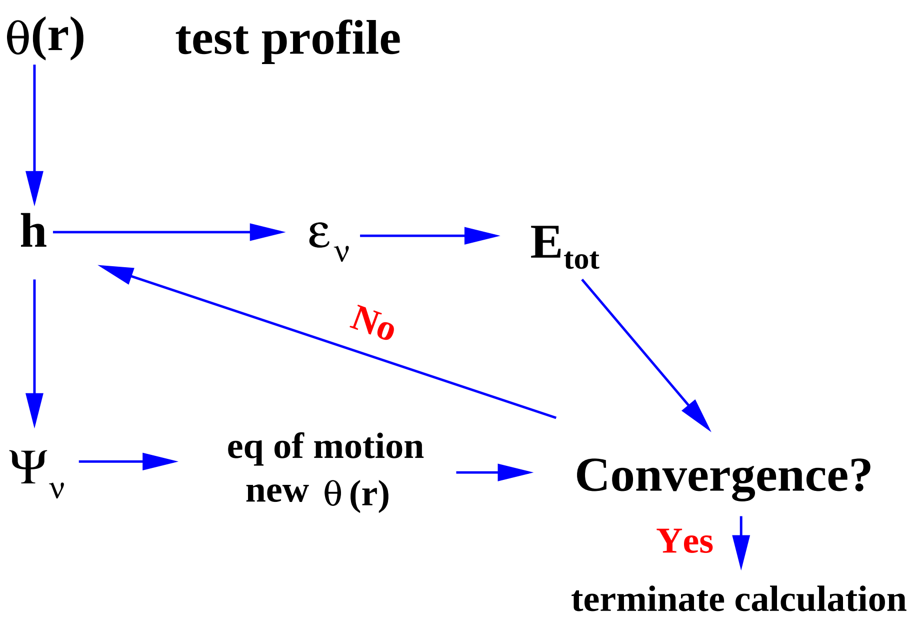
⋆ hedgehog ansatz

$$U(\vec{x}) = \exp\left(i\hat{r}\cdot\vec{\tau}\,\theta(r)\right) \qquad (r = |\vec{x}\,|)$$

$$h = \vec{\alpha}\cdot\vec{p} + \gamma_0\, m\,\left(\cos\theta(r) + i\gamma_5\,\hat{r}\cdot\vec{\tau}\sin\theta(r)\right)$$

⋆ iterative procedure for self–consistent solution

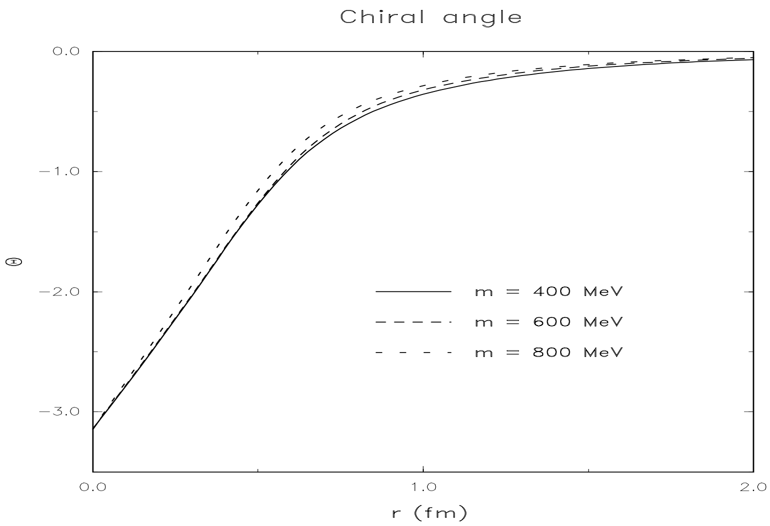
$$\theta(r) \sim \langle \Psi_\theta^\dagger(\vec{x})\Gamma\Psi_\theta^\dagger(\vec{x})\rangle$$



⋆ Numerical results

– soliton exists for $m \geq 325\text{MeV}$

– soliton true minimun for $m \geq 415\text{MeV}$



m	400	500	600
E_V	633	460	293
E_0	571	728	869
E_{local}	34	33	31
E_{tot}	1239	1221	1193

(numbers in MeV)

⋆ $\eta_{\text{val}} = 1$ for $m < 700\text{MeV}$

Connection to Skyrme type models

⋆ Skyrme type models

- soliton also of hedgehog form
- soliton build from meson clouds
  $i.e.\ \bar{q}q$ - pairs
- no valence degrees of freedom
- baryon number = topological charge W (winding number)

⋆ chiral quark (NJL) model soliton

- vacuum (polarization) contribution $\sim \bar{q}q$ - pairs
- valence contribution measured by $\eta_{\rm val} = 0, 1$
- baryon number $\eta_{\rm val} + B_{\rm vacuum}$
- gradient expansion $B_{\rm vacuum} = W$

⋆ extension of NJL model by vector mesons $\rho,\ \omega,\ a_1$

- self–consistent soliton: $\eta_{\rm val} = 0$
- polarized vacuum carries baryon number
- gradient expansion sensible, purely mesonic soliton
- phenomenological interpretation of $\rho,\ \omega,\ a_1$ fields:
  short range effects

Quantization of the chiral soliton

⋆ back to mesonic NJL chiral soliton: $\eta_{\rm val} = 1$

⋆ collective rotation

$$U(\vec{x}\,,t) = A(t)\,U_0(\vec{x}\,)\,A^\dagger(t) \qquad\qquad \underbrace{A(t) \in SU(2)}_{\text{collective coordinates}}$$

hedgehog

$$\mathcal{A}_{\rm NJL} = -T E_{\rm tot}[\theta] + \tfrac{T}{2}\,\alpha^2[\theta]\,\vec{\Omega}^{\,2} + \mathcal{O}\left(\vec{\Omega}^{\,4}\right)$$

moment of inertia

$$\underbrace{\vec{\Omega} = -i\mathrm{tr}\left[\vec{\tau}\,A^\dagger(t)\dot{A}(t)\right]}_{\text{angular velocities}}$$

⋆ spin operator

$$[\vec{x} \times \nabla, U] = \frac{\partial \dot{U}}{\partial \vec{\Omega}} \quad \Rightarrow \quad \vec{J} = \frac{\partial L}{\partial \vec{\Omega}} = \alpha^2[\theta]\,\vec{\Omega}$$

⋆ isospin operator

$$\text{hedgehog symmetry} \quad \Rightarrow \quad |\vec{J}\,| = |\vec{I}\,|$$

⋆ baryon masses (rigid top)

$$\boxed{M_\mathrm{B} = E_\mathrm{tot} + \frac{J(J+1)}{2\alpha^2} = E_\mathrm{tot} + \frac{I(I+1)}{2\alpha^2}}$$

⋆ $\alpha^2 \approx 5/\mathrm{GeV}$ for $420\mathrm{MeV} \lesssim m \lesssim 450\mathrm{MeV}$

⋆ finalizes parameter fixing

Further program

⋆ extract symmetry currents from coupling to external gauge fields v_μ , a_μ

⋆ Substitute rotating soliton

⋆ Employ quatization rules $\{A, \vec{\Omega}\} \longrightarrow \{\vec{J}, \vec{I}\}$

$$\Downarrow$$

Nucleon Form Factors

Reference on applications

(certainly incomplete)

⋆ axial charges, magnetic moments

M. Wakamatsu, H. Yoshiki, Nucl. Phys. **A524** (91) 561

A. Blotz, M. Praszałowicz, K. Goeke, Phys. Lett. **B317** (93) 195

⋆ $1/N_C$ corrections to g_a, μ_V, (in)consistency with PCAC

M. Wakamatsu, T. Watabe, Phys. Lett. **B312** (93) 184

R. Alkofer, HW, Phys. Lett. **B319** (93) 1

C. Christov, *et al.* Prog. Part. Nucl. Phys. **37** (96) 91

⋆ coll. coord. treatment of strange degrees of freedom

$$A(t) \in SU(3)$$

flavor symmetry breaking, a la Yabu & Ando

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⋆ bound state treatment of strangeness
heavy quark approach, a la Callan & Klebanov

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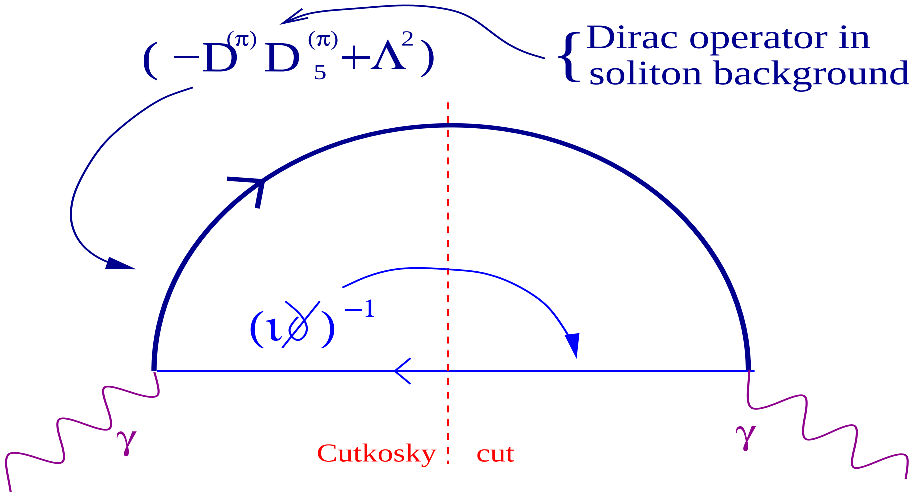
L. Gamberg, *et al.* , Phys. Rev. **D54** (96) 5812

Nucleon structure functions

⋆ absorptive part of Compton amplitude in Bjorken limit
here: to all orders in the pion field



⋆ scaling, sum rules √

⋆ hadronic tensor to leading order in $1/N_C$

$$
\begin{aligned}
W_{\mu\nu}(q) \;=\; & -iM_N \frac{N_C}{4} \int \frac{d\omega}{2\pi} \sum_\alpha \int d^3\xi \int \frac{d\lambda}{2\pi}\, \mathrm{e}^{iM_N x\lambda} \\
& \times \Big\langle N \Big| \Big\{ \Big[\bar{\Psi}_\alpha(\vec{\xi}) \mathcal{Q}_A^2 \gamma_\mu \not{n} \gamma_\nu \Psi_\alpha(\vec{\xi} + \lambda \hat{e}_3) \mathrm{e}^{-i\lambda\omega} \\
& \qquad - \bar{\Psi}_\alpha(\vec{\xi}) \mathcal{Q}_A^2 \gamma_\nu \not{n} \gamma_\mu \Psi_\alpha(\vec{\xi} - \lambda \hat{e}_3) \mathrm{e}^{i\lambda\omega} \Big] f_\alpha^+(\omega)\Big|_{\mathrm{pole}} \\
& + \Big[\bar{\Psi}_\alpha(\vec{\xi}) \mathcal{Q}_A^2 (\gamma_\mu \not{n} \gamma_\nu)_5 \Psi_\alpha(\vec{\xi} - \lambda \hat{e}_3) \mathrm{e}^{-i\lambda\omega} \\
& \qquad - \bar{\Psi}_\alpha(\vec{\xi}) \mathcal{Q}_A^2 (\gamma_\nu \not{n} \gamma_\mu)_5 \Psi_\alpha(\vec{\xi} + \lambda \hat{e}_3) \mathrm{e}^{i\lambda\omega} \Big] f_\alpha^-(\omega)\Big|_{\mathrm{pole}} \Big\} \Big| N \Big\rangle ,
\end{aligned}
$$

$n^\mu = (1,0,0,1)^\mu \qquad \mathcal{Q}_A = A^\dagger Q A \qquad$ Cutkosky rule

⋆ spectral functions

$$
f_\alpha^\pm(\omega) = \sum_{i=0}^{2} c_i \frac{\omega \pm \epsilon_\alpha}{\omega^2 - \epsilon_\alpha^2 - \Lambda_i^2 + i\epsilon} \pm \frac{\omega \pm \epsilon_\alpha}{\omega^2 - \epsilon_\alpha^2 + i\epsilon}
$$

⋆ without regularization

$$f^{+}_{\alpha}(\omega)\Big|_{\rm pole} = -4\pi i\delta(\omega-\epsilon_\alpha)\,, \qquad f^{-}_{\alpha}(\omega)\Big|_{\rm pole} = 0$$

$W_{\mu\nu}$ = sum of forward and backward distributions

⋆ unpolarized and polarized structure functions to leading order in $1/N_C$

$$\begin{aligned} f_1(x) &= \frac{5i}{36} M_N N_C \int \frac{d\omega}{2\pi} \sum_\alpha \int \frac{d\lambda}{2\pi}\, {\rm e}^{iM_N x\lambda} \left(\sum_{i=0}^{2} c_i \frac{\omega+\epsilon_\alpha}{\omega^2-\epsilon_\alpha^2-\Lambda_i^2+i\epsilon}\right)_{\rm pole} \\ &\quad \times \int d^3\xi\, \Big\{\Psi^\dagger_\alpha(\vec{\xi})\,(1-\alpha_3)\,\Psi_\alpha(\vec{\xi}+\lambda\hat{e}_3){\rm e}^{-i\omega\lambda} \\ &\qquad\qquad -\Psi^\dagger_\alpha(\vec{\xi})\,(1-\alpha_3)\,\Psi_\alpha(\vec{\xi}-\lambda\hat{e}_3){\rm e}^{i\omega\lambda}\Big\} \end{aligned}$$

$$\begin{aligned} g_1(x) &= \frac{M_N N_C}{36i}\Big\langle N\Big|I_3\Big|N\Big\rangle \int \frac{d\omega}{2\pi} \sum_\alpha \int \frac{d\lambda}{2\pi}\, {\rm e}^{iM_N x\lambda} \left(\sum_{i=0}^{2} c_i \frac{\omega+\epsilon_\alpha}{\omega^2-\epsilon_\alpha^2-\Lambda_i^2+i\epsilon}\right)_{\rm pole} \\ &\quad \times \int d^3\xi\, \Big[\Psi^\dagger_\alpha(\vec{\xi})\tau_3\,(1-\alpha_3)\,\gamma_5\Psi_\alpha(\vec{\xi}+\lambda\hat{e}_3){\rm e}^{-i\omega\lambda} \\ &\qquad\qquad +\Psi^\dagger_\alpha(\vec{\xi})\tau_3\,(1-\alpha_3)\,\gamma_5\Psi_\alpha(\vec{\xi}-\lambda\hat{e}_3){\rm e}^{i\omega\lambda}\Big] \end{aligned}$$

$$\begin{aligned} g_T(x) &= \frac{M_N N_C}{36i}\Big\langle N\Big|I_3\Big|N\Big\rangle \int \frac{d\omega}{2\pi} \sum_\alpha \int \frac{d\lambda}{2\pi}\, {\rm e}^{iM_N x\lambda} \left(\sum_{i=0}^{2} c_i \frac{\omega+\epsilon_\alpha}{\omega^2-\epsilon_\alpha^2-\Lambda_i^2+i\epsilon}\right)_{\rm pole} \\ &\quad \times \int d^3\xi\, \Big[\Psi^\dagger_\alpha(\vec{\xi})\tau_1\alpha_1\gamma_5\Psi_\alpha(\vec{\xi}+\lambda\hat{e}_3){\rm e}^{-i\omega\lambda} \\ &\qquad\qquad +\Psi^\dagger_\alpha(\vec{\xi})\tau_1\alpha_1\gamma_5\Psi_\alpha(\vec{\xi}-\lambda\hat{e}_3){\rm e}^{i\omega\lambda}\Big] \end{aligned}$$

⋆ 'baryon number' structure function

$$\begin{aligned} \bar{f}_1(x) &= -M_N \frac{N_C}{2} \sum_\alpha {\rm sign}(\epsilon_\alpha) \int d^3\xi \int \frac{d\lambda}{2\pi}\, {\rm e}^{iM_N x\lambda} \\ &\quad \times \Big[\Psi^\dagger_\alpha(\vec{\xi})\,(1-\alpha_3)\,\Psi_\alpha(\vec{\xi}+\lambda\hat{e}_3){\rm e}^{-i\epsilon_\alpha\lambda} + \Psi^\dagger_\alpha(\vec{\xi})\,(1-\alpha_3)\,\Psi_\alpha(\vec{\xi}-\lambda\hat{e}_3){\rm e}^{i\epsilon_\alpha\lambda}\Big] \end{aligned}$$

remains unregularized (from $\mathcal{A}_I$)!

Some numerical results

⋆ valence quark contributions dominate (saturate) spin dependent static nucleon properties (sum rules)

$m = 400\mathrm{MeV}$

leading terms in $1/N_C$

	valence	vacuum	val/tot
α^2	4.55/GeV	1.26/GeV	78%
g_A	0.73	0.03	96%
$g_A^{(0)}$	0.499	0.003	99%
$\Gamma_T^{(0)}$	0.72	0.04	95%
$\Gamma_T^{(V)}$	0.86	0.01	99%
$\mu^{(V)}$	1.63	0.61	73%
$\mu^{(V)}$	0.62	0.04	98%

⋆ substitute valence quark wave–function $\Psi_V(\vec{x},t)$ with rotational corrections into $W_{\mu\nu}$

$$h_{\mathrm{rot}} = \tfrac{1}{2}\vec{\tau}\cdot\vec{\Omega} \quad \longrightarrow \quad \tfrac{1}{2\alpha}\vec{\tau}\cdot\vec{J}$$

$$\Psi_V(\vec{x},t) \;=\; \mathrm{e}^{-i\epsilon_V}\, A(t)\Phi_V(\vec{x})$$

$$\Phi_V(\vec{x}) \;=\; \psi_V(\vec{x}) + \tfrac{1}{2}\sum_{\alpha\neq V}\psi_\alpha(\vec{x})\,\frac{\langle\alpha|\vec{\tau}\cdot\vec{\Omega}\,|V\rangle}{\epsilon_V-\epsilon_\alpha}$$

eigenfcts. of h

⋆ transformation to infinite momentum frame (IMF)

– proper support $x \in [0,1]$
– mitigate transverse momentum effects
– contact with parton model

$$f_i(x) \longrightarrow \frac{1}{1-x} f\left(-\ln\left[1-x\right]\right)$$

Jacobian

rapidity of Lorentz boost

⋆ DGLAP evolution

– logarithmic corrections to leading twist pieces within pertubative QCD
– integrate $Q_{\rm model} \longrightarrow Q_{\rm expt.}$

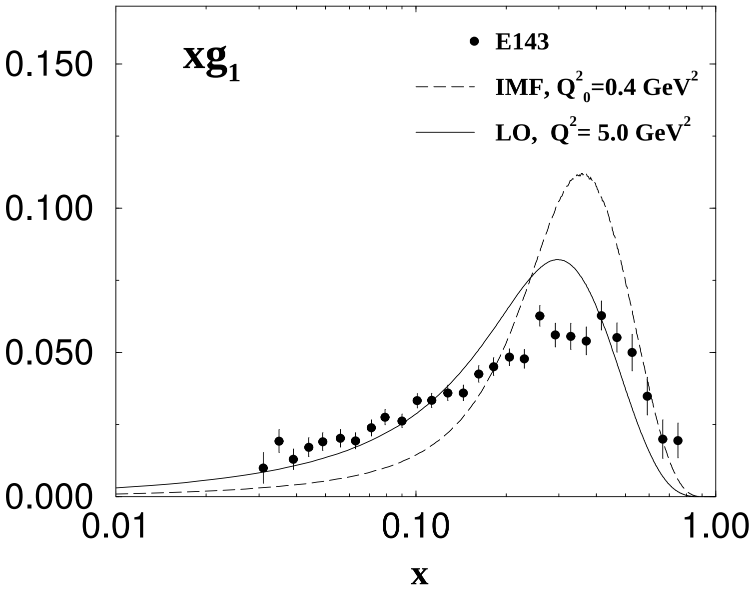
$$f_i(x, Q^2 + \delta Q^2) = f_i(x, Q^2) + \delta Q^2 \frac{df_i(x,Q^2)}{dQ^2}$$

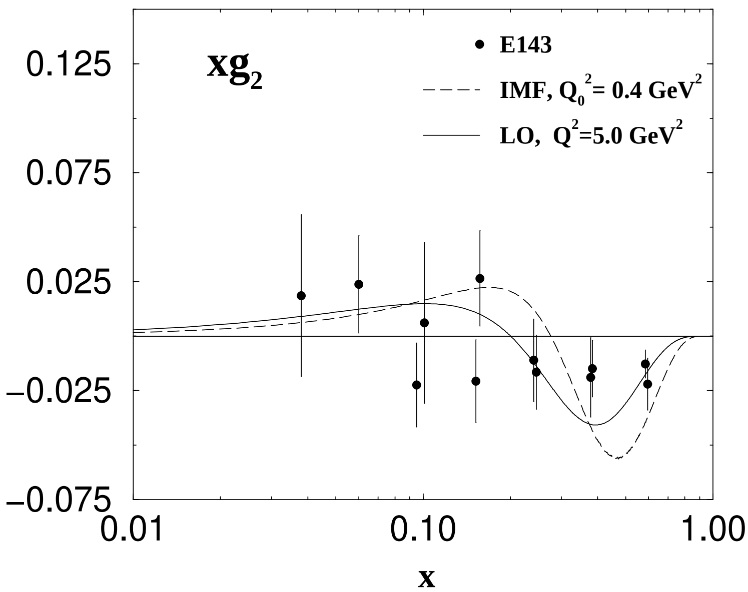
$$\frac{df_i(x,Q^2)}{dQ^2} \propto \alpha_{\rm QCD}(Q^2) \int_x^1 \frac{dy}{y} \mathcal{P}_{ij}(x) \, f_j\left(\frac{x}{y}, Q^2\right)$$

splitting fcts. from pQCD

⋆ determines $|Q_{\rm model}| \approx 0.7{\rm GeV}$

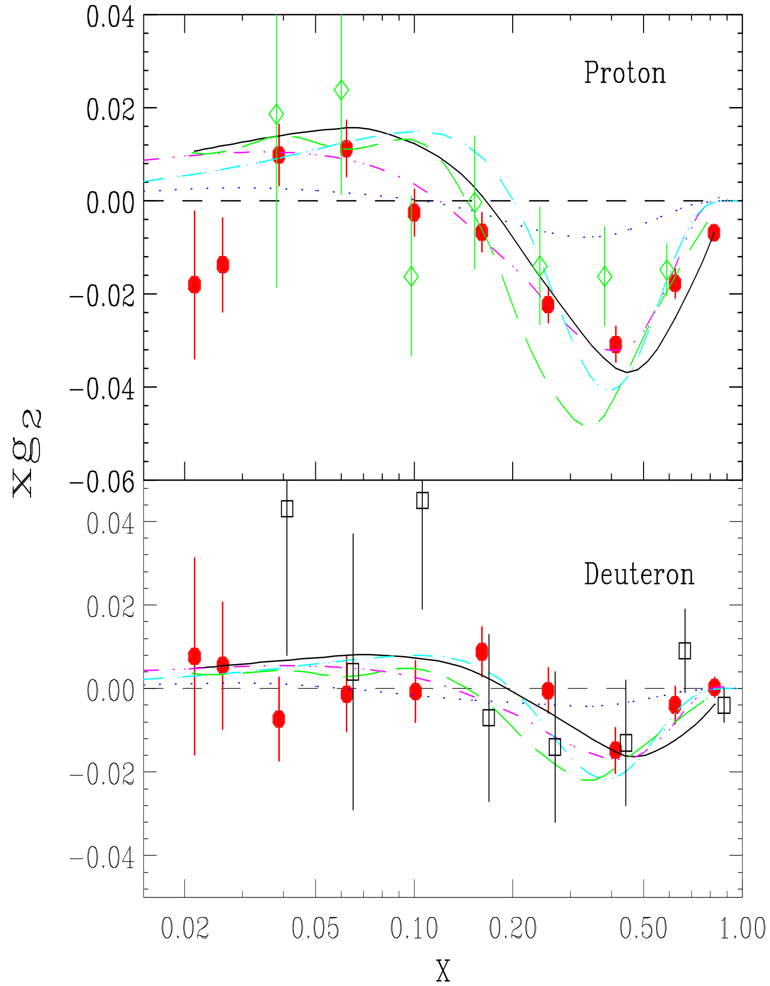
⋆ results for g_1 (proton) and g_2 (proton)
(E143: SLAC)





⋆ Q^2 evolution of $g_2 = g_T - g_1$ requires separation of twist–2 and twist–3 components
(Wandzura & Wilczek)

⋆ From E155X homepage



chiral quark model (Wakamatsu)

present model

bag model (Song)

bag model (Stratmann)

only twist−2 piece (Wandzura & Wilczek)

Related approaches

- Diakonov *et al.*, Nucl. Phys. **B480** (96) 341, Phys. Rev. **D56** (97) 4069
- Goeke *et al.*, Eur. Phys. J. **C14** (00) 147
- Wakamatsu *et al.*, Phys. Rev. **D60** (99) 034020, Phys. Lett. **B487** (00) 118

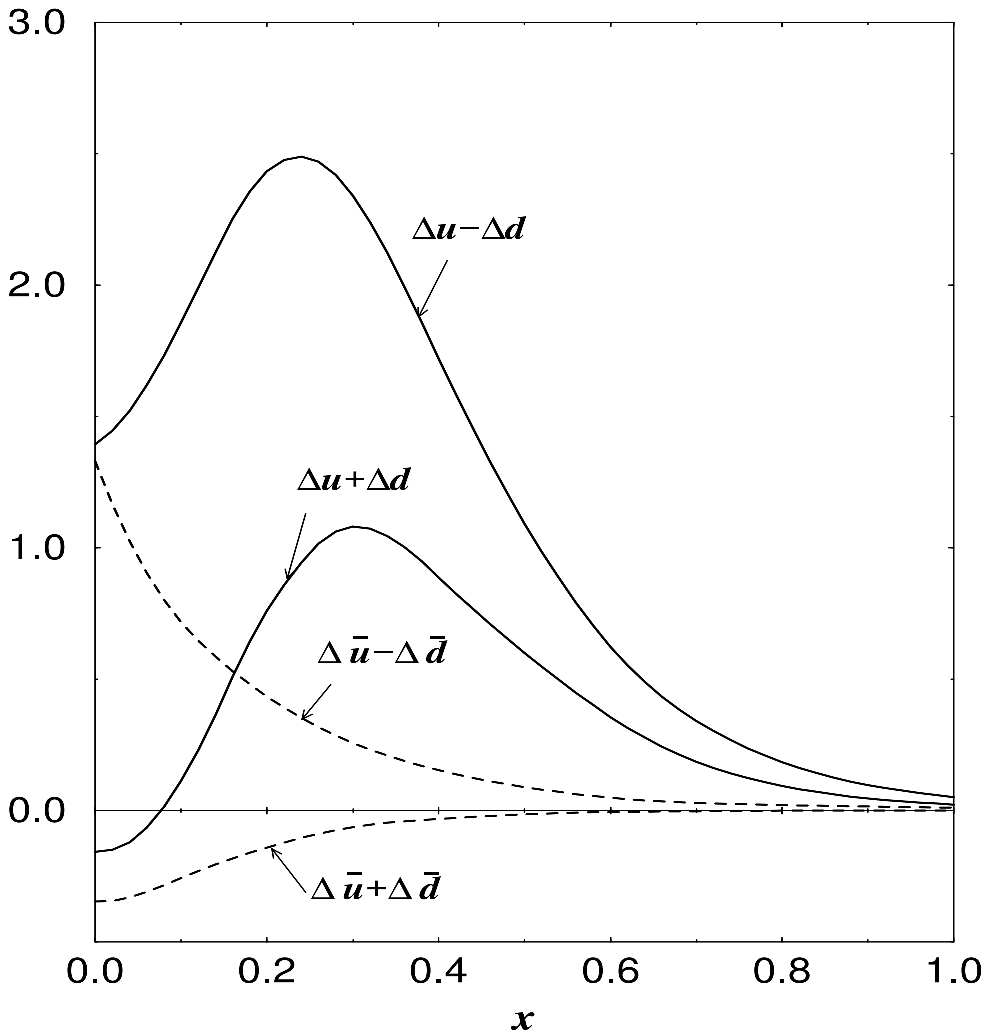
⋆ chiral quark soliton models

⋆ only a <u>single</u> PV–type subtraction
(works only in the chiral limit and on the chiral circle)

⋆ compute (regularized) <u>constituent</u> quark distributions
(including vacuum part $\sim$ Dirac sea)

⋆ interpretation as parton distributions
(identification of degrees of freedom,
not only of chiral symmetry)

Polarized *constituent* quark distributions



- ⋆ $\Delta u + \Delta d$: isoscalar combination
- ⋆ $\Delta u - \Delta d$: isovector combination
- ⋆ $\Delta \bar{u}(x) = \Delta u(-x)$, etc.: *anti–quark* distributions

(M. Wakamatsu, T. Kuboto, Phys. Rev. **D60** (99) 034020)

V) Conclusions

- ⋆ Large–N_C QCD $\longrightarrow$ effective meson theory
- ⋆ Chiral quark (NJL) model: simplified starting point to obtain effective meson theory
- ⋆ NJL model has dynamical chiral symmetry breaking: pion as (would–be) Goldstone boson
- ⋆ Structure functions from Compton amplitude (bosonization)
- ⋆ Regularization induces interactions
  PV-regularization special: sum rules, scaling
- ⋆ NJL–model chiral soliton: baryons
  applications:
  - hedgehog solution
  - quantization
  - static properties
  - structure functions
  - .....

  qualitatively good agreement with empirical data