NLO corrections to ultra-high energy neutrino-nucleon scattering, saturation and small x

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QCD 2002

IIT Kanpur, India

November 18-22

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Introduction

• An interaction of ultra high energy cosmic rays (UHECR) with either microwave background radiation or even the atmosphere would lead to generation of charged pions.

$$
\begin{aligned}
p\,(p/\gamma) \;\rightarrow\;& \pi^{\pm} + \text{anything} \\
& \hookrightarrow \mu\nu_\mu \\
& \qquad \hookrightarrow e\nu_e\nu_\mu\ .
\end{aligned}
\tag{1}
$$

• Decay of extremely massive primordial relic or even cosmic string

• Experimental detection of such neutrino fluxes is expected to provide insight

(1) origin of cosmic rays

(2) stellar structures

(3) probably to physics beyond standard model

• The interaction UHE-neutrino with matter is through deep inelastic scattering (DIS).

• The phenomenology of UHE-neutrino and their interction depends on the $\nu - N$ total cross section.

• It is more importantly to estimate more accurately of their interaction rate as well as their kinematic distributions.

• No data exists in the extreme region of the phase space (x, Q^2) to help in parametrisations of the parton distribution functions.

• Observation of the atmospheric neutrino flux with future neutrino telescope may probe parton distributions at much smaller x.

NLO corrections

- The general expression for the neutral current (NC) reaction

$$\nu_\mu(\bar\nu_\mu)\ N \to \nu_\mu(\bar\nu_\mu) + \textbf{anything},$$

is given by

$$\frac{d^2\sigma^{(\nu,\bar\nu)N}}{dxdy} = \frac{G_F^2}{2\pi}\left(\frac{M_Z^2}{Q^2+M_Z^2}\right)^2 s\ \Big[\mathsf{F}_2(1-y)+ \\ \mathsf{F}_1 xy^2 \pm \mathsf{F}_3 xy(1-\tfrac{1}{2}y)\Big] \qquad (2)$$

$G_F =$ Fermi constant ; $\qquad M_Z = Z$ boson mass.

$s =$ total CM energy ; $\qquad Q^2 = -\ q^2$

$x = Q^2/(2p.q)$; $\qquad y \equiv Q^2/(xs)$.

- The generic weak interaction vertex involving a quark f by $\gamma^\mu(c_V^f \pm c_A^f\gamma_5)$,

- The structure functions $\mathsf{F}_i \equiv \mathsf{F}_i(x,Q^2)$ $(i = 1,2,3)$ are given by

$$\mathsf{F_i}(x,Q^2) = {q_j}^{(0)} \otimes \hat{F}_i^{q_j} + \bar{q}_j^{(0)} \otimes \hat{F}_i^{\bar{q}_j} + g^{(0)} \otimes \hat{F}_i^{g},$$

where $\otimes$ denotes the convolution defined as

$${q_j}^{(0)} \otimes \hat{F}_i^{q_j} = \int_x^1 \frac{dy_p}{y_p}\ {q_j}^{(0)}(y_p)\ \hat{F}_i^{q_j}(\frac{x}{y_p}).$$

- For the quark initiated process, to order α_s, the partonic structure functions are given by

$$\hat{F}_i^{q}(z,Q^2) = K_i \left\{\delta(1-z) + \alpha_s\, (\frac{Q^2}{\mu^2})^{\frac{n}{2}-2} \Big[\ \zeta\ P_{qq}(z) + f_i^q(z)\ \Big]\right\}.$$

$$\begin{aligned} f_i^q(z) = {} & \frac{2}{3\pi}\Big[(1+z^2)\ (\frac{\ln(1-z)}{(1-z)})_+ - \frac{(1+z^2)}{(1-z)}\ \ln(z) \\ & -\frac{3}{2}\,\frac{1}{(1-z)_+} + 3 + 2z - \\ & \delta(1-z)(\frac{9}{2} + \frac{\pi^2}{3})\ \Big] + \Delta_i^q, \end{aligned}$$

with

$$\Delta_1^q = -\frac{2}{3\pi}K_1[2z]; \qquad \Delta_2^q = 0;$$

$$\Delta_3^q = -\frac{2}{3\pi}K_3[1+z],$$

and

$$P_{qq}(z) = \frac{2}{3\pi}\left[\frac{(1+z^2)}{(1-z)_+} + \frac{3}{2}\delta(1-z)\right]$$

$$\zeta = \left[\frac{2}{n-4} + \gamma - \ln(4\pi)\right].$$

• The UV singularities is removed by usual renormalisation procedure and IR singularities cancel between the real and virtual diagrams, leaving only the collinear (mass) divergence.

• The $\overline{MS}$ counter term for the quark process is given by

$$\hat{F}_i^{(q,ct)}(z, M^2) = K_i\ \alpha_s\ (\frac{M^2}{\mu^2})^{\frac{n}{2}-2}\ \left[\ -\zeta\ P_{qq}(z)\ \right]$$

where M is the factorization scale.

• For the anti-quark initiated processes, all the structure functions are same except for $\hat{F}_3^q(z, Q^2)$ which appears with the opposite sign in eqn.(2) .

• For the gluon initiated process, on the other hand, the parton structure functions are given by

$$\hat{F}_i^g(z,Q^2) \;=\; K_i\ \alpha_s\ (\frac{Q^2}{\mu^2})^{\frac{n}{2}-2}\ \Big[\ \zeta\ P_{gq}(z) + f_i^g(z)\ \Big].$$

where

$$f_i^g(z) \;=\; \frac{1}{4\pi}\Big[\{(1-z)^2+z^2\}\ \ln(\frac{1-z}{z}) + 6\ z\ (1-z)\Big] + \Delta_i^g$$

with

$$P_{gq}(z) = \frac{1}{4\pi}\Big[(1-z)^2+z^2\Big];$$

and

$$\Delta_1^g = \frac{1}{4\pi}\Big[4z(1-z)\Big]; \qquad \Delta_2^g = 0; \qquad \Delta_3^g = 0.$$

• The $\overline{MS}$ counter term for the gluon initiated process is given by

$$\hat{F}_i^{(g,ct)}(z,M^2) = K_i\ \alpha_s\ (\frac{M^2}{\mu^2})^{\frac{n}{2}-2}\ \Big[-\zeta\ P_{gq}(z)\ \Big]$$

• The Q^2 dependent structure function therefore is given by

$$\mathsf{F}_2(x,Q^2) \equiv \sum_{j=1}^{n_f} K_{q_j}^{(pc)}\; x\; \Big(q_j{}^{(2)}(x,Q^2) + \bar{q}_j^{(2)}(x,Q^2) \Big);$$

$$\begin{aligned} \mathrm{q}_\mathrm{j}^{(\mathrm{i})}(x,Q^2) \;=\; & \int_x^1 \frac{dy_p}{y_p}\; q_j{}^{(0)}(y_p)\; \hat{F}_i^q\big(\frac{x}{y_p},Q^2\big) + \\ & \int_x^1 \frac{dy_p}{y_p}\; g^{(0)}(y_p)\; \hat{F}_i^g\big(\frac{x}{y_p},Q^2\big), \end{aligned}$$

where $\mathrm{q}_\mathrm{j}^{(\mathrm{i})}(x,Q^2)$ are the renormalised parton distributions, $z = \frac{x}{y_p}$ and $i = 1,2,3$ corresponds to the three structure functions.

- The general expression for the charged current (CC) reaction

$$\nu_\mu(\bar{\nu}_\mu)N \rightarrow \mu^\pm + \textbf{anything},$$

is given by

$$\frac{d^2\sigma^{(\nu,\bar{\nu})N}}{dxdy} = \frac{2G_F^2}{\pi}\Big(\frac{M_W^2}{Q^2+M_W^2}\Big)^2 s\,\Big[\mathsf{F}_2(1-y) + \mathsf{F}_1 xy^2 \pm \mathsf{F}_3 xy(1-\frac{1}{2}y)\Big]$$

- All the notation being the same as NC except that the couplings are $K_1 = \frac{1}{4}, K_2 = K_3 = \frac{1}{2}$.

Parton Distributions

- Deep inelastic scattering (DIS) of UHE-neutrinos is unique in the sense that it explores extreme regions of the (Q^2, x) phase space.

- In the region between, say, $50\ \text{GeV}^2 < Q^2 < 10^5\ \text{GeV}^2$ standard parametrisations are expected to work reasonably well. All parametrisations in these regions use functions of the form $Ax^\alpha(1-x)^\beta f(\sqrt{x})$ where the last function is some polynomial in $\sqrt{x}$.

- The region above $Q^2 = 10^6\ \text{GeV}^2$ and low x, $x < 10^{-5}$ is beyond the region of validity of GRV. Here we use the Double Asymptotic Scaling (DAS) matched appropriately at the boundary with GRV.

- In the large $x > 10^{-5}$ and large $Q^2 > 10^6\ \text{GeV}^2$ region, we have used GRV with a fixed value of $Q^2 = 10^6\ \text{GeV}^2$.

• In DIS at very low x and low Q^2, we have used saturated densities.

• The structure function $F_2(x, Q^2)$ rises strongly at small x due to sea and gluon parton distributions for fixed Q^2 and hence cross-section violates unitarity.

• This can be interpreted as some important physical effect may be neglected by approximation in linear evolution of DGLAP Eqn.

• The linear evolution of DGLAP equation allow the scale dependent (logarithmic) in the structure functions in the pQCD. Now the question is how low Q^2 and low x, DGLAP equation is applicable and should be determined from the analysis of DIS-data.

• In principle the value of x and Q^2 indicating that boundary are correlated i.e. $Q^2 = Q_s^2(x)$.

Model

• The dipole cross-section is given by (which assures unitarity)

$$\hat{\sigma}(x,r) = \sigma_0 \left[1 - \exp\left(\frac{-r^2}{4R_0^2(x)}\right)\right]$$

where $R_0^2(x)$ is called the saturation radius given by

$$R_0^2(x) = \left(1 GeV\right)^{-1} \left(\frac{x}{x_0}\right)^{\frac{\lambda}{2}}$$

• The three parameters $\sigma_0 = 23$ mb (energy independent), $\lambda = 0.3$, and $x_0 = 3 \times 10^{-4}$ were fitted to all small x DIS data with $x < 10^{-2}$.

• At small r, $\hat{\sigma}$ features colour transparency, $\hat{\sigma} \sim r^2$ which is the pQCD phenomenon. At large r, $\hat{\sigma} \sim \sigma_0$.

- The gluon densities become so large that $gg \to g$ recombination processes become important. The regime in Q^2, x in which this happen is referred as "saturation region".

- Therefore the decreasing x the saturation radius $R_0^2(x)$ also decreases and consequently $\hat{\sigma}$ never exceeds σ_0. This is the way infra-red diffusion , which concerns large diploe is tamed by existence of saturetion scale:

$$Q_s^2(x) \equiv \frac{1}{R_0^2(x)} \sim x^{-\lambda}$$

Results and Discussion

- Figure 1 shows the numerical size of NLO corrections. This is for the isoscalar target.

- The K-factor is defined as

$$K = \frac{\sigma_{\text{NLO}}}{\sigma_{\text{LO}}}$$

- Figure 2 shows in region $E_\nu < 10^4$ that K-factor is almost one and rises between the energy range $10^4 < E_\nu \leq 10^6$, then again it falls below one.

- For low neutrino energies, the maximal energy transfer falls well short of M_Z and thus the neutrino-parton interaction is well described in terms of a $4-$fermi interaction.

- $s \sim 10^4$ GeV^2 , the natural scale of the problem, the contribution to the cross section is naturally dominated by virtual exchanges with $Q^2 \sim M_Z^2$.

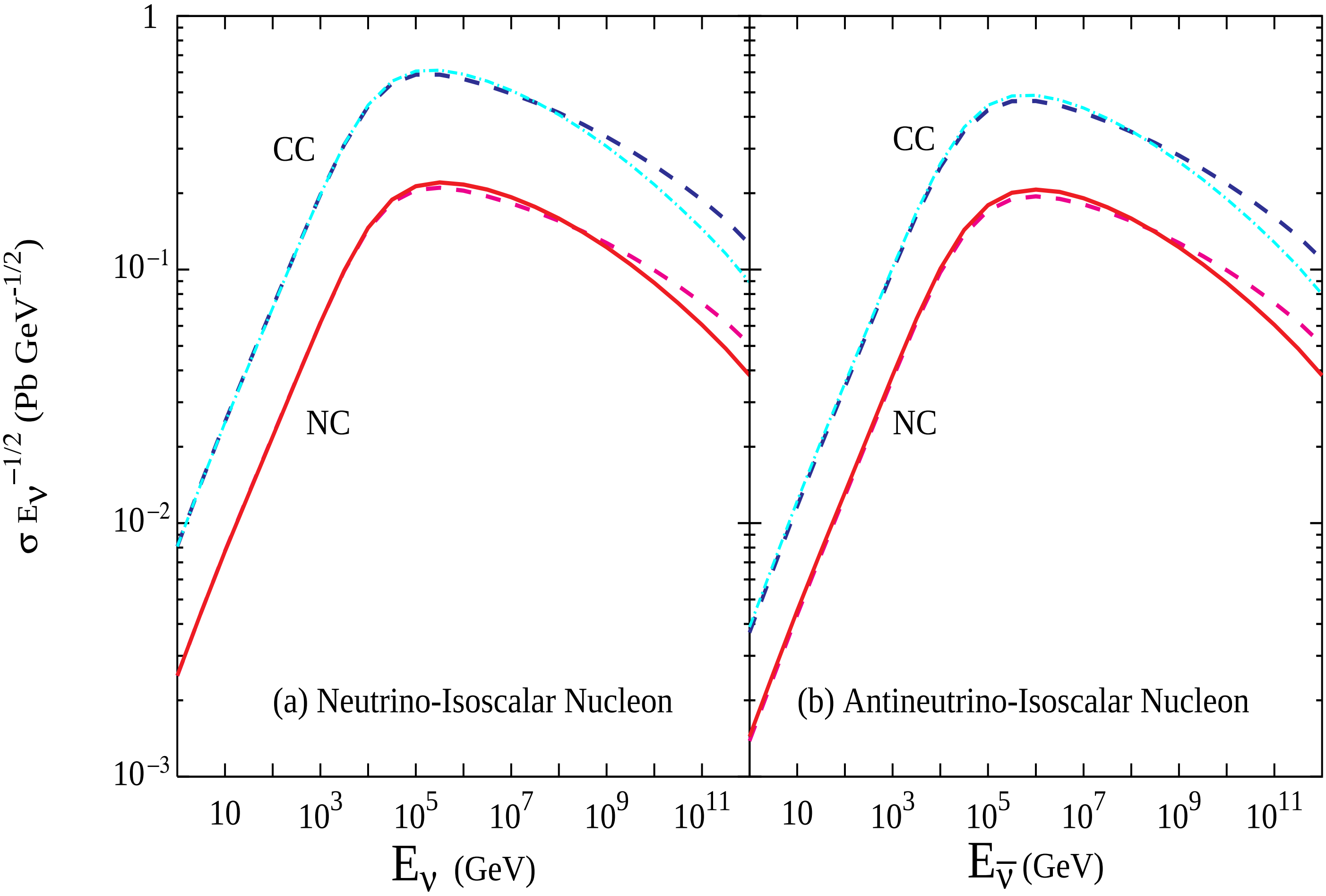
- The fast fall of the parton densities at large x, implies that such regions of the phase space should contribute very little to the cross section.

- In the region $\underline{Q^2 \sim M_Z^2}$ (and, hence, relatively small x) receives prominence in the integration. i.e. the maximum in distribution is far from $x \leq 1$.

- The normalized differential distributions, the LO and NLO curves for $\mathrm{d}\sigma/\mathrm{d}\log Q^2$, for a given E_ν, would be virtually indistinguishable (Figure3) from each other, while those for $\mathrm{d}\sigma/\mathrm{d}\log x$ would seem laterally displaced, with the effect more pronounced for larger E_ν (Figure 4).

- The main reason for the fall is that while at large E_ν and hence x is small, the partonic distributions are large, the rise of the distributions is much steeper in LO than in NLO.

Figure 1



CC
NC
(a) Neutrino-Isoscalar Nucleon
(b) Antineutrino-Isoscalar Nucleon
$\sigma\, E_\nu^{-1/2}$ (Pb GeV$^{-1/2}$)
E_ν (GeV)
$E_{\bar{\nu}}$ (GeV)
1
10^{-1}
10^{-2}
10^{-3}
10
10^3
10^5
10^7
10^9
10^{11}

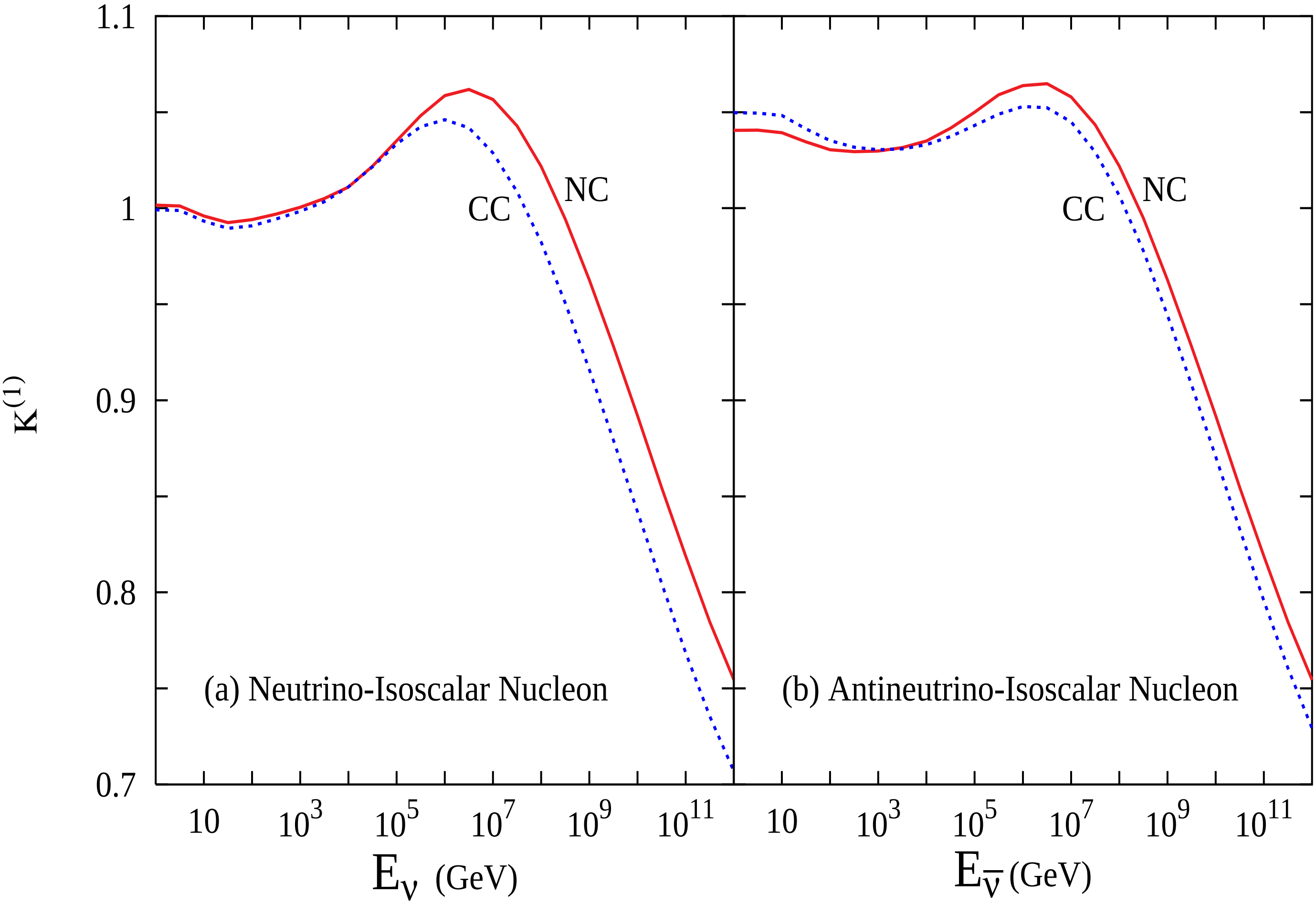
- The total cross-section in terms of neutrino energy can be expressed as

$$\sigma(\text{pb}) \approx \mathcal{A}\left(\frac{E_\nu}{1\ \text{GeV}}\right)^{\gamma}.$$

- In the different regions, the value of $\mathcal{A}$ and γ are given in the table 1.

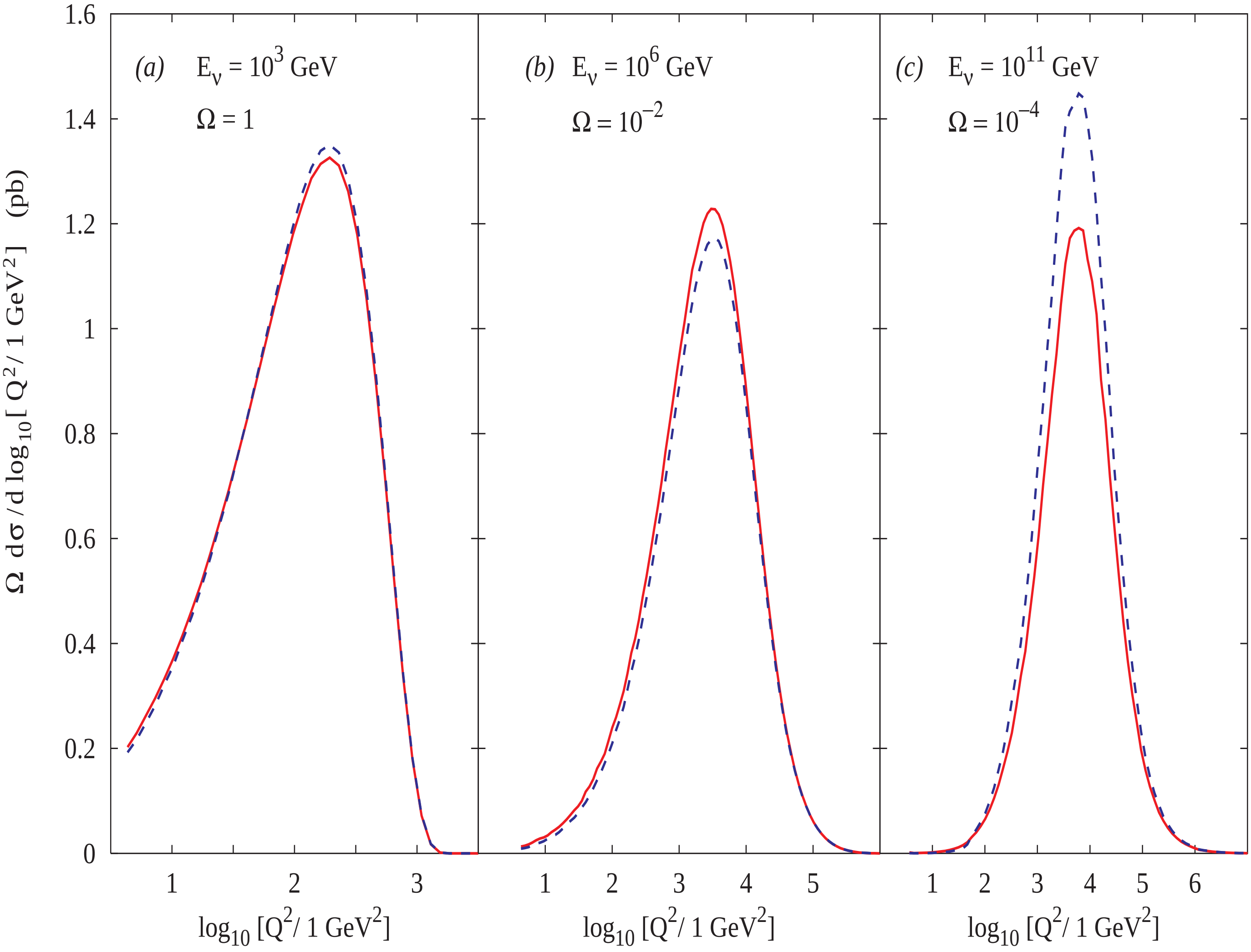
Process	Order	$1\ \mathrm{GeV} < E_\nu < 10^4\ \mathrm{GeV}$		$10^7\ \mathrm{GeV} < E_\nu < 10^{11}\ \mathrm{GeV}$	
		$\mathcal{A}$	γ	$\mathcal{A}$	γ
ν-NC	LO	2.72×10^{-3}	0.945	1.17	0.390
	NLO	2.71×10^{-3}	0.945	2.06	0.360
$\bar{\nu}$-NC	LO	1.44×10^{-3}	0.964	1.16	0.391
	NLO	1.52×10^{-3}	0.963	2.06	0.360
ν-CC	LO	8.89×10^{-3}	0.940	3.57	0.383
	NLO	8.82×10^{-3}	0.941	6.50	0.349
$\bar{\nu}$-CC	LO	3.95×10^{-3}	0.961	2.85	0.386
	NLO	4.15×10^{-3}	0.960	5.18	0.353

Figure 2



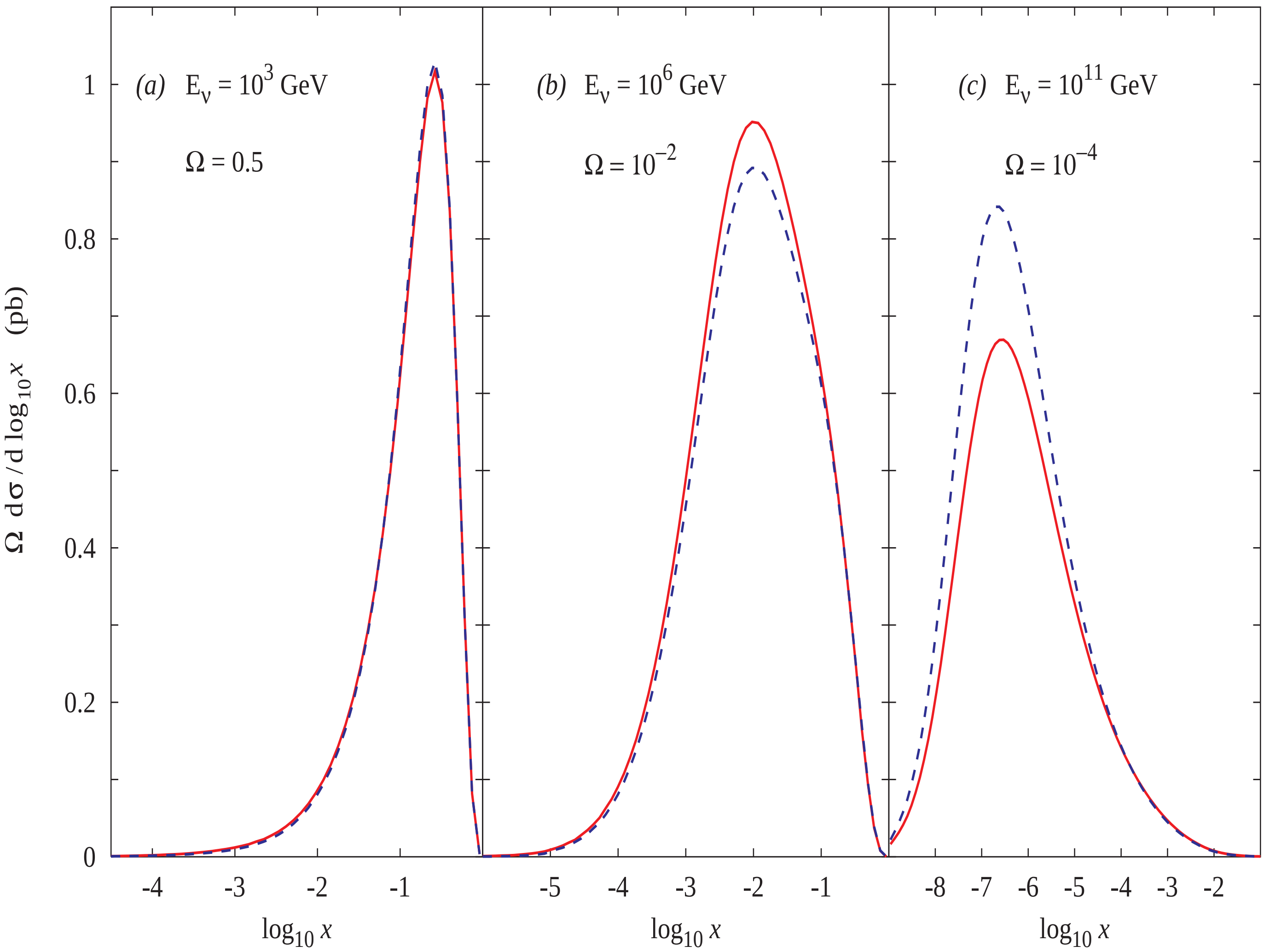
1.1
1
0.9
0.8
0.7
$K^{(1)}$
NC
CC
(a) Neutrino-Isoscalar Nucleon
(b) Antineutrino-Isoscalar Nucleon
10
10^3
10^5
10^7
10^9
10^{11}
E_ν (GeV)
$E_{\bar{\nu}}$ (GeV)

Figure 3



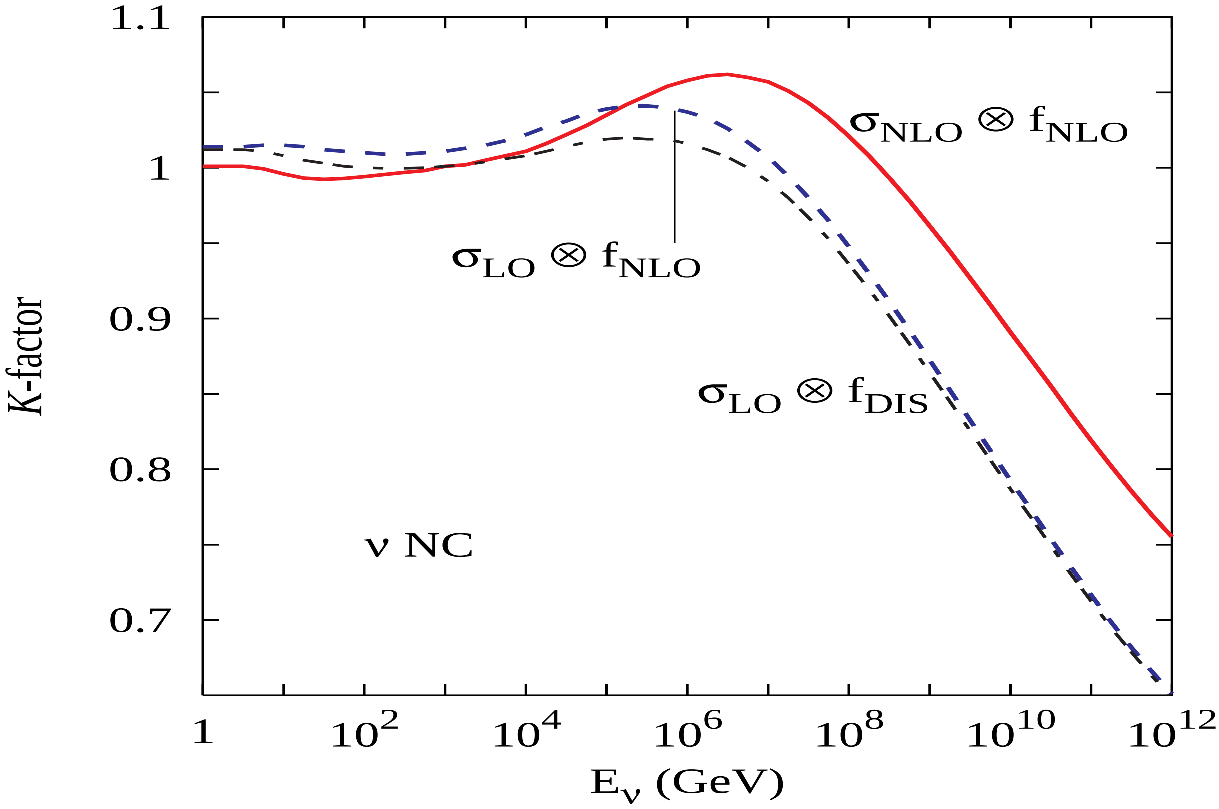
(a) $E_\nu = 10^3$ GeV
$\Omega = 1$
(b) $E_\nu = 10^6$ GeV
$\Omega = 10^{-2}$
(c) $E_\nu = 10^{11}$ GeV
$\Omega = 10^{-4}$
$\Omega\ d\sigma / d \log_{10}[Q^2/1\ \mathrm{GeV}^2]$ (pb)
$\log_{10}[Q^2/1\ \mathrm{GeV}^2]$
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1
2
3
4
5
6

Figure 4



(a) $E_\nu = 10^3$ GeV
$\Omega = 0.5$
(b) $E_\nu = 10^6$ GeV
$\Omega = 10^{-2}$
(c) $E_\nu = 10^{11}$ GeV
$\Omega = 10^{-4}$
$\Omega\ d\sigma / d \log_{10} x$ (pb)
0
0.2
0.4
0.6
0.8
1
-4
-3
-2
-1
-5
-4
-3
-2
-1
-8
-7
-6
-5
-4
-3
-2
$\log_{10} x$
$\log_{10} x$
$\log_{10} x$

Figure 5



1.1
1
0.9
0.8
0.7
K-factor
$\sigma_{NLO} \otimes f_{NLO}$
$\sigma_{LO} \otimes f_{NLO}$
$\sigma_{LO} \otimes f_{DIS}$
ν NC
1
10^2
10^4
10^6
10^8
10^{10}
10^{12}
E_ν (GeV)

Summary

- We have calculated the NLO corrections to UHE-neutrino-nucleon scattering and compared result with LO cross section.

- We identified the various regions in the $x - Q^2$ space where different parametrisations of the quark and gluon densities are valid and discuss their contributions in various regions.

- From our analysis, NLO calculations are necsssary for UHE-neutrino isoscalar scattering processes to get a complete picture.

- Finally it is possible to look at the NNLO (α_s^2) to see what effect this has at least upto values $x \sim 10^{-4}$ for future study.