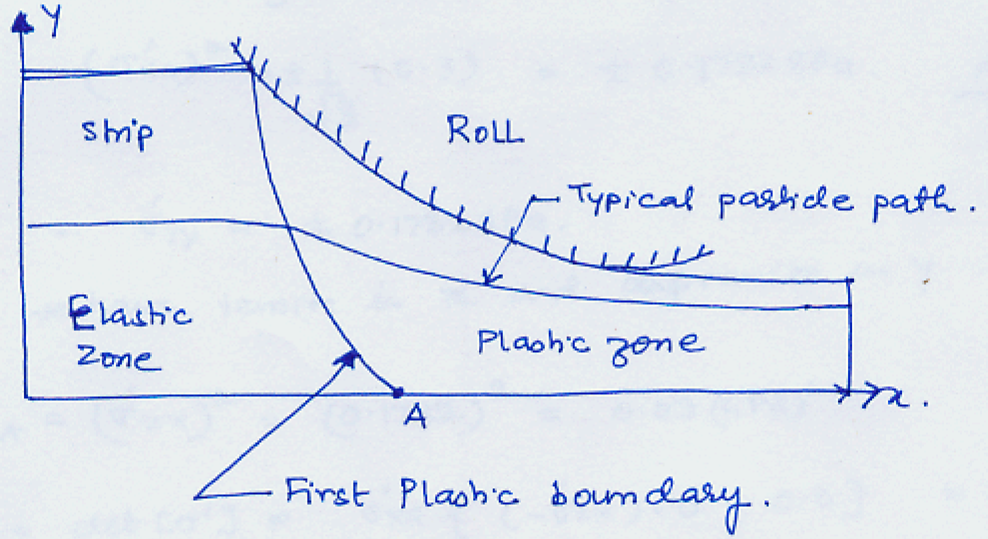
Problem 1 :



Given: $\sigma_y = 0.3$ GPa.

Material yields according to von Mises criterion.

Now $[\sigma'] = \begin{bmatrix} \sigma'_{xx} & \sigma'_{xy} & 0 \\ \sigma'_{xy} & \sigma'_{yy} & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

a) at Point A : $\sigma'_{xy} = 0$. $\therefore [\sigma']_A = \begin{bmatrix} \sigma'_{xx} & 0 & 0 \\ 0 & \sigma'_{yy} & 0 \\ 0 & 0 & 0 \end{bmatrix}$ — (1)

Now ~~do~~ $tr[\sigma']_A = 0$ ($\because$ deviatoric ~~compo~~ ~~nets~~ parts trace is zero).

$\therefore \sigma'_{xx} + \sigma'_{yy} = 0$

$\therefore \sigma'_{xx} = -\sigma'_{yy}$ — (2)

Using this in (1)

$$[\sigma']_A = \begin{bmatrix} +\sigma'_{xx} & 0 & 0 \\ 0 & -\sigma'_{xx} & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Now according to von Mises criterion

$J_2 - \frac{1}{3}\sigma_y^2 = 0$ at plastic points. — (3)

Now $J_2|_A = \frac{1}{2}\sigma'_{ij}\sigma'_{ij} = \frac{1}{2}\left[\sigma'_{xx}\sigma'_{xx} + (-\sigma'_{xx})(-\sigma'_{xx})\right]$

$= (\sigma'_{xx})^2$ — (4)

Substituting (4) and $\sigma_y = 0.3$ in (3) we get

$$(\sigma'_{xx})^2 - \frac{1}{3}(0.3)^2 = 0$$

$$(\sigma'_{xx}) = \pm\frac{1}{\sqrt{3}}(0.3) = \pm 0.1732\,GPa. \quad \underline{Ans}.$$

$$\therefore \sigma'_{yy} = \mp 0.1732\,GPa.$$

$\because$ at A we have tension in x and compression in y $\therefore$ $\sigma'_{xx} = 0.1732$ GPa, $\sigma'_{yy} = -0.1732$ GPa

b). $$J_2|_A = (\sigma'_{xx})^2 = (0.1732)^2 = 0.03\,(GPa)^2$$

$$J_3|_A = \det[\sigma']_A = \sigma'_{xx}\cdot[(-\sigma'_{xx})\cdot 0 - 0.0] = 0.$$

Tresca ~~criterion~~ yield function is

$$f(J_2, J_3) \equiv 4\left(J_2 - \frac{\sigma_y^2}{4}\right)(J_2 - \sigma_y^2)^2 - 27J_3^2$$

substituting values

$$f(J_2, J_3)|_A = 4\left(0.03 - \frac{0.09}{4}\right)(0.03 - 0.09)^2 - 0$$

$$f(J_2, J_3)|_A = 1.8\times 10^{-4} > 0.$$

$\because$ $f(J_2, J_3)|_A$ is positive $\therefore$ Point A is in plastic zone according to Tresca criterion.