Problem 1: axisymmetric problem, velocity component

\[ v_r = -r (A_3 + B) \quad v_\theta = 0 \quad v_z = A \left( \frac{r^2 + z^2}{2} \right) + 2Bz. \]

\( A, B \) constants.

a) \( \dot{\varepsilon}_{rr} = \frac{\partial v_r}{\partial r} = - (A_3 + B) \)

\( \dot{\varepsilon}_{\theta\theta} = \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r} = 0 + \left[ - (A_3 + B) \right] = - (A_3 + B) \)

\( \dot{\varepsilon}_{zz} = \frac{\partial v_z}{\partial z} = 2A_3 + 2B = 2 (A_3 + B) \)

\( \dot{\varepsilon}_{r\theta} = \dot{\varepsilon}_{\theta r} = 0 \)

\( \dot{\varepsilon}_{zz} = \frac{1}{2} \left( \frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z} \right) = \frac{1}{2} \left( A_3 - A_3 \right) = 0. \)

b) Volumetric strain rate \( \dot{\varepsilon}_V = \dot{\varepsilon}_{rr} + \dot{\varepsilon}_{\theta\theta} + \dot{\varepsilon}_{zz} \)

\[ \begin{align*}
\dot{\varepsilon}_V &= - (A_3 + B) - (A_3 + B) + 2 (A_3 + B) \\
&= - 2(A_3 + B) + 2(A_3 + B) \\
&= 0.
\end{align*} \]

c) \( du = u dt \)

\[ \begin{align*}
\Delta u_r &= u_r dt \\
\Delta u_\theta &= u_\theta dt \\
\Delta u_z &= u_z dt.
\end{align*} \]

\[ \begin{align*}
\Delta u_r &= -r(A_3 + B) dt \\
\Delta u_\theta &= 0 dt \\
\Delta u_z &= A \left( \frac{r^2 + z^2}{2} \right) + 2Bz dt.
\end{align*} \]

d) The incremental strain displacement relation in cylindrical coordinates.

\[ \begin{align*}
\Delta \varepsilon_{rr} &= \frac{\partial \Delta u_r}{\partial r} = - (A_3 + B) dt \\
\Delta \varepsilon_{\theta\theta} &= \frac{1}{r} \frac{\partial \Delta u_\theta}{\partial \theta} + \frac{\Delta u_r}{r} = 0 - (A_3 + B) dt \\
\Delta \varepsilon_{zz} &= \frac{\partial \Delta u_z}{\partial z} = 2(A_3 + B) dt \\
\Delta \varepsilon_{r\theta} = \Delta \varepsilon_{\theta r} &= 0 \\
\Delta \varepsilon_{zz} &= \frac{1}{2} \left( \frac{\partial \Delta u_z}{\partial r} + \frac{\partial \Delta u_r}{\partial z} \right) = \frac{1}{2} \left( A_3 - A_3 \right) dt = 0.
\end{align*} \]
\[\begin{align*}
\text{d} \varepsilon_r &= - (\varepsilon^2 + B) \text{d} t \\
\text{d} \varepsilon_{\theta r} &= - (\varepsilon^2 + B) \text{d} t \\
\text{d} \varepsilon_{\phi r} &= 2 (\varepsilon^2 + B) \text{d} t \\
\text{d} \varepsilon_{\theta \phi} &= \text{d} \phi = 0 \\
\text{d} \varepsilon_{\phi \phi} &= 0 \\
\text{e) Incremental volumetric strain} \ \varepsilon_{\text{v}} &= \text{d} \varepsilon_r + \text{d} \varepsilon_{\theta r} + \text{d} \varepsilon_{\phi r} \\
&= - (\varepsilon^2 + B) \text{d} t - (\varepsilon^2 + B) \text{d} t + 2 (\varepsilon^2 + B) \text{d} t \\
&= 0.
\end{align*}\]