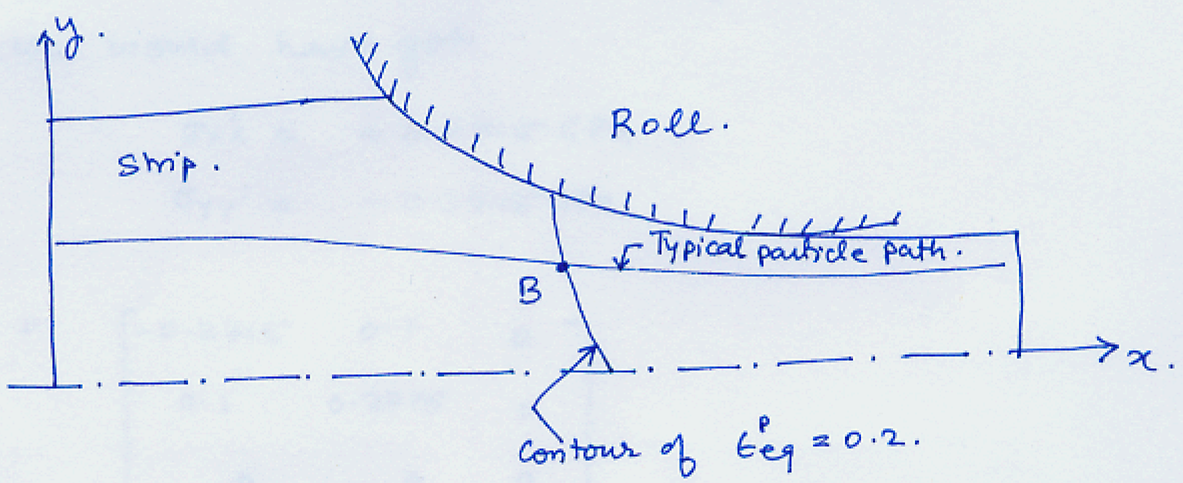
ME 721 Homework 3 Theory of Plasticity

Problem 1 :



Hardening relationship is $\sigma_{eq} = \sigma_y [1 + K(\epsilon^p_{eq})^n]$ — ①

Given, $\sigma_y = 0.3$ GPa $K = 1.5$ $n = 0.5$

Also given is $[\sigma'] = \begin{bmatrix} \sigma'_{xx} & \sigma'_{xy} & 0 \\ \sigma'_{yx} & \sigma'_{yy} & 0 \\ 0 & 0 & 0 \end{bmatrix}$ — ②

At B $\epsilon^p_{eq} = 0.2$ and $\sigma'_{xy} = 0.1$ GPa.

$\therefore$ at B from hardening law $\sigma_{eq} = 0.3[1 + 1.5(0.2)^{0.5}]$

$= 0.5012$ GPa. — ③

Now $\sigma'_{xx} + \sigma'_{yy} = 0 \Rightarrow \sigma'_{xx} = -\sigma'_{yy}$ — ④

and σ_{eq} is given by $\sigma_{eq} = \sqrt{\frac{3}{2}\sigma'_{ij}\sigma'_{ij}}$ — ⑤

$\therefore \sigma_{eq} = \sqrt{\frac{3}{2}(\sigma'_{xx}\sigma'_{xx} + \sigma'_{xy}\sigma'_{xy} + \sigma'_{yx}\sigma'_{yx} + \sigma'_{yy}\sigma'_{yy})}$ {Rest all are = 0} — ⑥

$= \sqrt{\frac{3}{2}(\sigma'_{xx}\sigma'_{xx} + 0.1^2 + 0.1^2 + \sigma'_{yy}\sigma'_{yy})}$

$= \sqrt{\frac{3}{2}(2\sigma'^2_{xx} + 0.02)}$ — ⑦ {From ④, $\because \sigma'_{xx} = -\sigma'_{yy}$}.

$\therefore$ From ③ and ⑦

$0.5012 = \sqrt{\frac{3}{2}(2\sigma'^2_{xx} + 0.02)}$

Solving we get $\sigma'_{xx} = 0.2715$ GPa.

$\therefore \sigma'_{yy} = -0.2715$ GPa.

Also ~~we~~ if we had taken σ_{xx} instead of σ_{yy} in (7)
we would have got

$$\sigma_{xx}' = +0.2715 \text{ GPa}$$
$$\sigma_{yy}' = -0.2715 \text{ GPa}.$$

$$[\sigma'] = \begin{bmatrix} -0.2715 & 0.1 & 0 \\ 0.1 & 0.2715 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

or

$$[\sigma'] = \begin{bmatrix} 0.2715 & 0.1 & 0 \\ 0.1 & -0.2715 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$