ME 721 Theory of Plasticity Homework 4.

Problem 1:

$$v_r = -r(Az+B) \; ; \quad v_\theta = 0 \; ; \quad v_z = A\left(\frac{r^2}{2}+z^2\right)+2Bz. \qquad — (1)$$

A, B are constants.

a) the ~~Levy Mises equation~~ strain rate - velocity relations are

$$\dot\epsilon_{rr} = \frac{\partial v_r}{\partial r} = -(Az+B)$$

$$\dot\epsilon_{\theta\theta} = \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r} = 0 - (Az+B) = -(Az+B)$$

$$\dot\epsilon_{zz} = \frac{\partial v_z}{\partial z} = 2Az+2B = 2(Az+B).$$

$$\dot\epsilon_{zr} = \frac{1}{2}\left(\frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z}\right) = \frac{1}{2}(Ar + -Ar) = 0. \qquad — (2)$$

Now the Levy Mises equation.

$$\sigma'_{ij} = \frac{2}{3}\frac{\sigma_Y}{\dot\epsilon_{eq}}\dot\epsilon_{ij} \qquad — (3)$$

$\therefore$ we have

$$\sigma'_{rr} = \frac{2}{3}\frac{\sigma_Y}{\dot\epsilon_{eq}}\dot\epsilon_{rr}$$

$$\sigma'_{\theta\theta} = \frac{2}{3}\frac{\sigma_Y}{\dot\epsilon_{eq}}\dot\epsilon_{\theta\theta}$$

$$\sigma'_{zz} = \frac{2}{3}\frac{\sigma_Y}{\dot\epsilon_{eq}}\dot\epsilon_{zz}$$

$$\sigma'_{zr} = \frac{2}{3}\frac{\sigma_Y}{\dot\epsilon_{eq}}\dot\epsilon_{zr} \qquad — (4)$$

Now $\dot\epsilon_{eq} = \sqrt{\frac{2}{3}\dot\epsilon_{ij}\dot\epsilon_{ij}} = \sqrt{\frac{2}{3}\left\{(Az+B)^2 + (Az+B)^2 + 4(Az+B)^2\right.}$

$$\dot\epsilon_{eq} = 2(Az+B). \qquad — (5)$$

Substituting ②, ⑤ in ④ we have

$$\left.\begin{aligned}
\sigma'_{rr} &= \frac{2}{3}\,\frac{\sigma_Y}{2(Az+B)}\,(-(Az+B)) = -\frac{\sigma_Y}{3}. \\
\sigma'_{\theta\theta} &= \frac{2}{3}\,\frac{\sigma_Y}{2(Az+B)}\,(-(Az+B)) = -\frac{\sigma_Y}{3}. \\
\sigma'_{zz} &= \frac{2}{3}\cdot\frac{\sigma_Y}{2(Az+B)}\cdot 2(Az+B) = \frac{2}{3}\sigma_Y. \\
\sigma'_{zr} &= \frac{2}{3}\,\frac{\sigma_Y}{2(Az+B)}\cdot 0 = 0.
\end{aligned}\right\}\ \underline{\text{Ans}}.$$

b). $\sigma_{ij} = \frac{1}{3}\sigma_{kk}\,\delta_{ij} + \sigma'_{ij}$

$\frac{1}{3}\sigma_{kk} = \frac{\sigma_Y}{3}$. (Given).

$\sigma_{ij} = \frac{\sigma_Y}{3}\,\delta_{ij} + \sigma'_{ij}$

$$[\sigma] = \begin{bmatrix} \sigma_Y/3 & 0 & 0 \\ 0 & \sigma_Y/3 & 0 \\ 0 & 0 & \sigma_Y/3 \end{bmatrix} + \begin{bmatrix} -\sigma_Y/3 & 0 & 0 \\ 0 & -\sigma_Y/3 & 0 \\ 0 & 0 & 2/3\,\sigma_Y \end{bmatrix}$$

$$[\sigma] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \sigma_Y. \end{bmatrix}$$ $\underline{\text{Ans}}$.