SURGE 2012

An Algorithm for fast and automatic high-order representation of complex 3-D surfaces via Fourier Continuation Analysis

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INTRODUCTION

- **AIM:**

To develop an algorithm for construction of high-order parameterizations of 3-D surfaces, fast and automatically i.e. given a point cloud (thousands of points) on the surface, how to produce the full surface parameterizations *automatically* and *efficiently*.

- **EARLIER WORK:**

The method presented for the construction of *high-order parameterization of surfaces* published in the **Journal of Computational Physics Vol. 227, Issue 2, 10 December 2007, Pages 1094-1125**, renders both smooth and non-smooth portions of the surface, starting from the point cloud, using the concept of Fourier Analysis, which yields super-algebraically Fourier approximation to a surface up to and including all points of geometric singularity, such as corners, edges, conical points etc.

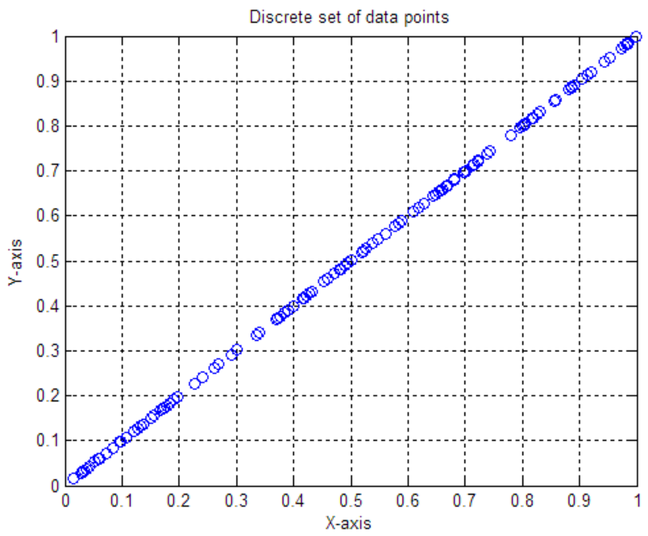
DRAWBACKS:

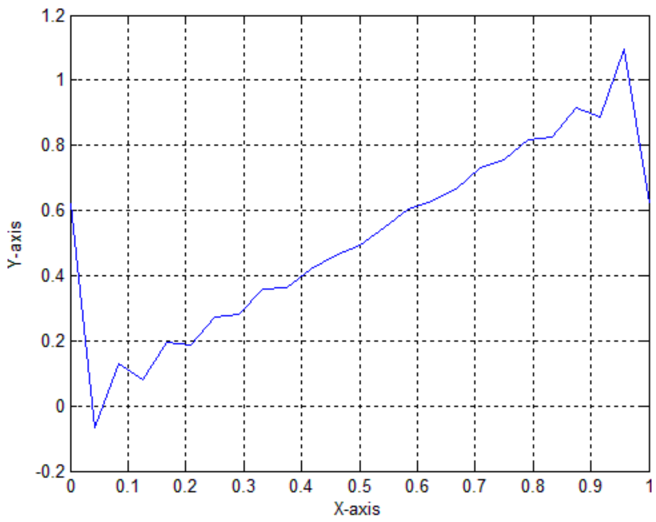
The method presented requires a lot of manual work. In the intermediate process of generation of good projection surface, the software designed requires a lot of human intervention. As a result the overall process of surface parameterizations becomes slow.

Fourier Continuation Method

- This method enables creation of rapidly converging Fourier series from discrete point values of functions that are smooth but not necessarily periodic. For this, we interpolate the given set of discrete data (not equally-spaced) by a trigonometric polynomial $\boldsymbol{F(x)}$ (finite Fourier series) using the **Method of Least Squares**.

$\boldsymbol{F(x) = \sum_{k=-m}^{m} c_k e^{\frac{i2\pi kx}{b}}}$ $where\ m \leq N$.For example consider the function *f(x) = x,*





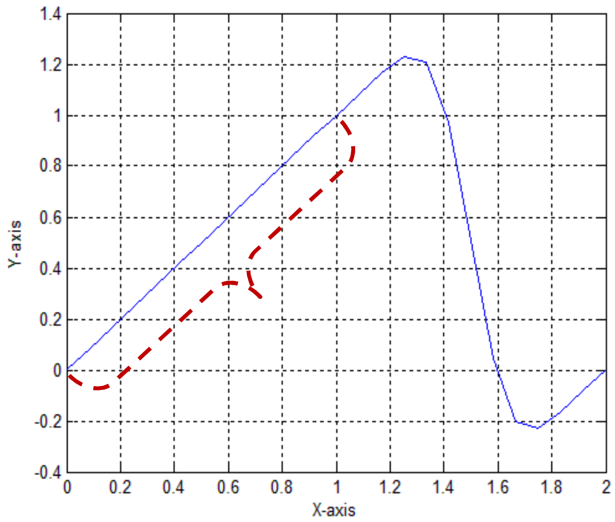


Figure 1: Randomly generated point cloud for the function f(x) = x in the interval$x \in [0,1]$.

Figure 2: Oscillating finite Fourier series of *f(x) = x* around the points of discontinuity, 0 & 1 (also known as ***Gibbs phenomenon)*** when the periodic extension F(x) is 1.

Figure 3: Exactly converging Fourier series in the interval [0, 1] when the periodic extension is greater than the domain interval.

Parameterization by Projections and Continuation

In order to construct the surface parameterizations for the given point cloud, we follow the following algorithm:

For the point cloud that represents a smooth surface, find a good projection surface/curve (not unique).

↓

Project each point orthogonally on the projection surface/curve and find the point of projection.

↓

Then, find the directed distance of each point from the point of orthogonal projection using:

$$d(t_j) = \left[\left(x_j, y_j\right) - P(t_j)\right].\ \overrightarrow{n(t_j)}$$

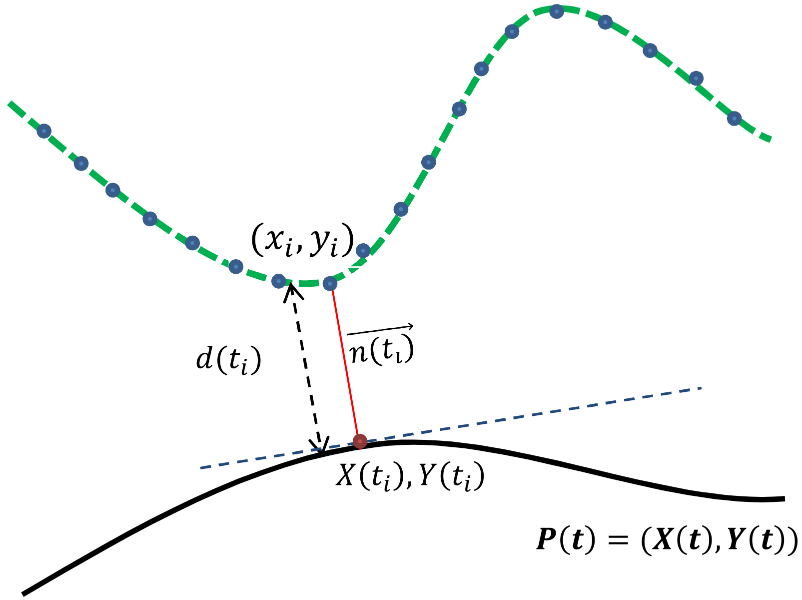
↓

Find the smooth distance function using Fourier Continuation method for the discrete directed distance values obtained above.

↓

Now, using the smooth distance function and the smooth projection surface/curve, determine the full surface parameterization for the given point cloud using:

$$\left(X(t), Y(t)\right) = P(t) + d(t).\overrightarrow{n(t)}$$



Algorithm for Finding the Projection Surface Automatically

The projection surface should be such that the resulting distance function is smooth and such a surface is not unique. To find a good projection surface automatically, we give the following algorithm:

Interpolate the given point cloud by the quadratic curves given by:
$Ax^2 + By^2 + Cz^2 + 2Dxy + 2Exz + 2Fyz + Gx + Hy + Iz + J = 0$
using the method of Least squares.

↓

Use *Single Value Decomposition Method* to find the solution to the resulting homogeneous system of equations.

↓

The above equation can be represented as :

$$\begin{pmatrix} x & y & z \end{pmatrix} \begin{pmatrix} A & D & E \\ D & B & F \\ E & F & C \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} G & H & I \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} + J = 0$$

. Now diagonalise the 3×3 matrix and write the equation in new coordinate system say (X, Y and Z).

↓

We then parameterize the resulting equation using the approach of completing the squares. The only possibilities are either an ellipsoid,

$$aX^2 + bY^2 + cZ^2 + dX + eY + fZ + g = 0$$

hyperboloid (one-sheet/two-sheet) or a paraboloid.

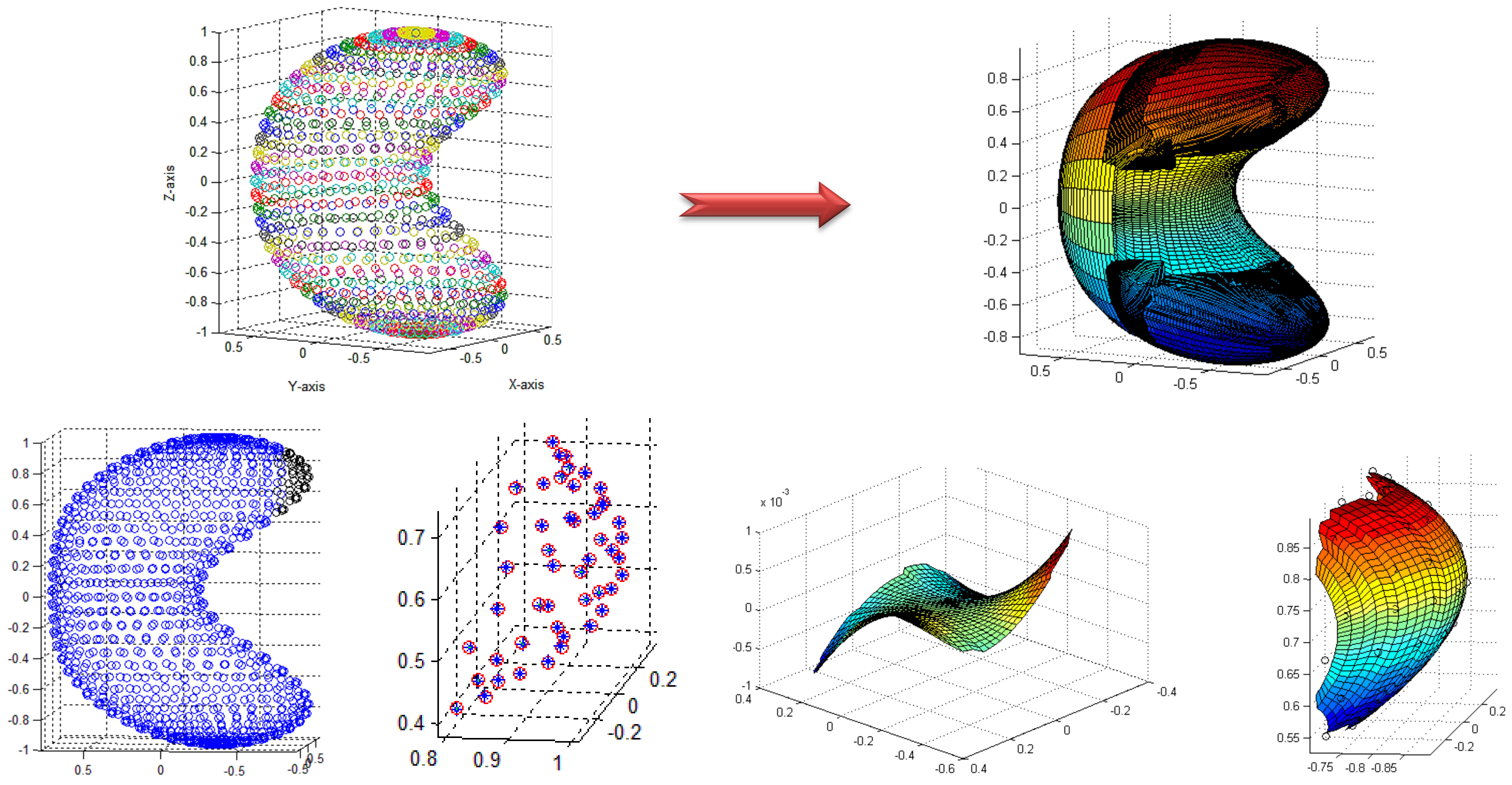
- To orthogonally project the points, solve the parametric equation

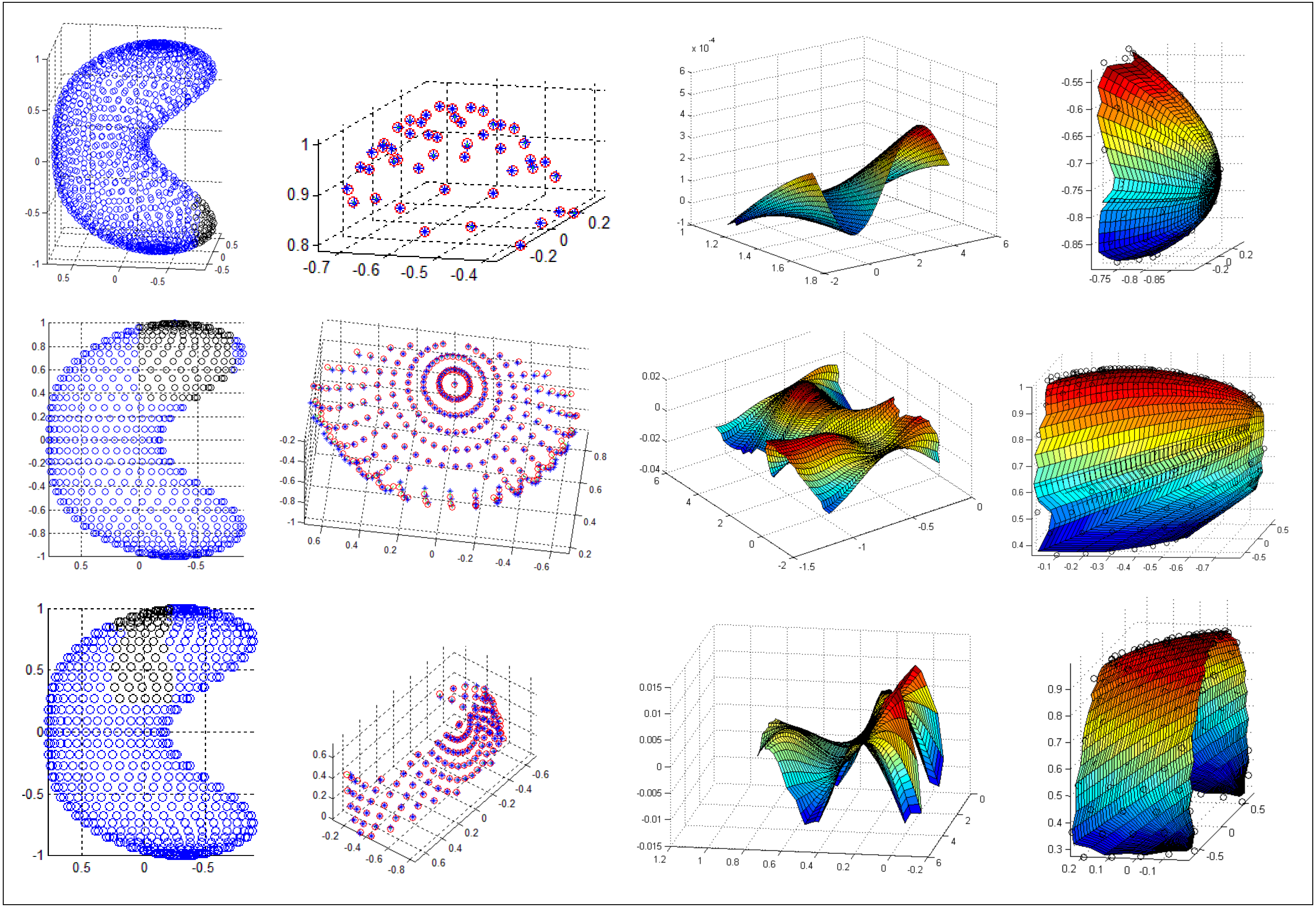
$$\left((\boldsymbol{X}_i - \boldsymbol{X}), (\boldsymbol{Y}_i - \boldsymbol{Y}), ((\boldsymbol{Z}_i - \boldsymbol{Z})\right) = \boldsymbol{\alpha}\nabla\boldsymbol{F}(\boldsymbol{X}, \boldsymbol{Y}, \boldsymbol{Z})$$

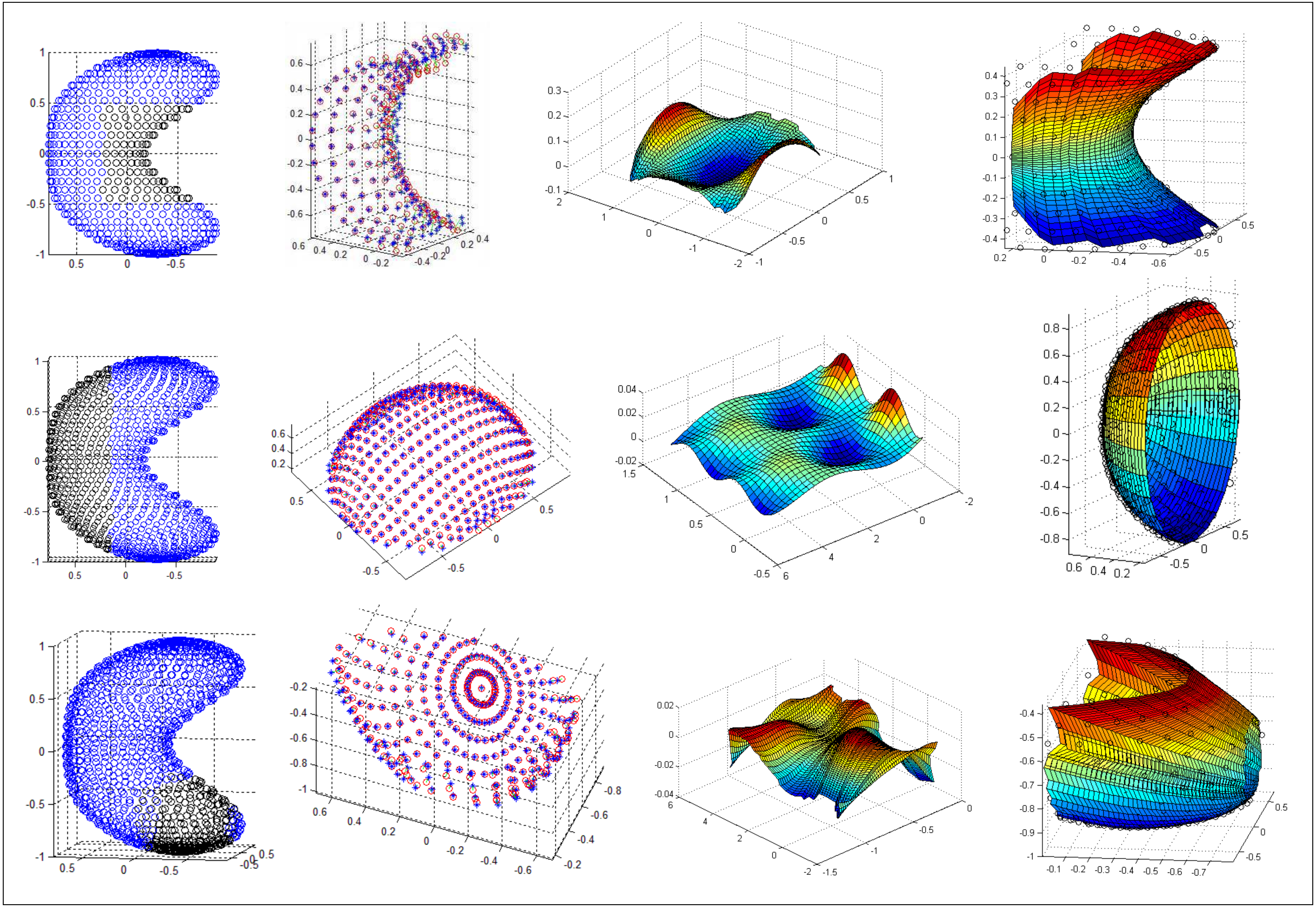
Implementation Results

We generated a point cloud randomly for the surface represented by the equation:

$$\frac{x^2}{a^2(1-\alpha_3\cos(\pi z))}+\frac{(\alpha_1\cos(\pi z)+y)^2}{b^2(1-\alpha_2\cos(\pi z))}+\frac{z^2}{c^2}=1$$

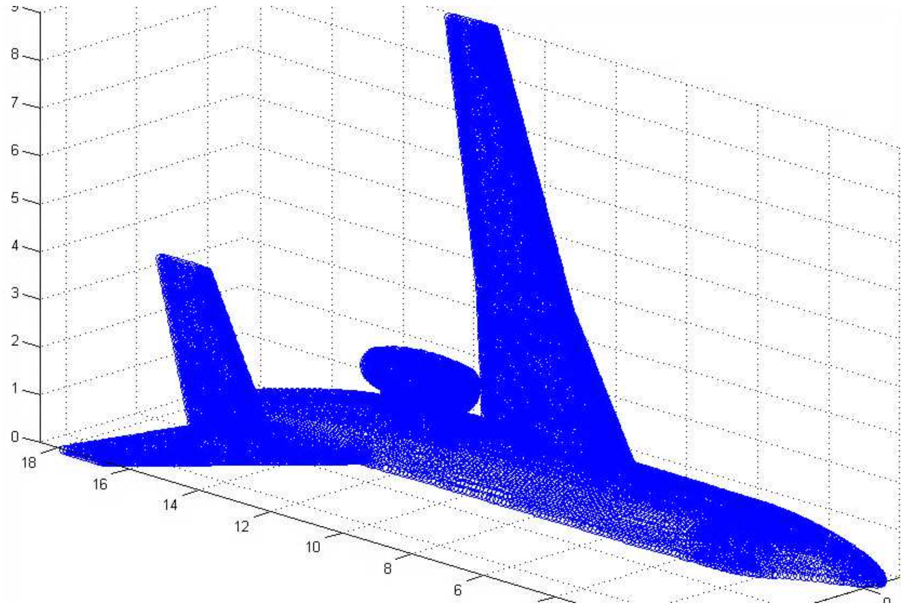



x 10^{-4}

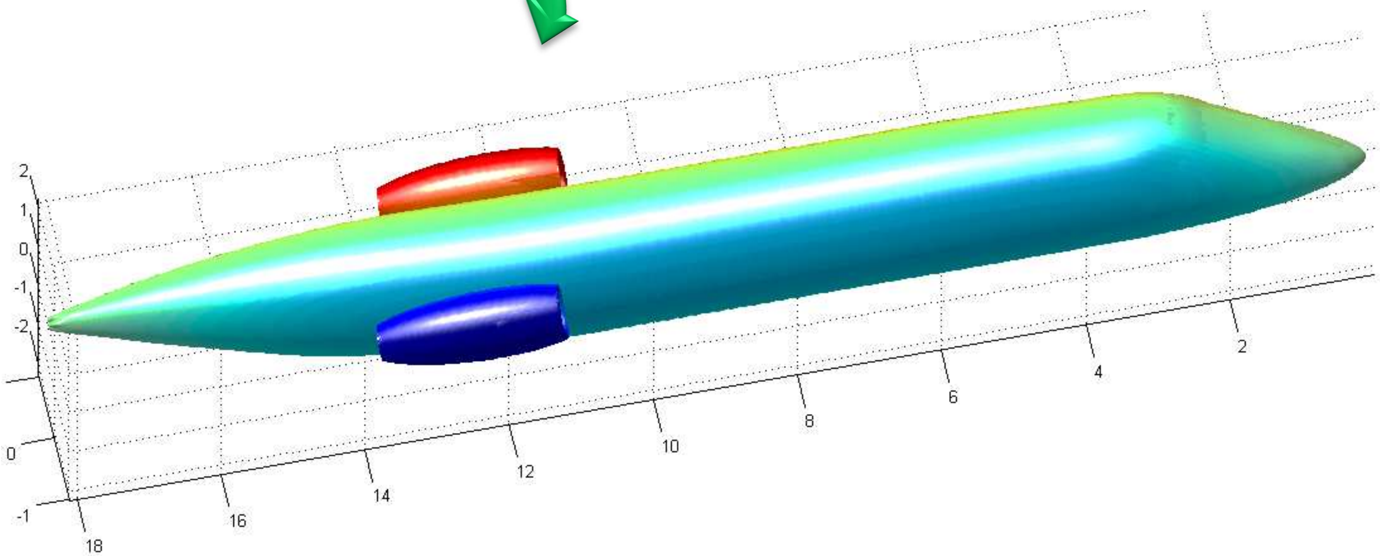


Falcon Aircraft

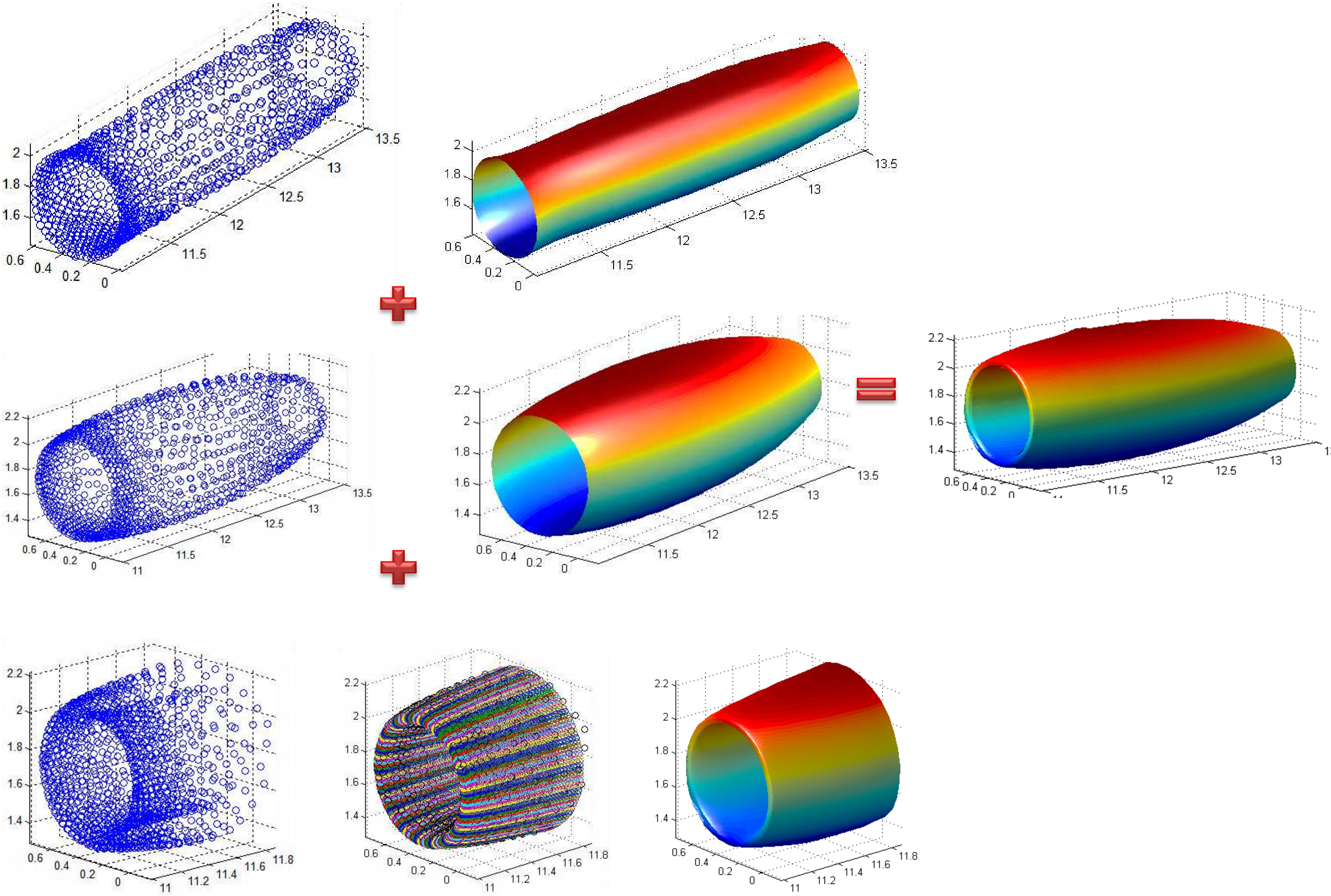
The following figures show our implementation results on the Falcon aircraft:

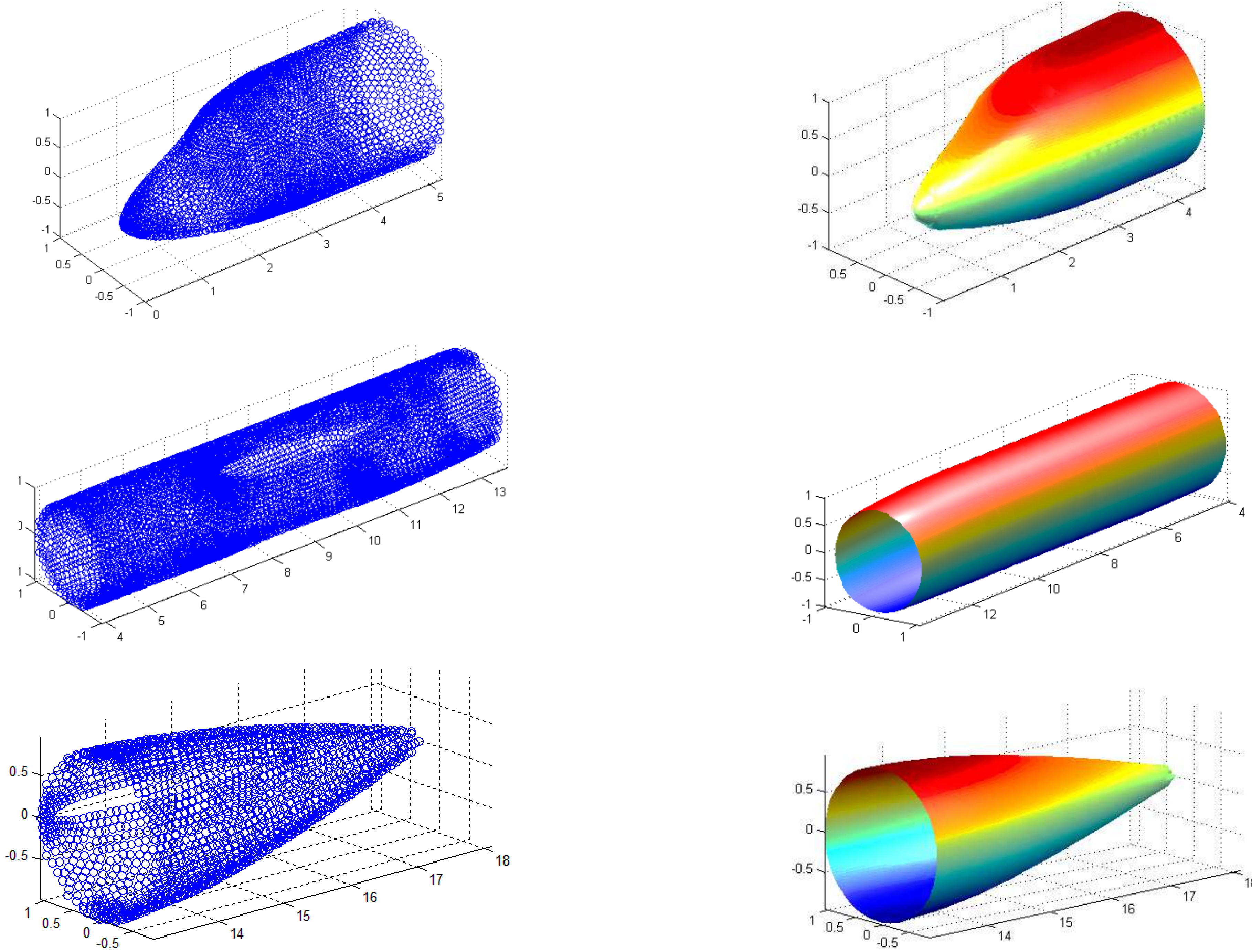


Given point cloud



Fuselage and Engines





Conclusion

- Presented an algorithm for finding the intermediate projection surfaces automatically.
- An algorithm for finding the parametric form of the projection surface.
- An algorithm for finding the orthogonal projection of the points efficiently.

Future Directions

- Automating the pre-processing of point cloud i.e. given a point cloud, extracting the point in cloud which will represent a smooth surface of piecewise smooth surface. (This takes into account dealing with geometric singularities).
- Implementation of all possible surfaces of revolution and finding an algorithm for selecting a suitable SOR for a given point cloud automatically.
- When the different patches of surface are overlapped, the overlapping may not be exact. For this we need to develop an algorithm to properly blend the surfaces of singularity.