Department of Mathematics and Statistics,  
Indian Institute of Technology Kanpur  
MTH 101A, 2013-2014, I Semester  

Instructor: Dr. P. Shunmugaraj,  
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Text: Thomas’ Calculus.  

Course web site: http://home.iitk.ac.in/~psraj/mth101  

Lecture-wise notes of all the lectures, assignments, practice problems with hints/solutions, course plan and necessary information will be available on this course web site.  

Some proofs in the lecture notes and some problems in the practice problems are marked (*). Such proofs and problems will not be asked in the exams and quizzes.  

The assignment problems will be discussed in the tutorial classes. The practice problems available on the course web site are usually not discussed in the tutorial classes; however, the students can discuss these problems with the tutors during office hours.  

Examinations: There will be two quizzes. Each one will be of 20 minutes duration and 15 marks. There will be one mid-semester examination which will be of 2 hours duration and 70 marks. End Semester Examination will be of 3 hours duration and 100 marks.  

Course Plan  

Lecture 1: Real number system: Completeness axiom, density of rationals (irrationals) in \( \mathbb{R} \).  

Lecture 2: (11.1) Convergence of a sequence, Sandwich theorem, Monotone sequences.  

Lecture 3: Cauchy criterion, Subsequence, Every bounded sequence has a convergent subsequence, convergence of a sequence satisfying Cauchy criterion.  

Lecture 4: (2.1-2.6) Limits and Continuity of functions, Boundedness of a continuous function on \([a,b]\).  

Lecture 5: (2.6,4.1) Existence of maximum of a continuous function on \([a,b]\), Intermediate value property, Differentiability.  

Lecture 6: (4.1-4.3) Necessary condition for local maxima, Rolle’s theorem and Mean value theorem.  

Lecture 7: (4.6) Cauchy mean value theorem, L’Hospital rule.  

Lecture 8: (4.7) Fixed point iteration method (Picard’s method), Newton’s method.  

Lecture 9: (4.3,4.4) Increasing and decreasing functions, Convexity, Second derivative test for max and min, Point of inflection, curve sketching.  

Lecture 10 (4.4,11.9) Curve sketching (contd.), Taylor’s theorem with remainder.  

Lecture 11: (11.2) Convergence of series, Geometric and Harmonic series, Absolute convergence.  

\(^1\)Section from XI th edition of the text book.
Lecture 12: (11.4) Comparison test, Cauchy condensation test: \( \sum a_n \text{ conv.} \Leftrightarrow \sum 2^k a_{2^k} \text{ conv.} \) for \( a_n \geq 0 \) and \( a_{n+1} \leq a_n \). Examples: \( \sum \frac{1}{n^p}, \sum \frac{1}{n(\log n)^p} \).

Lecture 13: (11.5) Ratio test, Root test, Examples, Leibniz’s theorem.

Lecture 14: (11.7,11.8) Power series, Radius of convergence, Taylor series, Maclaurin series.

Lecture 15: (5.3) Introduction to Riemann integration, Integrability.

Lecture 16: (5.3) The integral existence theorem for continuous functions and monotone functions, Elementary properties of integral.

Lecture 17: (5.4) Fundamental Theorems of calculus, Riemann Sum.

Lecture 18: (8.8) Improper integral of first & second kind, Comparison test, Absolute convergence.

Lecture 19: (5.6,10.5,10.6) Applications of definite integral: Area between two curves, Polar coordinates, Graphs of polar coordinates.

Mid Semester Examination: Sept. 16 - Sept. 21, 2013

Lecture 20: (10.7,6.1) Area between two curves when their equations are given in polar coordinates, Volumes by slicing.

Lecture 21: (6.2,6.3) Volumes by Shell and Washer methods, Length of a curve.

Lecture 22: (6.5) Area of surface of revolution, Pappus’s Theorem.

Lecture 23: Review of vector algebra, Equations of lines and planes.

Lecture 24: (13.1,13.3) Continuity and Differentiability of vector functions, Arc length for space curves, Unit tangent vector.

Lecture 25: (13.4) Unit normal and Curvature to plane and space curves, Binormal.

Lecture 26: (14.1-14.3) Functions of several variables, Continuity, Partial derivatives, differentiability.

Lecture 27: (14.4) Differentiability \( \Rightarrow \) Continuity, Increment theorem, Chain rule.

Lecture 28: (14.5,14.6) Gradient, Directional derivatives, Tangent plane and Normal line.

Lecture 29: (14.10) Mixed derivative theorem, Mean value theorem (MVT), Extended MVT, Hessian.

Lecture 30: (14.7) Necessary and sufficient conditions for Maxima, Minima and Saddle point.


Lecture 32: (15.1,15.2) Double integral, Fubini’s theorem, Volumes and Areas.

Lecture 33: (15.3,15.4) Change of variable in a double integral, special case: Polar coordinates, Triple integral, Applications.
Lecture 34: (15.6,15.7,16.5) Change of variables in a triple integral, Special cases: Cylindrical and Spherical coordinates, Surface area.

Lecture 35: (16.5,16.1) Surface area (contd.), Surface integrals, Line integrals.

Lecture 36: (16.4) Green’s Theorem.

Lecture 37: (16.2) Vector fields, Divergence and Curl of a vector field.

Lecture 38: (16.7) Stokes’ Theorem.

Lecture 39: (16.8) The divergence theorem.

End Semester Examination: Nov. 18 - Nov. 27, 2013