QUIZ 1

MTH 101A, 5TH OCTOBER 2020

Time: 25 mins **Full Marks: 16**

Instructions: Type ONLY the correct option(s) of all the questions in the box below the pdf file in the portal(for example if in question 3 the correct options are a,b,c then type 3. a,b,c in the box) then click save and submit.
Each question will be evaluated to the following marking scheme

- Full Marks: +4 if only (all) the correct option(s) is (are) chosen.
- Partial Marks: +2 if ONLY correct option (s) is (are) chosen but some correct option(s) is (are) left out.
- Zero Mark: If one wrong option is chosen.
- Negative Mark: -1 if more than one wrong options are chosen.

Notations:

$\mathbb{R}$ denotes set of reals and $\mathbb{Q}$ set of rational.
lub (sup) and glb (inf) stand for least upper bound (supremum) and greatest lower bound (infimum) respectively.

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(1) Which of the following statement(s) is (are) true.
 (a) There does NOT exist sets S and T with $\sup S \leq \inf T$ and $S \cap T \neq \emptyset$.
 (b) Let $\alpha = \inf S$ and $\beta > \alpha$ there exists an element $x \in S$ such that $x < \beta$.
 (c) Let $\alpha = \sup S$ and $\beta < \alpha$ there exists an element $x \in S$ such that $x > \beta$.
 (d) If $A \subset \mathbb{R} \setminus \mathbb{Q}$ and bounded then $\inf A \in \mathbb{Q}$.

Answer: b,c

(2) Which of the following statement(s) is (are) true.
 (a) $\{\frac{n+1}{n-1}\}_{n \geq 2}$ is a decreasing sequence and converges to 1 as $n \to \infty$.
 (b) $\lim\limits_{n\to\infty} n^{-n-\frac{1}{n}}(1+n)^n = \frac{1}{e}$.
 (c) Consider the recurrence relation $x_{n+1} = x_n + x_{n-1},\ n = 1, 2, \ldots,$ where $x_0 = x_1 = 1$. Define the sequence $y_n = \frac{x_n}{x_{n+1}}$. Then $\{y_n\}$ does not have any convergent subsequence.
 (d) If $x_1 = 2$ and x_n's are defined recursively as $x_{n+1} = 2 + \frac{1}{x_n}$ then $\{x_n\}$ is a Cauchy sequence.

Answer: a,d

(3) Which of the following statement(s) is (are) true.
 (a) Let $a \in \mathbb{R}$, $f(x) < g(x)$, and $\lim\limits_{x\to a} f(x) = l$, $\lim\limits_{x\to a} g(x) = m$. Then $l < m$.
 (b) Let $f : [0,1] \to \mathbb{R}$ be a continuous function and $f(\frac{1}{2}) = 1$ then there exists a closed interval $I \subset [0,1]$ such that $f(x) > \frac{1}{2}$ for all $x \in I$.
 (c) Let $f : [a,b] \to \mathbb{R}$ be continuous such that $f(a) < a$ and $f(b) > b$. Then there exists a $x \in [a,b]$ such that $f(x) = x$.
 (d) If f is a monotonically increasing function then $\lim\limits_{x\to a} f(x) = \sup\{f(x) : x < a\}$.

Answer: b,c,d

(4) Which of the following statement(s) is (are) true.
 (a) $f(x) = |x|^{\frac{1001}{1000}}$ is not differentiable at 0.

(b) Let $f : (-1, 1) \to \mathbb{R}$ be a differentiable function such that $f'(0) < 0$. There exists a $\delta > 0$ such that $f(x) < f(0)$ for all $x \in (0, \delta)$.

(c) $|\cos x - \cos y| \leq |x - y|$ for all $x, y \in \mathbb{R}$.

(d) There exists $x \in [0, \pi]$ such that $e^x \cos x + 1 = 0$.

Answer: b,c,d