MTH 101N : 2010 - 11
Midsem - I

Qn 1

a) For $K \geq 1$ $\quad \dfrac{1}{a_K a_{K+1}} = \dfrac{1}{3}\left(\dfrac{1}{a_K} - \dfrac{1}{a_{K+1}}\right)$ (2 marks)

Hence $\dfrac{1}{a_1 a_2} + \dfrac{1}{a_2 a_3} + \cdots + \dfrac{1}{a_{n-1} a_n} = \dfrac{1}{3}\left(\dfrac{1}{a_1} - \dfrac{1}{a_n}\right) \longrightarrow \dfrac{1}{3}$
(2 marks)

b) Let $a_n = \dfrac{(x-1)^{2n}}{2^n n^2}$. By root test $\sqrt[n]{a_n} \longrightarrow \dfrac{(x-1)^2}{2}$

Thus for $x \in (1-\sqrt{2}, 1+\sqrt{2})$, $\sum_n \dfrac{(x-1)^{2n}}{2^n n^2}$ Converges
(3 marks)

& at $x = 1 \pm \sqrt{2}$ $\quad \sum_n \dfrac{(x-1)^{2n}}{2^n n^2}$ Converges
(1-mark)

c) $\lim\limits_{x\to 0} \dfrac{1}{x}\left[f(x) + f(x/2) + \cdots + f(x/k)\right]$

$$= \lim_{x\to 0}\left[\frac{f(x)-f(0)}{x} + \frac{1}{2}\cdot\frac{f(x/2)-f(0)}{x/2} + \cdots + \frac{1}{k}\frac{f(x/k)-f(0)}{x/k}\right]$$

$$= \left(1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{k}\right) f'(0).$$

-(2 marks

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<u>n 2</u>

a) By induction $\frac{3}{2} \le a_n \le 2$

(2 marks)

Also $a_n \ge a_{n-1}$ (by induction: Let $a_{n-1} \ge a_{n-2}$

Then $a_n^2 - a_{n-1}^2 = 3(a_{n-1} - a_{n-2}) \ge 0$

Hence $(a_n - a_{n-1})(a_n + a_{n-1}) \ge 0$

$\Rightarrow a_n - a_{n-1} \ge 0$)

(2 marks)

$\{a_n\}$ is a monotone increasing sequence which is bounded above. Hence converges.

Let $l = \lim_n a_n$. Then $l = \sqrt{3l-2}$

$\Rightarrow l = 1$ or 2. Since $a_n \ge 3/2 \ \forall n$, $l = 2$

(1 mark)

b) Let $b_n = a_n - \frac{1}{2^{n-1}}$.

(2 marks)

Then $\{b_n\}$ is bounded above as $\{a_n\}$ is so.

$b_{n+1} - b_n = a_{n+1} - a_n - \frac{1}{2^n} + \frac{1}{2^{n-1}} = a_{n+1} - a_n + \frac{1}{2^n} \ge 0$

Hence $\{b_n\}$ is monotone increasing

(2 mark)

Hence $\{b_n\}$ converges and $a_n = b_n + \frac{1}{2^{n-1}}$ converges.

(1 mark)

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Midsem-I

Qn 3.

a) Consider $f(x) = \frac{1}{x^{32}} \left| x^{31} + x^{8} + 2010 \right| - 1$ - (2 marks)

As $x \longrightarrow 0$ $f(x) \longrightarrow +\infty$

as $x \longrightarrow +\infty$ $f(x) \longrightarrow -1$. - (2 marks)

Hence there exists, by IMV property

$\exists$ $x_0 \in (0, \infty)$ such that $f(x_0) = 0$.

that is $\left| x_0^{31} + x_0^{8} + 2010 \right| = x_0^{32}$.

(1-mark)

b) Let $Q(x) = \frac{a_0}{n+1} x^{n+1} + \frac{a_1}{n} x^{n} + \cdots + a_n x$

(2 marks)

Then $Q(0) = 0$

$Q(1) = \frac{a_0}{n+1} + \frac{a_1}{n} + \cdots + a_n = 0$ (2 marks)

By Rolle's Theorem $\exists$ $x_0 \in (0,1)$ such that

$Q'(x_0) = a_0 x_0^{n} + a_1 x_0^{n-1} + \cdots + a_n = 0$

(1-mark)

Marking scheme

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Qn. 4 $\quad f(x) = \dfrac{2x^2}{1-x^2}$

a) Asymptotes : $x = \pm 1$ (1 mark)

$f(x) = -2 + \dfrac{1}{1-x^2}$

$y = -2$ is also asymptote (2 mark)

b) $f'(x) = \dfrac{4x}{(1-x^2)^2}$ > 0 on $(0,1) \cup (1,\infty)$

f is increasing (1 mark)

< 0 on $(-\infty,-1) \cup (-1,0)$

f is decreasing (1 mark)

c) 0 is a local minimum (1 mark)

d) $f''(x) = \dfrac{4(3x^2+1)}{(1-x^2)^3}$ > 0 on $(-1,1)$

f is Convex (1 mark)

< 0 on $(-\infty,-1) \cup (1,\infty)$

f is Concave (1 mark)

e) Sketch :