Department of Mathematics, Indian Institute of Technology, Kanpur
MTH101A: Quiz 2 A- 31-10-2014

Maximum Marks- 15 **5:15-5:35 p.m.**

Name: Roll No: Section:

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(1) A particle moves in space according to $R(t) = (cos\, t, sin\, t, cos\, 2t)$. Find the tangential and normal components of the acceleration at $t = \frac{\pi}{4}$. Further, find the curvature at $t = \frac{\pi}{4}$. [7]

Solution: We observe that the velocity vector $v(t) = (-\sin\, t, \cos\, t, -2sin\, 2t)$ and the acceleration vector $a(t) = (-cos\, t, -\sin\, t, -4cos\, 2t)$ [2]

At $t = \pi/4$, $v = (\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, -2)$ and $a = (-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0)$

The speed $\frac{ds}{dt} = \sqrt{5}$.

Since $a(t) = a_T T(t) + a_N N(t)$, we observe that since $a.v = 0$, $a_T = 0$.

Thus $a_N = |a| = 1$ [3]

The curvature $\kappa(\frac{\pi}{4}) = a_N/(\frac{ds}{dt})^2 = \frac{1}{5}$ [2]

(2) 1. The position of a fly flying in a room at time t is given by $R(t) = (cos\, t, sin\, t, t)$. The temperature in the room is given by $f(x, y, z) = xyz$. What is the rate of change of the temperature experienced by the fly at time t. [4]

Solution: Let the temperature be written as $f(x, y) = xyz$. The rate of change of temperature experienced by the fly at any time t will be $\frac{d}{dt}(f(R(t))$.

By the Chain Rule $\frac{d}{dt}(f(R(t)) = \nabla f(R(t)).R'(t)$. $\nabla f = (yz, xz, xy)$ and $\nabla f(R(t)).R'(t) = (t(sin\, t), t(cos\, t), \cos\, t\, sin\, t)$. [2]

$R'(t) = (-sin\, t, cos\, t, 1)$

$\frac{d}{dt}(f(R(t)) = -tsin^2t + tcos^2t + cos\, tsin\, t$. [2]

2. Find the equation of the tangent plane to the surface $z = xsin\,(x+y)$ at $(-1, 1, 0)$. [4]

Solution: Let $f(x, y) = xsin\,(x + y)$.

Then $f_x = sin(x + y) + xcos(x + y)$ and $f_y = xcos(x + y)$ [1]

Equation of the tangent plane at $(-1, 1, 0)$ is

$z - f(-1, 1) = f_x(-1, 1)(x + 1) + f_y(-1, 1)(y - 1)$. [1]

Equation of the tangent plane is : $z = -x - y$ [2]