(1) Let \( A : \{ (x, y) \in \mathbb{R}^2 : x \geq 0 \text{ and } y \in \mathbb{R} \} \) and \( f : A \to \mathbb{R} \) be the function defined as

\[
f(x, y) = \begin{cases} 
xy \sin(xy) & \text{if } x \geq 0 \text{ and } y \geq 0 \\
-xy \sin(xy) & \text{if } x \geq 0 \text{ and } y < 0 
\end{cases}
\]

Determine the points on \( A \) where \( f \) is continuous.

(2) Let \( f(x, y) = \frac{1}{2}||x| - |y|| - |x| - |y|| - xy. \)

(a) Does the directional derivative of \( f \) exist at \((0, 0)\) in the direction \((\frac{3}{5}, \frac{4}{5})\)? Justify your answer.

(b) Is \( f \) differentiable at \((0, 0)\)? Justify your answer.

(3) Let \( S \) be the solid obtained by revolving the region bounded by the curves \( f_1(x) = x^2 - 4 \) and \( f_2(x) = 4 - x^2 \) about the line \( x = 2 \). Using the shell method determine the volume of the solid \( S \).

Solutions:

(1) \textbf{Case 1:} Let \( x = 0 \) or \( y = 0 \).

Suppose \( x_n \to x \) and \( y_n \to y. \)

Then \( |f(x_n, y_n) - f(x, y)| = |f(x_n, y_n)| \leq |x_n y_n| \to 0. \)

Hence \( f \) is continuous at \((x, y)\). \[2\]

\textbf{Case 2:} Let \( x > 0 \) and \( y > 0 \).

Suppose \( x_n \to x \) and \( y_n \to y. \)

Then \( x_n > 0 \) and \( y_n > 0 \) eventually and

\[
|f(x_n, y_n) - f(x, y)| = |x_n y_n \sin(x_n y_n) - x y \sin(xy)| \to 0.
\]

Hence \( f \) is continuous at \((x, y)\). \[2\]

\textbf{Case 3:} Let \( x > 0 \) and \( y > 0 \).

This case is similar to Case 1.

Therefore \( f \) is continuous at points of \( A \). \[1\]

(2) Note that 

\[
\frac{f(t^\frac{3}{5}, t^\frac{4}{5}) - f(0, 0)}{t} = \frac{|t| \left|| \frac{3}{5} - \frac{4}{5} \right|| - \left| -\frac{3}{5} - \frac{4}{5} \right|| - \left| -\frac{3}{5} - \frac{4}{5} \right|}{t} = \frac{3}{5} \frac{4}{5}.
\]

Hence \( f(t^\frac{3}{5}, t^\frac{4}{5}) - f(0, 0) \) does not exist. \[3\]

Since the directional directive of \( f \) at \((0, 0)\) in a direction does not exist, \( f \) is NOT differentiable at \((0, 0)\). \[2\]
(3) Volume = \[ \int_{-2}^{2} 2\pi (2 - x)((4 - x^2) - (x^2 - 4))dx \] 

Volume = \[ \frac{256\pi}{4} \].