1. Find the area of the region enclosed by $y = \cos x$, $y = \sin x$, $x = \frac{\pi}{2}$ and $x = 0$.

2. Consider the curves $y = x^3 - 9x$ and $y = 9 - x^2$.
   (a) Show that the curves intersect at $(-3, 0), (-1, 8)$ and $(3, 0)$.
   (b) Find the area of the region bounded by the curves.

3. Sketch the graphs of the following polar equations:
   (a) $r = \cos \theta$
   (b) $r = -\cos \theta$
   (c) $r = \sin \theta$
   (d) $r = -\sin \theta$.

4. Sketch the limacons (convex or oval limacons, limacons with dimples, cardioids and limacons with inner loops).
   (a) $r = 3 + \cos \theta$
   (b) $r = \frac{3}{2} + \cos \theta$
   (c) $r = 1 + \cos \theta$
   (d) $r = \frac{1}{2} + \cos \theta$
   (e) $r = 3 - \cos \theta$
   (f) $r = \frac{3}{2} - \cos \theta$
   (g) $r = 1 - \cos \theta$
   (h) $r = \frac{1}{2} - \cos \theta$
   (i) $r = 3 + \sin \theta$
   (j) $r = \frac{3}{2} + \sin \theta$
   (k) $r = 1 + \sin \theta$
   (l) $r = \frac{1}{2} + \sin \theta$
   (m) $r = 3 - \sin \theta$
   (n) $r = \frac{3}{2} - \sin \theta$
   (o) $r = 1 - \sin \theta$
   (p) $r = \frac{1}{2} - \sin \theta$

5. Sketch the roses:
   (a) $r = \sin 2\theta$
   (b) $r = \sin 3\theta$
   (c) $r = \sin 4\theta$
   (d) $r = \sin 5\theta$
   (e) $r = \cos 2\theta$
   (f) $r = \cos 3\theta$
   (g) $r = \cos 4\theta$
   (h) $r = \cos 5\theta$

6. Consider the equations $r = 2 + \sin \theta$ and $r = -2 + \sin \theta$.
   (a) Show that both the equations describe the same curve.
   (b) Sketch the curve.

7. Consider the equations $r = \sin \frac{\theta}{2}$ and $r = \cos \frac{\theta}{2}$.
   (a) Show that if $(r, \theta)$ satisfies the equation $r = \sin \frac{\theta}{2}$, then its one of the other representations $(-r, \theta + \pi)$ satisfies the equation $r = \cos \frac{\theta}{2}$.
   (b) Show that both the equations describe the same curve and sketch the curve.
   (c) Observe from the graph that the curve is symmetric with respect to both x-axis and y-axis.

8. Sketch the following curves:
   (a) $r = 2 + \sin(2\theta)$
   (b) $r^2 = -\sin \theta$
   (c) $r = \theta$, $\theta \geq 0$
   (d) $r = \theta$, $\theta \leq 0$
   (e) $r = \theta$
   (f) $r = -\theta$

9. Consider the equation $r = \theta + 2\pi$.
   (a) Observe that the equation changes if $(r, \theta)$ is replaced by $(r, \pi - \theta)$ or $(-r, -\theta)$.
   (b) Show that the equation given above and $r = \theta$ describe the same curve (Spiral of Archimedes).
   (c) Show that the curve obtained is symmetric with respect to the y-axis.
10. Sketch the regions described by the following sets.
   
   \[(a) \{(r, \theta) : 1 \leq r \leq 1 - 2 \cos \theta, \ 0 \leq \theta \leq \frac{3 \pi}{2}\}\]
   \[(b) \{(r, \theta) : 1 + \cos \theta \leq r \leq 3 \cos \theta, \ -\frac{\pi}{3} \leq \theta \leq 0\}\]

11. Replace the equation \(x^2 + y^2 - 4y = 0\) by equivalent polar equation.

12. Replace the equation \(r = 6 \cos \theta + 8 \sin \theta\) by equivalent Cartesian equation and show that the equation describe a circle.

Practice Problems 19 : Hints/Solutions

1. Solving \(\sin x = \cos x\) implies that \(x = \frac{\pi}{4} \in [0, \frac{\pi}{2}]\) (see Figure 1). Therefore the required area is \( \int_{0}^{\frac{\pi}{4}} (\cos x - \sin x) \, dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\sin x - \cos x) \, dx = 2\sqrt{2} - 2 \).

2. (a) Note that \(x^3 - 9x - 9 + x^2 = (x + 3)(x + 1)(x - 3)\).
   
   (b) Area = \( \int_{1}^{-3} [(x^3 - 9x) - (9 - x^2)] \, dx + \int_{3}^{-1} [(9 - x^2) - (x^3 - 9x)] \, dx \) (see Figure 2).

3. See Figure 3.

4. See Figure 4 for the graphs of the equations given in (a)-(d).

   Observe that the graphs of the equations given in (e)-(h) are obtained by rotating the curves described by the equations given in (a)-(d) counterclockwise by \(\pi\). For example, \(r = 3 - \cos \theta = 3 + \cos(\theta - \pi)\).

   Similarly, the graphs of the equations given in (i)-(p) are obtained by rotating the curves described by the equations given in (a)-(h) counterclockwise by \(\frac{\pi}{2}\). For example, \(r = 3 + \sin \theta = 3 + \cos(\theta - \frac{\pi}{2})\).

5. See Figure 5.

6. (a) Observe that both the curves are symmetric with respect to the y-axis. Moreover, if \((r, \theta)\) satisfies the equation \(r = 2 + \sin \theta\) then \((-r, -\theta)\) satisfies the equation \(r = -2 + \sin \theta\) and vice versa. Therefore both the equations describe the same curve.
   
   (b) Refer Figure 6 for the graph.

7. (a) Easy to verify.
   
   (b) From (a), it follows that both the equations describe the same curve. Refer Figure 7 for the graph.
   
   (c) The symmetry is shown in the figure.

8. See Figure 8.

9. It is easy to verify.

10. See Figure 10.

11. Substituting \(x = r \cos \theta\) and \(y = r \sin \theta\) in the given equation leads to the equation \(r(r - 4 \sin \theta) = 0\). The equation \(r = 0\) represents the origin which is included in the curve described by the equation \(r = 4 \sin \theta\). The required equation is \(r = 4 \sin \theta\).

12. The given equation can be written as \(r^2 - 6r \cos \theta - 8r \sin \theta = 0\). The substitutions, \(x = r \cos \theta, y = r \sin \theta\) and \(r^2 = x^2 + y^2\) lead to the equation \((x - 3)^2 + (y - 4)^2 = 25\).