1. Let $D$ denote the solid bounded by the surfaces $y = x$, $y = x^2$, $z = x$ and $z = 0$. Evaluate $\iiint_D y \, dxdydz$.

2. Let $D$ denote the solid bounded below by the plane $z + y = 2$, above by the cylinder $z + y^2 = 4$ and on the sides $x = 0$ and $x = 2$. Evaluate $\iiint_D x \, dxdydz$.

3. Suppose $\int_0^2 \int_0^{\sqrt{x^2 - y^2}} dz \, dy \, dx$ for some region $D \subset \mathbb{R}^3$.
   (a) Sketch the region $D$.
   (b) Sketch the projections of $D$ on the $xy$, $yz$ and $xz$ planes.
   (c) Write $\int_0^2 \int_0^{\sqrt{x^2 - y^2}} dz \, dy \, dx$ as iterated integrals of other orders.

4. Let $D = \{(x, y, z) \in \mathbb{R}^3 : \frac{x^2}{4} + \frac{y^2}{16} + \frac{z^2}{9} \leq 1\}$ and $E = \{(u, v, w) \in \mathbb{R}^3 : u^2 + v^2 + w^2 \leq 1\}$. Show that $\iiint_D dxdydz = \iiint_E 24dudvdw$.

5. In each of the following cases, describe the solid $D$ in terms of the cylindrical coordinates.
   (a) Let $D$ be the solid that is bounded by the paraboloids $z = x^2 + y^2$ and $z = 36 - 3x^2 - 3y^2$.
   (b) Let $D$ be the solid that lies within the cylinder $x^2 + (y - 1)^2 = 1$ below the paraboloid $z = x^2 + y^2$ and above the plane $z = 0$.
   (c) Let $S$ denote the torus generated by revolving the circle $\{(x, z) : (x - 2)^2 + z^2 = 1\}$ about the $z$-axis. Let $D$ be the solid that is bounded above by the surface $S$ and below by $z = 0$.

6. Let $D$ be the solid that lies inside the cylinder $x^2 + y^2 = 1$, below the cone $z = \sqrt{4(x^2 + y^2)}$ and above the plane $z = 0$. Evaluate $\iiint_D x^2 dxdydz$.

7. Evaluate $\int_{-2}^{2} \int_{\sqrt{4-x^2}}^{4} zdxdydz$.

8. Describe the following regions in terms of the spherical coordinates.
   (a) The region that lies inside the sphere $x^2 + y^2 + (z - 2)^2 = 4$ and outside the sphere $x^2 + y^2 + z^2 = 1$.
   (b) The region that lies below the sphere $x^2 + y^2 + z^2 = 2$ and above the cone $z = \sqrt{x^2 + y^2}$.
   (c) The region that is enclosed by the cone $z = \sqrt{3(x^2 + y^2)}$ and the planes $z = 1$ and $z = 2$.

9. Let $D$ denote the solid bounded above by the plane $z = 4$ and below by the cone $z = \sqrt{x^2 + y^2}$. Evaluate $\iiint_D \sqrt{x^2 + y^2 + z^2} dxdydz$.

10. Let $D$ denote the solid enclosed by the spheres $x^2 + y^2 + (z - 1)^2 = 1$ and $x^2 + y^2 + z^2 = 3$. Using spherical coordinates, set up iterated integrals that gives the volume of $D$. 
Consider the change of variables $D$ given by

$$
\begin{align*}
\frac{1}{2} \int_0^2 \int y \, dx dy dz &= \int_0^2 \int y dx dy \int y dz = \int_0^2 \int y dx dy dz.
\end{align*}
$$

2. See Figure 1. Solving $4 - y^2 = 2 - y$ implies $y = -1, 2.$ The projection of the solid $D$ on the $xy$-plane is given by $R = [0, 2] \times [-1, 2].$ The solid lies above $z = f_1(x, y) = 2 - y$ and below $z = f_2(x, y) = 4 - y^2.$ Therefore

$$
\begin{align*}
\int_0^2 \int_0^2 \int x dy dz dx &= \int_0^2 \int_0^2 \int x dx dy dz = \int_0^2 \int_0^2 \int x dy dz dx.
\end{align*}
$$

3. (a) See Figure 2.

(b) See Figure 3, Figure 4 and Figure 5.

(c) \[
\begin{align*}
\int_0^2 \int_0^2 dz dy dx &= \int_0^2 \int_0^2 dz dy dx = \int_0^2 \int_0^2 dz dy dx = \int_0^2 \int_0^2 dz dy dx.
\end{align*}
\]

4. Consider the change of variables $x = 2u, y = 4v$ and $z = 3w.$ Note that the transformation $T(u, v, w) = (2u, 4v, 3w)$ maps $E$ onto $D$ and the Jacobian $J(u, v, w) = 24.$

5. (a) Solving $x^2 + y^2 = 36 - 3(x^2 + y^2)$ implies that $x^2 + y^2 = 9.$ The projection of the solid $D$ on the $xy$-plane is the circular disk \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 9\}. The solid is bounded by $z = r^2$ and $z = 36 - 3r^2.$ Therefore $D = \{(r, \theta, z) : 0 \leq \theta \leq 2\pi, 0 \leq r \leq 3, 3r^2 \leq z \leq 36 - 3r^2\}.$

(b) The projection of $D$ on the $xy$-plane is given by $\{(x, y) \in \mathbb{R}^2 : x^2 + (y - 1)^2 \leq 1\}$ which is described in cylindrical coordinates as $\{(r, \theta, z) : 0 \leq \theta \leq \pi, 0 \leq r \leq 2 \sin \theta, 0 \leq z \leq r^2\}$.

(c) The projection of the solid $D$ on the $xy$-plane is the region between the circles $r = 1$ and $r = 3.$ Allow $\theta$ to run from $0$ to $2\pi$ and consider the cross section of the solid, perpendicular to the $xy$-plane, corresponding to a fixed $\theta.$ The cross section is a circle which is shown in Figure 6. The equation of the circle can be considered as $(r - 2)^2 + z^2 = 1$ for $1 \leq r \leq 3.$ Therefore $D = \{(r, \theta, z) : 0 \leq \theta \leq 2\pi, 1 \leq r \leq 3, 0 \leq z \leq \sqrt{1 - (r - 2)^2}\}.$

6. The projection of the solid $D$ on the $xy$-plane is the circular disk $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1\}.$ We will use the cylindrical coordinates. The solid $D$ is bounded by $z = 0$ and $z = 2r.$ Therefore

$$
\begin{align*}
\int_0^2 \int_0^2 x^2 dz dy dx &= \int_0^2 \int_0^2 r^2 \cos^2 \theta dz dr d\theta.
\end{align*}
$$

7. Note that

$$
\begin{align*}
\int_0^2 \int_{-\sqrt{4 - z^2}}^{\sqrt{4 - z^2}} x dz dx &= \int_0^2 \int_{-\sqrt{4 - z^2}}^{\sqrt{4 - z^2}} x dz dx.
\end{align*}
$$

by $z = x^2 + y^2$ and above by $z = 4.$ The projection of the solid on the $xy$-plane is given by $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 4\}.$ By the cylindrical coordinates

$$
\begin{align*}
\int_0^2 \int_0^2 \int_0^{2\pi} x dz dr d\theta.
\end{align*}
$$
8. (a) See Figure 7. The sphere \( x^2 + y^2 + z^2 = 1 \) is expressed as \( \rho = 1 \) where as \( x^2 + y^2 + (z - 2)^2 = 4 \) is expressed as \( \rho = 4 \cos \phi \). The two spheres intersect at \( \cos \phi = \frac{1}{4} \). For a fixed \( \phi \in [0, \cos^{-1} \frac{1}{4}] \), \( \rho \) varies from 1 to \( 4 \cos \phi \) in the given region. Therefore the region is given by \( \{ (\rho, \theta, \phi) : 0 \leq \theta \leq 2\pi, 0 \leq \phi \leq \cos^{-1} \frac{1}{4}, 1 \leq \rho \leq 4 \cos \phi \} \).

(b) See Figure 8. The sphere is expressed as \( \rho = \cos \phi \). The cone is expressed as \( \rho \cos \phi = \rho \sin \phi \) that is \( \phi = \frac{\pi}{4} \). For a fixed \( \phi \in [0, \frac{\pi}{4}] \), \( \rho \) varies from 0 to \( \cos \phi \) in the given region. Therefore the region is given by \( \{ (\rho, \theta, \phi) : 0 \leq \theta \leq 2\pi, 0 \leq \phi \leq \frac{\pi}{4}, 0 \leq \rho \leq \cos \phi \} \).

(c) See Figure 9. The cone is written as \( \rho \cos \phi = \sqrt{3} \rho \sin \phi \); that is \( \phi = \frac{\pi}{6} \). For a fixed \( \phi \in [0, \frac{\pi}{6}] \), \( \rho \) varies from \( \sec \phi \) to \( 2 \sec \phi \) in the given region. Therefore the region is given by \( \{ (\rho, \theta, \phi) : 0 \leq \theta \leq 2\pi, 0 \leq \phi \leq \frac{\pi}{6}, \sec \phi \leq \rho \leq 2 \sec \phi \} \).

9. See Figure 10. Let us use the spherical coordinates. The equation \( z = \sqrt{x^2 + y^2} \) is written as \( \rho \cos \phi = \rho \sin \phi \). This implies that \( \tan \phi = 1 \), i.e., \( \phi = \frac{\pi}{4} \). The equation \( z = 4 \) is written as \( 4 = \rho \cos \phi \) that is \( \rho = \frac{4}{\cos \phi} \). Therefore \( \iiint_D \sqrt{x^2 + y^2 + z^2} \, dxdydz = 2\pi \frac{\pi}{4} \int_0^\pi \int_0^\frac{\pi}{4} \int_0^{\frac{\pi}{4}} \rho^2 \sin \phi \sin \phi d\rho d\phi d\theta = 2\pi \frac{\pi}{4} \int_0^\pi \int_0^\frac{\pi}{4} \frac{\pi}{4} \sin^2 \phi d\phi \).

10. See Figure 11. Solving \( x^2 + y^2 + (z - 1)^2 = 1 \) and \( x^2 + y^2 + z^2 = 3 \) implies that \( z = \frac{3}{2} \), i.e., \( \rho \cos \phi = \frac{3}{2} \). The equation \( x^2 + y^2 + (z - 1)^2 = 1 \) becomes \( \rho = 2 \cos \phi \) in the spherical coordinates. The required volumes is the sum of the volume of the portion of the region \( x^2 + y^2 + z^2 \leq 3 \) that lies inside the cone \( \phi = \frac{\pi}{6} \) and the volume of the portion of the region \( x^2 + y^2 + (z - 1)^2 \leq 1 \) that lies inside the sphere \( x^2 + y^2 + z^2 = 3 \). Therefore the required volume is given by \( \int_0^\frac{\pi}{6} \int_0^\frac{\pi}{4} \int_0^\frac{\pi}{4} \rho^2 \sin \phi \rho^2 \sin \phi d\rho d\phi d\theta + \int_0^\frac{\pi}{6} \int_0^\frac{\pi}{4} \int_0^{2 \cos \phi} \rho^2 \sin \phi \rho^2 \sin \phi d\rho d\phi d\theta \).