MTH102N
ASSIGNMENT–LA 1

(1) Let A, B be 2×2 real matrices such that $A\begin{pmatrix} x \\ y \end{pmatrix} = B\begin{pmatrix} x \\ y \end{pmatrix}$ for all $(x, y) \in \mathbb{R}^2$. Prove that $A = B$.

(2) It is easy to check that all the matrices given below

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},$$

satisfy the equation $A^2 = I_2$. Explain geometrically why it is happening.

(3) Explain geometrically, why

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

(4) Show that the matrix multiplication is associative and distributive over addition of matrices.

(5) Given $A = (a_{ij})$ define $A^t = (b_{ij})$ where $b_{ij} = a_{ji}$.
   (a) For two matrices A and B show that $(A + B)^t = A^t + B^t$ if $A + B$ is defined.
   (b) $(AB)^t = B^t A^t$ if AB is defined.

(6) Show that every square matrix can be written as a sum of a symmetric and a skew symmetric matrices. Further show that if A and B are symmetric then AB is symmetric if and only if $AB = BA$.
(A matrix A is called symmetric if $A = A^t$ and skew symmetric if $A = -A^t$)

(7) Let A and B be two $n \times n$ invertible matrices. Show that $(AB)^{-1} = B^{-1}A^{-1}$.

(8) Apply Gauss elimination to the following system

$$\begin{aligned} 2x + y + 2z &= 3 \\ 3x - y + 4z &= 7 \\ 4x + 3y + 6z &= 5 \end{aligned}$$

(9) Let A be a 2×2 real invertible matrices. Show that the image under A of
   (a) any straight line is a straight line
   (b) any straight line passing through origin is a straight line passing through origin.
   (c) any two parallel straight lines are parallel straight lines.

(10) Let A be a nilpotent ($A^m = 0,$ for some $m \geq 1$) matrix. Show that $I + A$ is invertible.

(11) If a $n \times n$ real matrix A satisfies the relation $AA^t = 0$ then show that $A = 0$. Is the same true if A is a complex matrix? Show that if A is a $n \times n$ complex matrix and $A\bar{A}^t = 0$ then $A = 0$.

(12) Let A and B be two $n \times n$ matrices.

(a) If $AB = BA$ then show that $(A + B)^m = \sum_{i=1}^{m} \binom{m}{i} A^{m-i} B^i$.

(b) Show by an example that if $AB \neq BA$ then a) need not hold.

(c) If Tr $(A) = \sum_{i=1}^{n} a_{ii}$ then show that Tr $(AB) =$ Tr (BA). Hence show that if A is invertible then Tr $(ABA^{-1}) =$ Tr (B).

(13) Find the inverse of

$$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}.$$