MTH102N
ASSIGNMENT–LA 3

(1) Show that the space of all real (respectively complex) matrices is a vector space over $\mathbb{R}$ (respectively $\mathbb{C}$) with respect to the usual addition and scalar multiplication.

(2) Let S =The set of all $n \times n$ skew hermitian matrices. Check whether S is a real (or complex) vector space under usual addition and scalar multiplication of matrices.

(3) In $\mathbb{R}$, consider the addition $x \oplus y = x + y - 1$ and $a.x = a(x-1)+1$. Show that $\mathbb{R}$ is a real vector space with respect to these operations with additive identity 1.

(4) Which of the following are subspaces of $\mathbb{R}^3$:

$$(a)\ \{(x,y,z) \mid x \geq 0\},\ (b)\ \{(x,y,z) \mid x+y=z\},\ (c)\ \{(x,y,z) \mid x=y^2\}.$$

(5) Which of the following are the subspaces of $\mathbb{C}^3$ (over $\mathbb{C}$):

$$(a)\ \{(z_1,z_2,z_3) \mid z_1 \text{ is real}\},\ (b)\ \{(z_1,z_2,z_3) \mid z_1+z_2=10z_3\}.$$

(6) Find the condition on real numbers a,b,c,d so that the set $\{(x,y,z) \mid ax + by + cz = d\}$ is a subspace of $\mathbb{R}^3$.

(7) Show that $W = \{(x_1,x_2,x_3,x_4) : x_4 - x_3 = x_2 - x_1\}$ is a subspace of $\mathbb{R}^4$, spanned by $(1,0,0,-1),\ (0,1,0,1)$ and $(0,0,1,1)$.

(8) Let W_1 and W_2 be subspaces of a vector space V such that $W_1 \cup W_2$ is also a subspace. Prove that one of the spaces $W_i,\ i = 1,2$ is contained in the other.

(9) Find all the subspaces of $\mathbb{R}^2$.

(10) Let $x,y,z \in V$ such that $x+2y+7z=0$. Show that $L(\{x,y\}) = L(\{y,z\}) = L(\{z,x\})$.

(11) Show that $(1,0),(i,0) \in \mathbb{C}^2$ are linearly independent over $\mathbb{R}$ and linearly dependent over $\mathbb{C}$.

(12) Let V be a vector space over $\mathbb{C}$ and let $x_1, \ldots, x_r \in V$ be linearly independent. Show that $x_1, \ldots, x_r, ix_1, \ldots, ix_r$ are linearly independent over $\mathbb{R}$.

(13) Discuss the linear dependence/linear independence of the following sets:

(a) $S = \{(1, 0, 0), (1, 1, 0), (1, 1, 1)\}$ of $\mathbb{R}^3$.

(b) $S = \{(1, 0, 0, 0), (1, 1, 0, 0), (1, 2, 0, 0), (1, 1, 1, 1)\}$ of $\mathbb{R}^4$.

(c) $S = \{(1, i, 0), (1, 0, 1), (i + 2, -1, 2)\}$ of $\mathbb{C}^3(\mathbb{C})$.

(d) $S = \{(1, i, 0), (1, 0, 1), (i + 2, -1, 2)\}$ of $\mathbb{C}^3(\mathbb{R})$.

(14) Under what conditions on α are the vectors $(1 + \alpha, 1 - \alpha)$ and $(1 - \alpha, 1 + \alpha)$ of $\mathbb{C}^2$ are linearly independent over $\mathbb{R}$?