SECOND MIDSEMESTER EXAMINATION
MTH102N
SECOND SEMESTER, 2008-09
TIME: 1 HOUR

Marks: 40

Note: *a) All matrices have real entries. b) If A is a $n \times n$ matrix then $|A|$ stands for the determinant of A. c) I_n stands for the identity matrix of order n.*

1.a) Describe the image of the set $\{z \in \mathbb{C} : |z| \leq 1, \mathrm{Re}(z) \geq 0\}$ under the Möbius transformation $f(z) = \dfrac{1+z}{1-z}$.

b) Find the residue of the function $f(z) = \dfrac{\sinh z}{z^3}$ at $z = 0$. [5+5]

2.a) Evaluate the integral $\displaystyle\int_0^{2\pi} \frac{d\theta}{1 + a\cos\theta}$, where $|a| < 1$.

b) Let A and B be symmetric $n \times n$ matrices. Prove that AB is a skew symmetric matrix if and only if $AB = -BA$. [5+5]

3.a) Given a $n \times n$ matrix A define $\mathrm{Tr}(A) = \sum_{i=1}^{n} a_{ii}$. Prove that $\mathrm{Tr}(A - B) = \mathrm{Tr}(A) - \mathrm{Tr}(B)$. Hence prove that there does not exist any pair of $n \times n$ matrices A, B such that $AB - BA = I_n$.

b) Let A and B be $n \times n$ matrices. Evaluate the determinant of the $2n \times 2n$ matrix

$C = \begin{pmatrix} A & 0 \\ 0 & B \end{pmatrix}$ in terms of determinant of A and B.

(0 *stands for* $n \times n$ *zero matrix*). [5+5]

4.a) Let A be a $n \times n$ matrix with $|A| = 1$. Evaluate $\mathrm{Adj}(\mathrm{Adj}(A))$ in term of A.

b) Let A be a $n \times n$ matrix. Prove that, if the equation $Ax = 0$ has $x = 0$ as the only solution then A is invertible. [5+5]