TUTORIAL 2 EXERCISES MTH 301 (NEED NOT BE SUBMITTED)

August 4, 2011

Fix $b \in \mathbb{R}$, $b > 1$

(i) If m, n, p, q be integers $n > 0$, $q > 0$ and $r = \dfrac{m}{n} = \dfrac{p}{q}$, then show that
$$(b^m)^{1/n} = (b^p)^{1/q}.$$
Hence define $b^r = (b^m)^{1/n}$.

(ii) Prove that $b^{r+s} = b^r.b^s$ if r and s are rational.

(iii) If x is real, then define $B(x) = \{b^t \mid t \in \mathbb{Q}, t \leq x\}$. Prove that if r is rational then $\sup B(r) = b^r$.
Define, $b^x = \sup B(x)$.

(iv) Prove that $b^{x+y} = b^x.b^y$ for all x, $y \in \mathbb{R}$.