## Tutorial I

1. Use induction to show that $n! \geq 2^n$ for all $n > 4$.

2. Use induction to prove the following statement also known as the Well Ordering Principle. Every non-empty subset of natural numbers has a least number, namely, if $A \neq \emptyset$ is a subset of $\mathbb{N}$ then there exists $a \in A$ such that $a \leq k$ for all $k \in A$. (Hint: Assume that $A$ has no least number. Consider the set $B$ of lower bounds of $A$. Note that $B$ and $A$ cannot have any numbers in common. Now use induction to complete proof by contradiction.)

3. Let $a$,$b$ and $c$ be rational numbers. Use the fact that $\mathbb{Q}$ is an ordered field to prove the following (this means you can use the 11 properties we stated for a field in class and the 2 properties needed for a field to be ordered).
   (i) $-(-a) = a$.
   (ii) If $a \neq 0$ and $ab = ac$ then $b = c$.
   (iii) $a.0 = 0$.
   (iv) If $a > 0$ then $ab > 0$ implies $b > 0$. (Prove by contradiction)
   (v) If $0 < a < b$ then $0 < \frac{1}{b} < \frac{1}{a}$. (Hint:1 $> 0$ and previous argument)

4. Let $A = \{p \in \mathbb{Q} \mid p > 0, p^2 < 2\}$ and $B = \{p \in \mathbb{Q} \mid 2 < p^2, p > 0\}$.
   (i) Let $q = p - \dfrac{p^2 - 2}{(p+2)}$. Show that $q > p$ if $p \in A$ and $q < p$ if $p \in B$.
   (ii) Show that $q^2 - 2 = \dfrac{2(p^2-2)}{(p+2)}$.
   (iii) Show that $A$ does not have least upper bound in $\mathbb{Q}$ and $B$ does not have greatest lower bound in $\mathbb{Q}$.

Following at problems from Chapter 1, in the textbook

Chapter 1, Pb 4 Let $A$ be an nonempty subset of $\mathbb{R}$ that is bounded above. Show that there is a sequence $(x_n)$ of elements of $A$ that converges to $supA$.

Chapter 1, Pb. 7 If $a < b$ then there is also an irrational $x \in \mathbb{R}/\mathbb{Q}$ with $a < x < b$.[ Hint: Find an irrational of the form $\dfrac{p\sqrt{2}}{q}$]