MTH 301 Tutorial V
Due on September 6, 2011
Submit only the starred exercises

1. Show that every open interval (and hence every open set) in $\mathbb{R}$ is a countable union of closed itnervals and that every closed interval in $\mathbb{R}$ is a countable intersection of open intervals.

2*. Let $e^{(k)} = (0, \cdots, 1, \cdots, 0)$ where the kth entry is 1 and the rest are 0. Show that $\{e^{(k)} \mid k \geq 1\}$ is a closed subset of l_1.

3.* Show that the set $A = \{x \in l_2 \mid |x_n| leq 1/n, n \in \mathbb{N}\}$ is a closed subset of l_2 but that $B = \{x \in l_2 \mid |x_n| < 1/n\}$ is not an open set. (Hint: Does $B(0, \epsilon) \subset B$?).

4. Given a nonempty bounded subset E of $\mathbb{R}$ show that $\sup E$ and $\inf E$ are elements of $\overline{E}$. Thus $\sup E$ and $\inf E$ are in E wherenever E is closed.

5.* Let A and B be subsets in a metric space (M, d).Show that $\overline{A \cap B} \subset \overline{A} \cap \overline{B}$. Give an example showing that this inclusion can be proper.

6.* If $x \neq y$ in a metric space (M, d) show that there exist disjoint open sets U, V with $x \in U$ and $y \in V$ such that U and V are disjoint.

7.* Let A be subset of a metric space (M, d). Show that if x is a limit point of A then every neighbourhood of x contains infinitely many points of A.

8. A metric space is separable if it has a countable dense subset. Show that $\mathbb{R}^2$ and l_2 are separable. (Look at problem 49 on page 59 in the book for hint).