Math 301 Tutorial 7

(1) If $A \subset B \subset \overline{A} \subset (M, d)$, and if A is connected, show that B is connected. In particular, $\overline{A}$ is connected. (Problem 9 page 82).

(2) Prove that there does not exist a continuous function $f : \mathbb{R} \to \mathbb{R}$ satisfying $f(\mathbb{Q}) \subset \mathbb{R} - \mathbb{Q}$ and $f(\mathbb{R} - \mathbb{Q}) \subset \mathbb{Q}$. (Problem 17, page 83).

(3) If $f : \mathbb{R} \to \mathbb{R}$ is continous and one-one, show that f is strictly monotone. (Problem 16, page 83)

(4) Suppose that M is compact and that $f : M \to N$ is continuous, one-one and onto. Prove that f is a homeomorphism. (Problem 23, page 111)

(5) Show that a subset A of $\mathbb{R}$ is totally bounded if and only if it is bounded. (Problem 2, page 90)

(6) Give an example of a closed bounded subset of ℓ_∞ that is not totally bounded. (Problem 9, page 92)

(7) If K is a non-empty compact subset of $\mathbb{R}$, show that $\sup K$ and $\inf K$ are elements of K. (Problem 1, page 109)

(8) Let $E = \{x \in \mathbb{Q} : 2 < x^2 < 3\}$ considered as a subset of $\mathbb{Q}$ (with its usual metric). Show that E is closed and bounded but not compact. (Problem 2, Page 109)

(9) Show that the Hilbert cube H^∞ is compact.(Problem 14, page 110)