

MATH 301 TUTORIAL 9

- (1) Page 133, Problem 14. Prove that A has an empty interior in M if and only if A^c is dense in M .
- (2) * Page 133, Problem 15. If G is open and dense in \mathbb{R} , show that the same is true of $G - x$ for any $x \in \mathbb{R}$. Is this true in any metric space? Explain.
- (3) * Prove that a complete metric space without isolated points is uncountable. In particular, this gives another proof that Δ is uncountable.
- (4) Show that \mathbb{N} is first category in \mathbb{R} but second category in itself. Show that \mathbb{Q} is first category in itself. (check for definition of first category and second category on page 132)
- (5) * Page 149, Problem 7. Let (f_n) and (g_n) be sequences of real-valued functions on a set X and suppose that (f_n) and (g_n) converge uniformly. Show that $(f_n + g_n)$ converges uniformly. Given an example showing that $f_n g_n$ may not converge uniformly (it will of course converge pointwise).
- (6) * Page 149, Problem 9.
- (7) * Page 151, Problem 18.
- (8) Page 153, Problem 19.
- (9) * Page 159, Problem 29.
- (10) Page 159, Problem 31.