MTH 751 Assignment 1
to be submitted on January 17, 2012

(1) Let K is a subgroup of finite index in G and $H < G$. Show that $[H : H \cap K] = [G : K]$ if and only if $G = KH$.

(2) Show that if H and K are subgroups of finite index of a group G. Then $[G : H \cap K]$ is finite and $[G : H \cap K] \leq [G : H][G : K]$. Further show that $G = HK$ if and only if $[G : H \cap K] = [G : H][G : K]$.

(3) Let H and K be groups of finite index of a group G. Given that $[G : H]$ and $[G : K]$ are relatively prime show that $G = HK$. What can you say about the isomorphism classes of groups of order pq where $p \neq q$ are both prime?

(4) Problems from Herstein, Chapter 2

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