MTH 751 Assignment 2

to be submitted on January 27, 2012 before 11 AM

(1) Let G be a group such that $\mathrm{Aut}(G)$ is cyclic. Show that G is abelian.

(2) Show that a group of order 40 cannot be simple.

(3) Let G be a simple group of order 60.
   (a) Show that the action of G by conjugation on the set of Sylow subgroups gives an imbedding $G \hookrightarrow A_6$.
   (b) Use part (a) to show that G is isomorphic to A_5.
   (c) Show that A_6 has an automorphism which is not induced by an inner automorphism of S_6.

(4) Let S be a set with at least 2 elements and G be a finite group acting on S. If this action has only one orbit show that there exists $g \in G$ such that $g.s \neq s$ for all $s \in S$.

(5) Show that if H is a proper subgroup of a finite group G then G cannot be written as a union of the conjugates of H.