MTH 751 Tutorial 2
Discussion problems

(1) Read sections 2.8 and 2.9.

(2) If $H < G$ then the factor group $N_G(H)/C_G(H)$ is isomorphic to a subgroup of $\mathrm{Aut}(H)$.

(3) Let G be a group acting on a set S containing at least two elements. Assume that G is transitive; that is, given any $x, y \in S$, there exists $g \in G$ such that $gx = y$. Prove
   (a) for $x \in S$, the orbit x of x is S;
   (b) all the stabilizers G_x (for $x \in S$) are conjugate;
   (c) if G has the property: $\{g \in G \mid gx = x, \forall x \in S\} = \{e\}$ (which is the case if $G < S_n$ for some n and $S = \{1, 2, ..., n\}$) and if $N \triangle G$ and $N < G_x$ for some $x \in S$, then $N = \{e\}$;
   (d) for $x \in S$, $|S| = [G : G_x]$; hence $|S|$ divides $|G|$.

(4) Exhibit an automorphism of $\mathbb{Z}_6$ that is not an inner automorphism.

(5) If $G/Z(G)$ is cyclic, then G is abelian.

(6) Problems from Herstein, Chapter 2

   (a) Page 71; Problems 1, 2 , 5, 16, 19.