Financial Constraints, Inventory Investment, and Fixed Capital

by
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Abstract
In a series of empirical studies Fazzari and Petersen (FP) and their associates examined the changes in the stock of fixed capital and inventory investment of firms when they encounter short run and/or sporadic financial constraints. In general, they consider the balance sheet constraint as the source of adjustments. In particular, they argue that the cost differential between external and internal finances makes adjustments in capital investments difficult. Hence, inventory investment bears the brunt of the adjustment. However, a myopic firm may prefer to increase sales and augment short term cashflows when confronted with a financial constraint. Inventory investment may decrease along with a reduction in fixed capital investments. The firm has many more options if the financial constraints persist. A more satisfactory theoretical explanation for the relationship between the financial constraints and investments in inventories and fixed capital is therefore necessary. This study sets up a comprehensive theoretical framework and demonstrates that changes in cost of production and other logistic costs will be the primary channel through which financial constraints affect investment in inventories and fixed capital. Many other important insights into the transmission mechanism have been highlighted.

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1. The Background

Theoretical studies generally postulate that financial markets are competitive. Under these conditions firms initially choose a time profile of production, inventories, and investment in fixed capital assets to satisfy the demand for their products\(^2\). This will define their financial requirements over time. The firms can obtain these finances at the market determined interest rate. As such, the cost of financing inventory investment and fixed capital assets, as reflected in the market rate of interest, is the only financial constraint. In general, it is expected that there will be a decrease in both types of investment, via the cost effect, if there is an increase in the interest rate.

By way of contrast, much of the current literature acknowledges that capital markets are imperfect. Fundamentally, capital market imperfection results in differential credit ratings of firms based on the market value of their assets. Hence, both the interest rate and the quantum of finances available to the firm will be affected. Thus, it is necessary to acknowledge that quantitative limits on finances, in addition to the interest rate, affect both types of investment.

The usual assumption is that the changes in investment in fixed capital and inventory investment are determined by the balance sheet constraint\(^3\). One possibility is that product markets are oligopolistic or monopolistic competition. Hence, it may be argued that firms must utilize, on a priority basis, any long term investment opportunities they can identify. For, there is a danger of decreasing their market value if there is any delay. Further, from a practical viewpoint, there can be several institutional and structural rigidities that prevent the firm from canceling orders for machinery and equipment at

\(^1\) I benefited from discussions with Surajit Sinha and Surajit Bhattacharyya. The responsibility for the contents is, however, my own.

\(^2\) It is generally assumed that the demand must be satisfied as and when it arises. However, Blinder (1982) makes the sales decision endogenous. This more general approach will be followed in this study.

\(^3\) Analytical studies dealing with financial constraints, such as Calomiris and Hubbard (1990), Bernanke and Gertler (1995), and Hubbard (1998), identified the interest rate, bank credit, and the balance sheet constraint as the primary channels through which capital market imperfections are transmitted to investment decisions.
short notice. In general, even when the firm experiences a financial constraint, it may be unrealistic to expect it to forego such opportunities for investment and divert long term finances to working capital. Fazzari et al (1988), Fazzari and Petersen (1993), and Carpenter et al (1998) argue that firms tend to reduce inventory investment\(^4\) and forego short term profits, if necessary, to maintain their long run market share\(^5\).

It is equally plausible to argue that when confronted with unfavorable market conditions\(^6\) myopic firms will assign a priority to the utilization of the existing capital stock to augment cashflows. In general, the firm may reduce the volume of inventories by either increasing sales (that augment the availability of short term finances)\(^7\) or by

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\(^4\) Fazzari and Petersen (1993, p.329) argued that financially constrained firms “can offset the impact of cashflow shocks on fixed investment by adjusting working capital, even setting working capital investment at negative levels. These actions release short-run liquidity, allowing firms to smooth fixed investment relative to cashflow shocks. The marginal opportunity cost of adjusting working capital in this manner, and therefore the extent of investment smoothing, should depend on the firm’s initial stock of working capital, a variable related to the strength of its balance sheet.” It is important to recognize, following Fazzari and Petersen (1993, p.331) that “working capital investment may be temporarily negative if firms consume raw material inventories faster than they are replaced. Firms can also liquidate working capital by intensifying efforts to collect accounts receivable … or by tightening credit policies on new sales, resulting in lower-than-normal accounts receivable per dollar of sales.”

\(^5\) When production is flexible ex post, i.e., after the market demand is realized, firms have a better chance of catering to the demand as it arises. For, such flexibility allows the firm to reduce the marginal cost of production. See, for instance, Flacco and Kroetch (1986) and Aiginger (1985, 1987). The consumer may, in this sense, consider the firm with a larger capital stock as a more reliable supplier. Firms make attempts to augment production capacity for this reason as well. On the contrary, firms may find it difficult to adjust production, at short notice, to changing market conditions. In such a case, they may sell out of inventory stock. Perforce, the stability of the market share depends on the stock of inventory. Further, as Langlois (1989, p.50) noted “the holding of inventory, by making a product available to the consumers on demand (or within a short period of time), generates in and of itself a positive demand for a firm’s products.” Similarly, Ware (1985, p.93) argued that a large inventory stock can have a barrier to entry effect. For, the threat to sell off inventory by temporarily reducing price can deter entry. In such a case, fixed capital investments may not receive the priority implied here.

\(^6\) Clearly, the cashflow and financial constraints are a result of these market conditions.

\(^7\) This is generally achieved by reducing price. However, this may not be sufficient in the context of durable goods industries. For, the firms may be forced to extend credit to conduct sales. When there is a financial crunch, the inability of the firm to conduct sales also implies that there is a further reduction in the flow of finances. See, for instance, Darbha (2000) and Fafchamps et al (2000).
reducing production (thereby reducing the demand for such finances). The priority to fixed capital investment is neither necessary nor sufficient to observe a reduction in inventory investment if this pattern is observed. This alternative must be evaluated against the above argument.

Assume that adequate market opportunities exist. In such a case, the reduction in cashflow may be entirely due to an inefficient operation of the firm. One source of such inefficiency may be the low stock of capital. It is possible to reduce costs by increasing investments in fixed capital if the firm is operating on the decreasing portion of the long run average cost curve. The firm may choose this even if inventory investment must be reduced temporarily. The other possibility is that the firm is undertaking more activities (of production and distribution) than it can manage efficiently. In such a case some organizational restructuring may reduce costs. For instance, the firm may divest its marketing network and entrust it to a subcontractor. This decision improves efficiency and augments cashflows. It also has implications for productive fixed capital and inventory investment of the firm.

Consider the credit ratings and market valuation of the common stock of the firm in secondary markets. The shareholders will not, in general, know the efficiency of operations of the firm. They utilize signals like dividend payments and announcements about fixed capital investments to calibrate the financial wellbeing of the firm. The management may assign priority to such decisions even when product markets are not favorable and cashflows are deficient. This may not be desirable from an efficiency point of view. But it can constitute an explanation for an increase in capital investments along with a reduction in inventory investment.

For all practical purposes it can be concluded that invoking the balance sheet constraint alone does not provide a satisfactory explanation for the observed patterns of adjustment. It is necessary to identify deeper causative relationships.

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8 Benito and Hernando (2002, p.3) noted the following. As they put it, ”in a mechanical sense, a key channel for the operation of monetary policy through the corporate sector is by altering borrowing costs. Monetary policy thereby imposes, or alleviates, financial pressure on firms. But in a behavioral sense, relatively little is known about how this affects firms and the actions taken by companies as a response. Adjustments by companies can potentially involve a wide range of activities with the most prominent relating to their investment decisions, human resource policies and financial policies.”
The basic purpose of the present study is to offer a theoretically satisfactory explanation of the channels through which financial constraints affect investment in fixed capital and inventory investment. It is a fundamental contribution in so far as it demonstrates, in a comprehensive theoretical framework, the necessity for cost reduction, achieved with an increase in the stock of capital, as the primary channel through which financial constraints affect investments in fixed and working capital.

The rest of the study is organized as follows. Section 2 examines the basic model in which efficiency considerations are paramount. Section 3 highlights organizational changes, in addition to augmentation of production capacity, as a response to financial constraints. This approach is also a result of efficiency considerations. Section 4 examines the possibility that efficiency considerations are not the primary consideration. Instead, the firm may consider the valuation of its common stock in secondary markets as the major concern. The possibility of increases in fixed capital investments along with a reduction in inventory investment is once again discernible. Section 5 contrasts these results and identifies the necessity for empirical evaluation. For, the increase in fixed capital will not be sustainable in the long run if it is not a result of efficiency considerations.

2. The Basic Model

Most of the theoretical studies consider inventory investment and investment in fixed capital assets as unrelated phenomenon. For example, the Blinder (1982) model and its variants examine only inventory investment. Similarly, Jorgenson (1963) and related studies emphasize capital investments. Vickers (1968, 1987) provides some analytical extensions that include the financial constraint. But, the reasons for the substitutability between fixed capital assets and inventory investment have been scarcely addressed. However, both the models alluded to above are amenable to extension and synthesis. This section presents a fundamental approach to this issue.

2.1. Some Notation

Similarly, Chirinko (1993, section V) alluded to the necessity for theoretical research on the several margins at which financially constrained firms operate. It is rather surprising that the development of such a theory alluded the economic theorists for so long.
Consider the fixed capital of the firm. A part of this is generally necessary to undertake production. Let
\[ K = \text{stock of fixed capital assets used in production} \]
Firms generally own and utilize other fixed assets as well. Land and buildings, marketing and other logistic networks are the other major component. Let
\[ X = \text{other fixed assets of the firm} \]
The short term assets of the firm can be represented by
\[ I = \text{inventories + net trade credit} \]
Long term assets will be generally financed by
\[ E = \text{equity capital} \]
\[ D = \text{long term borrowing or debt}^{10} \]
and internal sources. Let
\[ Z = \text{reserves and surpluses of the firm}^{11} \]
Assume that a fraction \( \beta \) of \( Z \) is used to finance fixed capital and inventories. Clearly, the rest of \( Z \) is a precautionary holding to tide over unexpected contingencies\(^{12} \). Assume further that a fraction \( (1 - \phi) \) of \( \beta Z \) is marked for financing fixed capital. Then, it follows that
\[ D = K + X - E - (1 - \phi)\beta Z \leq M^* \]
where
\[ M^* = \text{maximum long term borrowing available to the firm} \]

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9 Gurley and Shaw (1960) acknowledged the interrelationship between the real and financial variables. However, their models do not seem to be amenable to answer the questions that need to be considered in the present context.

10 It will be instructive to consider the endogenous choice of financial instruments. In particular, the decision concerning the capital structure represented by the choice of the debt equity ratio. For, it has important implications for the cost of capital and the market value of the firm. A somewhat different modeling framework will be necessary to incorporate this effect.

11 As Chandra (1997, p.541) observed, \( Z \) is held in different accounts. Each of these represents a different use of reserves and surpluses of the firm. In general, capital reserves may be used exclusively to finance fixed capital. But revenue reserves can be used with greater flexibility.

12 Hence, either the market demand and/or the cost may be affected by the choice of \( \beta \). An increase in \( \beta \), and the resulting change in \( I \) and \( K \), has its own effects on the revenue
Short term assets are financed by short term borrowing and the remaining reserves and surpluses. Hence,

\[ B = I - \phi \beta Z \leq M \]

where

\( M \) = limit on the availability of short term borrowing

Note that the sources of \( M^* \) and \( M \) can be different. For instance, \( M \) is primarily borrowing from banks. On the other hand, \( M^* \) is acquired from the public (by issuing bonds) and/or term lending institutions. The terms and conditions of these borrowings may be different\(^{13}\). Hence, it will be realistic to assume that each of these sources can be used only for specific purposes. The substitution between capital assets and inventories can be brought about by adjusting \( \phi \) alone.

Assume that

\( R = \) sales revenue of the firm

\( C = \) cost of production

\( G = \) gross investment in fixed capital

It should be acknowledged that some organizational rearrangements will be necessary to introduce new capital into the existing assembly lines. Hence, in addition to the cost of acquiring physical capital there will be fairly significant adjustment costs as well. An attempt can be made to account for this by writing

\[ g = \text{cost of acquisition of fixed capital plus adjustment cost} \]

and cost of the firm. An optimal value of \( \beta \) can be conceptualized by taking these into account.

\(^{13}\) Fazzari et al (1988, p.151) noted the following. “Because creditors understand the conflicts of interest that exist between themselves and equity holders, they demand covenants that restrict the behavior of managers, particularly with respect to new debt issues. As a result, covenants typically stipulate target debt-equity ratios. While they may provide a second best solution to the contracting problem given the potential for opportunism, they are not costless, and their restrictions on financial flexibility limit management’s choices of investment opportunities, as well as the ability to finance investment opportunities when internal funds are low. If covenants impose working capital requirements, for example, the supply of internal funds available to finance investment may be reduced. Hence, shocks to working capital, such as a debt deflation or a decline in internal finance, will make debt finance more expensive at the margin, probable at a time when the need for new debt is most acute.”
= g(G); g₁ > 0, g₁₁ > 0\(^{14}\)

Let

\(f(I) = \text{cost of holding inventory}\)

It is reasonable to assume that \(f₁, f₁₁ > 0\)

Let

\(r = \text{interest rate on borrowing of both types}\)^\(^{15}\)

It is then obvious that

\(F = \text{cashflow of the firm}\)

\[= R - C - g(G) - r(K + X + I - E - \beta Z) - f(I)\]

Let

\(\theta = \text{fraction of the cashflow paid out as dividends to shareholders}\)

Clearly,

\[\frac{dZ}{dt} = (1 - \theta)F - \beta Z\]

### 2.2. Effect of Capital Stock

In consonance with the FP argument it will be postulated that there will be a reduction in the market share of the firm if it does not take advantage of investment opportunities as they arise. Or, stated more positively, the consumers consider a firm to be a more reliable supplier if it has a larger productive capacity. Hence, it will have the effect of shifting the revenue function up. This will be represented by

\(R = \text{revenue of the firm}\)

\[= R(S, K); R₁ > 0, R₂ > 0\]

where

\(^{14}\) Clearly, \(g₁ = \frac{dg}{dG}\). Similarly, \(g₁₁ = \frac{dg₁}{dG}\). This notation will be maintained throughout.

\(^{15}\) Fazzari et al (1988, p.142) argue that the differences in \(r\) across different sources of finance determine the optimal financial mix and, in particular, the choice of \(\beta\). As they put it, “if the cost disadvantage of external finance is small, retention practices should reveal little or nothing about investment: firms will simply use external funds to smooth investment when internal finance fluctuates, regardless of their dividend policy. If the cost disadvantage is significant, firms that retain and invest most of their income may have no low cost source of investment finance, and their investment should be driven by fluctuations in cashflow.” Benito (2002) and Benito and Hernando (2002) offered some empirical evidence on this simultaneous determination of several financial and other
S = volume of sales
It will also be postulated that $R_{11}, R_{12} < 0$, and $R_{22} > 0$. As usual, $R_{11} < 0$ indicates a negative slope of the demand curve, $R_2 > 0$ a shift to the right of the demand curve as the stock of capital increases, and $R_{22} > 0$ is an acknowledgement of increasing returns to K in the relevant range. Similarly, $R_{12} < 0$ implies that firms have greater monopoly power if they have a larger stock of capital.

It is equally important to recognize that the cost of production depends on the volume of output as well as the stock of capital. In general,

$$C = \text{cost of production of the firm}$$

$$= C(Y,K) ; C_1 > 0, C_{11} > 0, C_{12} < 0, \text{and } C_{22} < 0$$

where

$Y = \text{volume of production}$

That is, it will be assumed that the firm experiences positive and increasing marginal cost of production. However, the most favorable conditions, for the increase in K, are those in which the long run average cost curve is decreasing. Hence, the specification $C_{12}, C_{22} < 0$ reflects this. The other cost effect is through the financial requirements. For, the cost of finances utilized by the firm will be $r(I + K + X - E - \beta Z)$, where

$r = \text{rate of interest on borrowings}$

2.3. The Specification

Much of the literature, including the formulations of Jorgenson (1963) and Blinder (1982), consider the market value of the firm as the primary concern. Consequently, the ability to generate cashflows through the use of the different assets of the firm will be of critical importance since the payments the firm can make at the time of liquidation depend on it.

The firm therefore

$$\text{Maximizes } \int_{0}^{\infty} e^{-\alpha t} F \, dt$$

where

policies of firms. However, this line of reasoning will not be pursued any further in the rest of the study.
\( \alpha = \) rate of discount applicable to the firm

Clearly, this quantity represents the present discounted value of the expected cashflows.

For all practical purposes it defines the market value of the firm.

The optimization is subject to the following constraints. First, the capital stock depreciates over time. Hence,

\[ \frac{dK}{dt} = \text{net addition to the stock of capital} \]

\[ = G - \delta K \]

where

\( \delta = \) rate of depreciation of the capital stock

Second, the output produced will accumulate as inventory if it is not sold. That is,

\[ \frac{dI}{dt} = Y - S \]

Third, the reserves and surpluses of the firm will be governed by

\[ \frac{dZ}{dt} = (1 - \theta)F - \beta Z \]

Fourth, the financial constraints are

\[ I - \phi \beta Z \leq M \]

and

\[ K + X - E - (1 - \phi) \beta Z \leq M^* \]

It is equally important to note that \( \phi, Y, S, G, \) and \( X \) are the basic decision variables\(^{17}\). As noted earlier, whenever the financial constraint is binding, the nature of substitution between \( K \) and \( I \) depends on whether \( G \) or \( X \) is the active decision. \( G \) will be considered as the active decision to begin with.

The complete specification of the problem is to choose \( \phi, Y, S, \) and \( G \) so as to

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\(^{16}\) By convention, the balance sheet records the book value of the firm. As such the market value of the common stock of the firm in secondary markets is not accounted for.

\(^{17}\) Note that \( \beta \) is taken to be exogenous. \( Z \) may appear in the R and C functions depending on the nature of contingencies and the use of reserves and surpluses. As \( M \) and/or \( M^* \) increases the firm may increase \( Z \) and create some profit opportunities. However, the firm may also increase \( K \) and/or \( I \) depending on the relative valuation of profit opportunities. The channels of transmission remain the same. Hence, making \( \beta \) endogenous will not change the results substantially. However, note that there is hardly any theoretical and/or empirical information about the reasons for the firm holding reserves and surpluses even when it is experiencing financial constraints. This is a worthwhile area for future research.
Max \( \int_0 e^{-\alpha t} [R(S,K) - C(Y,K) - r(I + K + X - E - \beta Z) - g(G) - f(I)] dt \)

Subject to
\[
\begin{align*}
\frac{dK}{dt} &= G - \delta K \\
\frac{dI}{dt} &= Y - S \\
\frac{dZ}{dt} &= (1 - \theta)F - \beta Z \\
I - \phi \beta Z &\leq M \\
K + X - E - (1 - \phi) \beta Z &\leq M^* 
\end{align*}
\]

### 2.4. Optimal Choices

The solution can be constructed as follows. Write the Hamiltonian as
\[
H = e^{-\alpha t} [R(S,K) - C(Y,K) - r(I + K + X - E - \beta Z) - g(G) - f(I)] + \lambda e^{-\alpha t} (G - \delta K) + \mu e^{-\alpha t} (Y - S) + \psi e^{-\alpha t} [(1 - \theta)F - \beta Z] + ve^{-\alpha t} (I - \phi \beta Z - M) + v^* e^{-\alpha t} [K + X - E - \beta(1 - \phi)Z - M^*]
\]

The expected market value of the firm consists of the valuation of all its assets. Current cashflow is the first expression. \( \lambda \) can be interpreted as the expected market value of a unit of fixed capital of the firm. For, at any point of time it is basically the resale value of the capital assets that is available to pay the shareholders in case of liquidation. Similarly, \( \mu \) is the market value generating potential of a unit of inventories and \( \psi \) is the market value that can be generated by the use of a unit of reserves and surpluses. The ability of the firm to utilize credit to conduct its operations can also be viewed as an asset. Hence, \( v \) and \( v^* \) represent the expected market value generating potential of a unit of finances.

From Pontryagin’s maximum principle it is well known that
\[
\begin{align*}
\frac{d(\lambda e^{-\alpha t})}{dt} &= e^{-\alpha t} (-R_2 + C_2 + r + \lambda \delta - v^*) \\
\frac{d(\mu e^{-\alpha t})}{dt} &= e^{-\alpha t} (f_1 + r - v) \\
\frac{d(\psi e^{-\alpha t})}{dt} &= e^{-\alpha t} \beta [ -r + \psi + v \phi + v^* (1 - \phi)]
\end{align*}
\]

Suppose \( I - \phi \beta Z < M \). Then, the finance that the firm is able to utilize is less than the amount available. That is, the firm feels that additional use of finance cannot add to the market value of the firm. On the other hand, if \( I - \phi \beta Z = M \) the firm can utilize the last unit of finance productively and generate additional market value. Hence, \( v = 0 \) if \( I - \phi \beta Z < M \)
> 0 if \( I - \phi \beta Z = M \)

This is the Kuhn-Tucker condition. Further, note that diminishing returns to the use of finances can be expected even if the financial constraint is binding. That is, in general, \( \frac{dv}{dM} < 0 \)

A similar condition holds with respect to \( \nu^* \) as well. Note further that \( \phi \) is optimum only if \( \nu = \nu^* \). For, as Fazzari and Petersen (1993, p.331) noted, “finance constraints pose no barrier to equating the marginal returns across different assets, net of adjustment costs, at each point of time. That is, the firm will equate marginal returns on all assets to a shadow value of finance.”

Observe that the cost of using a unit of finance is \( r \), the rate of interest. Hence, when the financial constraint is binding, the net additional value of the use of finances must be equal to \( r \). That is, \( \nu = r = \nu^* \) for all values of \( M \) when the financial constraint is binding.

Consider the choice of \( \beta \). It will be such that

\[ I - \phi \beta Z = M \]

if the short term financial constraint is binding. The value of \( X \) will be determined residually if, in addition, the long term financial constraint is binding.

Turning to the optimal choices of \( Y, S, \) and \( G \) note that they will be determined by the following equations.

\[ C_1(Y,K) = \mu(I,\nu) \]
\[ R_1(S,K) = \mu(I,\nu), \text{ and} \]
\[ g_1(G) = \lambda(Y,S,K,\nu) \]

Similarly, the relevant second order conditions for maximum are

\[ C_{11} + E_1f_{11} > 0 \]
\[ R_{11} - E_1f_{11} < 0, \text{ and} \]
\[- (C_{11} + E_1f_{11}) (R_{11} - E_1f_{11}) - E_1^2f_{11} > 0 \]

Observe that

\[ \lambda = \lambda_0 e^{(\alpha + \delta)t} - \frac{1}{(\alpha + \delta)} [C_2 - R_2 + r - \nu^*/(\alpha + \delta)] [1 - e^{(\alpha + \delta)t}] \]
\[ \mu = \mu_0 e^{at} - [(f_1 + r - \nu)/\alpha] (1 - e^{at}) \]

The optimal quantities \( Y, S, \) and \( G \) can be determined from the above three optimality conditions and the two differential equations governing \( K \) and \( I \). Write
\[
d\frac{K}{dt} = K - K_0 = G - \delta K
\]

Hence,
\[
(1 + \delta) \frac{dK}{dM} = \frac{dG}{dM}
\]

Similarly,
\[
\frac{dI}{dt} = I - I_0 = Y - S
\]

so that
\[
\frac{dI}{dM} = \frac{dY}{dM} - \frac{dS}{dM}
\]

Let
\[
E_1 = \frac{(1 - e^{\alpha t})}{\alpha}, \text{ and}
\]
\[
E_2 = \frac{[1 - e^{(\alpha + \delta)t}]}{(\alpha + \delta)}
\]

From the equation
\[
C_1(Y,K) = \mu(I,\nu)
\]

it follows that
\[
C_{11} \frac{dY}{dM} + C_{12} \frac{dK}{dM} = - E_1[f_{11} \frac{dI}{dM} - (d\nu/dM)]
\]

Similarly, differentiate
\[
R(S,K) = \mu(I,\nu)
\]

with respect to M. It can be verified that
\[
R_{11} \frac{dS}{dM} + R_{12} \frac{dK}{dM} = - E_1[f_{11} \frac{dI}{dM} - (d\nu/dM)]
\]

The optimality condition with respect to G yeilds
\[
g_{11} \frac{dG}{dM} = E_2 [-C_{12} \frac{dY}{dM} + R_{12} \frac{dS}{dM} + (R_{22} - C_{22}) \frac{dK}{dM} + (d\nu/dM)]
\]

Straightforward algebraic manipulations suggest that \(\frac{dY}{dM}\) and \(\frac{dS}{dM}\) can be solved from the equations\(^{18}\)
\[
a_{11} \frac{dY}{dM} + a_{12} \frac{dS}{dM} = b_1
\]
\[
a_{12} \frac{dY}{dM} + a_{22} \frac{dS}{dM} = b_2
\]

where
\[
a_{11} = C_{11} + f_{11} E_1 - C_{12} \frac{2E_2}{E}
\]
\[
E = (1 + \delta) g_{11} - (R_{22} - C_{22})E_2
\]
\[
a_{12} = a_{21} = - f_{11} E_1 + (C_{12}R_{12}E_2/E)
\]
\[
a_{22} = - R_{11} + f_{11} E_1 - (R_{12} \frac{2E_2}{E})
\]
\[ b_1 = [E_1 - (C_{12}E_2/E)] \frac{d\nu}{dM}, \text{ and} \]
\[ b_2 = [ - E_1 + (R_{12}E_2/E)] \frac{d\nu}{dM} \]

It can also be verified that
\[ \frac{dK}{dM} = E_2 \left[- C_{12} \frac{dY}{dM} + R_{12} \frac{dS}{dM} + \frac{d\nu}{dM}\right]/E \]

To begin with note that both \( E_1 \) and \( E_2 \) are negative. Assume that the firm is operating in the decreasing portion of the long run average cost curve. Then, \( C_{22} < 0 \). From this it follows that
\[ E > 0 \]

Similarly, utilizing the second order conditions it can be verified that
\[ a_{11}, a_{22} > 0, \text{ and} \]
\[ a_{12} > 0 \]

whenever \( g_{11} \) is relatively large.

Further, it can also be shown that
\[ D = a_{11}a_{22} - a_{12}^2 > 0 \]

Hence, it can be deduced that
\[ \frac{dY}{dM} = \frac{b_1a_{22} - b_2a_{12}}{D} > 0 \]
\[ \frac{dS}{dM} = \frac{b_2a_{11} - b_1a_{12}}{D} < 0 \]

It follows that
\[ \frac{dl}{dM} > 0 \]

Given these assumptions it can also be verified that
\[ \frac{dK}{dM} < 0 \]

if \( \frac{d\nu}{dM} \) is sufficiently small\(^{19}\).

Hence, as \( M \) decreases the firm increases \( K \) and reduces \( I \) so long as production is flexible ex post, the cost of making rapid adjustments to capital stock are high\(^{20}\), and it is operating on the decreasing portion of the long run average cost curve. The effects of a

\(^{18}\) The details of the algebra are available in a separate appendix. The interested reader can obtain the same from the author.

\(^{19}\) Note that \( E_2 \) is approximately equal to \(-t\). Hence, when \( t \) is small \( \frac{dK}{dM} < 0 \). In other words, small firms, in their early growth phase, are more likely to exhibit the property alluded to above. See, for example, Angelini et al (1998) and Kadapakkam et al (1998).

\(^{20}\) Unlike the FP argument this is the major reason why the firms cannot postpone investments in fixed capital to a future date. They tend to smooth capital formation as far as possible to keep the adjustment costs at a minimum.
change in $\Delta M^*$ will be identically the same since the optimal values of $Y,S,$ and $G$ have not been derived from the financial constraints per se.

In general, a higher cost of inventory holding, rather than other considerations, explains the reduction in $I$. Similarly, a large adjustment cost compels the firm to smooth $K$. The output market, in itself, has little effect.

Three further observations are in order. First, if $C_{22} > 0$ and the firm is operating on the increasing portion of the long run average cost curve, it is still possible that $E > 0$ so long as the cost of adding new capital equipment is significantly large. In other words, there may be a relatively small range of values in the increasing portion of the long run average cost curve where the above phenomenon will still be observed. Second, $R_{12} > 0$ is not a compulsion to infer its validity. Stated differently, the FP argument, that the prospect of market share reduction induces the firm to implement investment opportunities as they arise, is not crucial to their conclusion. The possibility of achieving cost reduction by increasing the stock of fixed capital is at the apex of the argument. Third, suppose production is inflexible ex post and the firm cannot afford to ignore the demand as it arises. Then, the firm will be under compulsion to sell out of inventory. It is reasonable to expect $R = R(S,I)$. The firm will give preference to $I$ and the above hypothesis will not hold. The argument is similar in the context of durable goods industries. For, the firm may need to extend credit to conduct sales in such markets. It cannot give priority to fixed capital accumulation when there is a financial constraint. In sum, there are several instances where the above phenomenon will not be observed.

Upto this point in the analysis the role of the interest rate, on the optimal choices of the firm, has not been apparent. Kuznets (1964, p.335 ff) documented the existence of an interest rate effect. However, as reported in Blinder (1981), and Maccini and Rosanna (1981), subsequent studies could not find any effect of interest rates on inventory investment. Ahmad (1998) argued that, in the presence of financial constraints, the interest rate itself has no effect. In general, two arguments have been offered to support this. First, the interest cost is a small fraction of the total cost. As such the effect of interest rate will be negligible. Second, since the product markets are imperfect, the firm can pass on most of the increase in cost due to higher interest rate to the consumer.
However, as noted above, when the financial constraint is binding, the firm will utilize credit to a point where the interest rate is equal to its implicit market value generating potential given by \( \nu \). Consider the optimal choice of \( Y, S, \) and \( G \). In equilibrium these values will be determined by \( \lambda \) and \( \mu \). Now, consider the changes in \( \lambda \) and \( \mu \) over time. Both of them contain the expression \( (r - \nu) \). This quantity is equal to zero. Hence, there is no interest rate effect in the presence of a financial constraint. A more fundamental economic reasoning is therefore available.

3. **Organizational Restructuring**

Consider the possibility that there is a reduction in the demand for the goods and services of the firm. There will be a corresponding reduction in the cashflow and a financial constraint results. The firm has to make appropriate adjustments if it considers this to be a long term change. In particular, it will consider the possibility of reducing the stock of inventory held by entrusting some of its activities (like the acquisition of raw materials and components, and marketing and distribution) to outside agents. For all practical purposes, these changes may precede any thoughts about increasing the stock of fixed capital. For, the cost reduction that can be achieved from the augmentation of productive capacity may not be realized at low levels of utilization. The basic choices for the firm will be the level of production, sales, and other assets (\( X \)) of the firm.

To formalize the argument assume that

\[
R = \text{revenue of the firm} \\
= R(S, \varepsilon) ; R_1, R_2, R_{22} > 0, R_{11} < 0
\]

represents the revenue of the firm. In this specification, \( \varepsilon \) represents an exogenous shift in the demand curve of the firm. It will be postulated that an increase in \( \varepsilon \) shifts the demand curve to the right without altering its slope. The firm organizing its own marketing or producing parts and components (i.e., the choice of \( X \)) is not an essential aspect of the consumers’ evaluation of the product. Similarly, in a declining market the firm does not have any specific advantage even if it has an adequate production capacity. Hence, \( R \) is not a function of \( K \) either.

Clearly, in this situation, an increase in \( \varepsilon \) augments the cashflows and the internally generated financial resources. Hence, ceteris paribus, it will be expected that

\[
M = M(\varepsilon) ; M_1 > 0
\]
\( M^* = M^*(\varepsilon) ; M_1^* > 0 \)

For all practical purposes the financial constraint is endogenous to the firm.

The firm is expected to react by increasing \( X \). That is, when the market improves, the firm is emboldened to take up some of its marketing and other logistic activities. This has two effects. First, the acquisition of the requisite assets \( (X) \) increases the cost of a unit of sales. Basically, this can be reflected in the specification

\[ C = \text{cost of production and sales} \]
\[ = C(Y,X) ; C_1, C_2, C_{11}, C_{12}, C_{22} > 0 \]

Second, the cost increase has an advantage. For, now the firm can reduce the share of revenue that was otherwise going to the franchisees and subcontractors. Let

\( s = \text{share of revenue accruing to the franchisees} \)
\[ = s(X) ; s_1, s_{11} < 0 \]

That is, the optimal choice of \( X \) depends on the increase in revenue relative to the cost that it entails.

With these modifications in place the problem for the firm is to

\[ \text{Max } \int_0^\infty e^{-\alpha t} \left[ \{1 - s(X)\} R(S,\varepsilon) - C(Y,X) - r(I + K + X - E - \beta Z) - g(G) - f(I) \right] dt \]

subject to

\[ \frac{dK}{dt} = G - \delta K \]
\[ \frac{dI}{dt} = Y - S \]
\[ \frac{dZ}{dt} = (1 - \theta)F - \beta Z - \phi \beta Z \leq M(\varepsilon), \text{ and} \]
\[ K + X - E - (1 - \phi)\beta Z \leq M^* \]

Construct the Hamiltonian as before. The optimal solution satisfies the equations

\( \mu = \mu_0 e^{\alpha t} - E_1 [f_1(I) + r - \nu] \)
\[ C_1(Y,X) = \mu(I,\nu) \]
\[ \{1 - s(X)\} R_1(S,\varepsilon) = \mu(I,\nu) \]
\[- s_1(X) R(S,\varepsilon) = C_2(Y,X) + r - \nu, \text{ and} \]
\[ \frac{dl}{d\varepsilon} = \frac{dY}{d\varepsilon} - \frac{dS}{d\varepsilon} \]

The second order conditions for maximum yield
\[ C_{11} + E_{1f_{11}} > 0 \]
\[ \{1 - s(X)\} R_{11} - E_{1f_{11}} < 0 \]
\[ s_{11} R + C_{22} > 0 \], and

the determinant of the second order Hessian \( D < 0 \).

From these conditions it can be verified that

\[-(s_{11}R + C_{22}) \left( \frac{dX}{d\varepsilon} \right) = C_{12} \left( \frac{dY}{d\varepsilon} \right) + s_{1}R_{2} - \left( \frac{d\nu}{d\varepsilon} \right) \]
\[ a_{11} \left( \frac{dY}{d\varepsilon} \right) + a_{12} \left( \frac{dS}{d\varepsilon} \right) = b_{1} \]
\[ a_{12} \left( \frac{dY}{d\varepsilon} \right) + a_{22} \left( \frac{dS}{d\varepsilon} \right) = b_{2} \]

where

\[ a_{11} = (C_{11} + E_{1f_{11}}) - C_{12}^2/(s_{11}R + C_{22}) \]
\[ a_{12} = -E_{1f_{11}} + s_{1}R_{1}C_{12}/(s_{11}R + C_{22}) \]
\[ a_{22} = [E_{1f_{11}} - (1-s)R_{11}] - s_{1}^2R_{1}^2/(s_{11}R + C_{22}) \]
\[ b_{1} = [s_{1}R_{2}C_{12}/(s_{11}R + C_{22})] + [E_{1} - C_{12}/(s_{11}R + C_{22})] \left( \frac{dv}{d\varepsilon} \right), \text{and} \]
\[ b_{2} = [s_{1}^2R_{1}R_{2}/(s_{11}R + C_{22})] + (1-s)R_{12} - [E_{1} + s_{1}R_{1}/(s_{11}R + C_{22})] \left( \frac{dv}{d\varepsilon} \right) \]

Consider \( D^* = a_{11}a_{22} - a_{12}^2 \)

It can be verified that

\[ D^* = -\frac{D}{(s_{11}R + C_{22})} - E_{1}s_{1}f_{11}R_{1}C_{12}/(s_{11}R + C_{22}) > 0 \]

only if \( f_{11} \) is relatively small. Under these conditions it can be shown that

\( \frac{dY}{d\varepsilon} < 0, \frac{dS}{d\varepsilon} > 0, \) and \( \frac{dl}{d\varepsilon} < 0. \)

Further, \( \frac{dX}{d\varepsilon} > 0 \) if \( dv/d\varepsilon \) is sufficiently small.

In other words, whenever the market conditions deteriorate the firm sells a lower quantity of output, decides to reduce the stock of inventory and increase production in the hope of reducing average costs. It also reduces \( X \). That is, as surmised above, it will leave marketing to a franchisee and so on. This is basically a cost saving mechanism.

When the financial constraint is binding, it is obvious that

\[ \frac{dK}{d\varepsilon} = \frac{dM}{d\varepsilon} + (1 - \phi)\beta \frac{dZ}{d\varepsilon} - \frac{dX}{d\varepsilon} \]

However, the model does not specify the change in \( M^* \) with \( \varepsilon \). Hence, in the absence of information about \( dM^*/d\varepsilon \) it is not possible to assert that \( \frac{dK}{d\varepsilon} < 0. \)

4. Share Prices in Secondary Markets
There is always a possibility that the shareholders of a firm experience information asymmetry with respect to the efficiency of its decision making process. In such situations, they evaluate the performance of the firm through proximate and visible signals like the dividend payments and investments in capital assets. The management may then be tempted to give priority to these aspects at the expense of cost reduction and maximization of cashflows of the firm\textsuperscript{21}.

It is now possible to argue that the changes in the share prices in the secondary markets depend on $\theta F$ and $K$\textsuperscript{22}. Let $p$ be the current market price of a unit of common stock. Then,

$$\frac{dp}{dt} = h(\theta F, K) ; h_1, h_2 > 0, h_{11}, h_{12} > 0, \text{ and } h_{22} < 0$$

The share price at any point of time $t$ is

$$\int_{0}^{t} e^{-\alpha t} h(\theta F, K) \, dt$$

At every point of time the shareholders take into account the dividends paid and expected capital gains while evaluating the value of the common stock of the firm\textsuperscript{23}. However, for purposes of comparison with the results of the foregoing sections assume that the common stock is held in perpetuity. That is, the maximization of the market value of the firm is reflected by

$$\max_{\theta, \phi} \int_{0}^{\infty} e^{-\alpha t} h(\theta F, K) \, dt$$

The decision variables are $Y$, $S$, and $G$. The firm can also choose $\theta$ and $\phi$ representing the use of finances. The constraints for optimization are

$$\frac{dK}{dt} = G - \delta K$$

$$\frac{dI}{dt} = Y - S$$

\textsuperscript{21} Clearly, there are limits on the extent to which the internal sources of finance can be reallocated between capital formation and working capital. Similarly, there will be significant institutional constraints on the use of long term debt and short term borrowings.

\textsuperscript{22} Since $h$ represents an increase in the share prices it may be a function of $G$, the addition to the capital stock rather than the entire $K$. However, it can be verified that the results are not substantially different.

\textsuperscript{23} The function $h$ may also depend on $\beta Z$. It is possible to conceptualize the optimal value of $\beta$ if this alternative is utilized.
\begin{equation}
dZ/dt = (1 - \theta)F - \beta Z \tag{1}
\end{equation}

\begin{equation}
I - \phi\beta Z \leq M, \text{ and} \tag{2}
\end{equation}

\begin{equation}
K + A - E - (1 - \phi)\beta Z \leq M^* \tag{3}
\end{equation}

To approach the solution to this problem construct the Hamiltonian
\begin{equation}
H = e^{-\alpha t} h (\theta F, K) + \lambda e^{-\alpha t} (G - \delta K) + \mu e^{-\alpha t} (Y - S) + \psi e^{-\alpha t} [(1 - \theta)F - \beta Z] + \nu e^{-\alpha t} (I - \phi\beta Z - M) + \nu^* e^{-\alpha t} [K + A - E - (1 - \phi)\beta Z - M^*] \tag{4}
\end{equation}

From the Pontryagin’s maximum principle it follows that
\begin{equation}
\lambda = \lambda_0 e^{-(\alpha - \delta)t} - E_2 (r h_1 + h_2 - \nu^*) \tag{5}
\end{equation}

\begin{equation}
\mu = \mu_0 e^{-(\alpha - r)t} - E_1 [h_1(f_1 + r) - \nu] \tag{6}
\end{equation}

\begin{equation}
\psi = \psi_0 e^{-(\alpha - r)t} - E_3 [\nu\phi + \nu^* (1 - \phi)] \tag{7}
\end{equation}

where
\begin{equation}
E_3 = \left[1 - e^{-(\alpha - r)t}\right] / (\alpha - r) \tag{8}
\end{equation}

The optimal values of Y, S, and G satisfy the equations
\begin{equation}
C_1 \left[\theta h_1 + \psi (1 - \theta)\right] = \mu (I, \nu, \nu^*) \tag{9}
\end{equation}

\begin{equation}
R_1 \left[\theta h_1 + \psi (1 - \theta)\right] = \mu (I, \nu, \nu^*) \tag{10}
\end{equation}

\begin{equation}
g_1 \left[\theta h_1 + \psi (1 - \theta)\right] = \lambda (F, K, \nu, \nu^*) \tag{11}
\end{equation}

Further, the optimal value of the payout to the shareholders will be determined by
\begin{equation}
h_1 = \psi \tag{12}
\end{equation}

and the allocation of internal finances will be optimal if
\begin{equation}
\nu = \nu^* \tag{13}
\end{equation}

The relevant second order conditions for maximum are
\begin{equation}
C_{11} + E_1 f_{11} > 0 \tag{14}
\end{equation}

\begin{equation}
R_{11} - E_1 f_{11} < 0, \text{ and} \tag{15}
\end{equation}

\begin{equation}
D = -(C_{11} + E_1 f_{11}) (R_{11} - E_1 f_{11}) - E_1^2 f_{11}^2 > 0 \tag{16}
\end{equation}

It can be now be verified that
\begin{equation}
a_{11} (dY/dM) + a_{12} (dS/dM) = b_1 \tag{17}
\end{equation}

\begin{equation}
a_{12} (dY/dM) + a_{22} (dS/dM) = b_2 \tag{18}
\end{equation}

where
\begin{equation}
a_{11} = -\psi (C_{11} + E_1 f_{11}) \tag{19}
\end{equation}

\begin{equation}
a_{12} = E_1 f_{11} \psi \tag{20}
\end{equation}
\[ a_{22} = (R_{11} - E_{1}f_{11}) \psi \]

\[ b_{1} = - [C_{1}E_{3}\phi + E_{1}(f_{1} + r)E_{3}\phi + E_{1}] \frac{d\nu}{dM} - [E_{1}(f_{1} + r) + C_{1}]E_{3}(1 - \phi) \frac{d\psi}{dM} \text{, and} \]

\[ b_{2} = [R_{1}E_{3}\phi + E_{1}(f_{1} + r)E_{3}\phi + E_{1}] \frac{d\nu}{dM} + [E_{1}(f_{1} + r) + R_{1}]E_{3}(1 - \phi) \frac{d\psi}{dM} \]

From this it follows that

\[ \frac{dY}{dM} = \frac{a_{22}b_{1} - a_{12}b_{2}}{D} > 0 \]

and

\[ \frac{dS}{dM} = \frac{-a_{12}b_{1} + a_{11}b_{2}}{D} < 0 \]

if both \((f_{1} + r)\) and \(f_{11}\) are relatively small. As a result

\[ \frac{dI}{dM} > 0 \]

Similarly, from the equation

\[ g_{1}\psi = \lambda \]

it can be deduced that

\[ \frac{dK}{dM} > 0 \]

provided \(g_{1}\) is small relative to \(r\).

In general, there will be an increase in \(K\) under these conditions\(^{24}\). Intuitively, it is obvious that a low enough adjustment cost is conducive to investment in fixed capital assets. Similarly, low inventory costs will encourage inventory investment. As noted above, efficiency considerations are not the channel of transmission. However, as in the FP argument, the firm considers it important to utilize exogenously determined investment opportunities as they arise.

5. Conclusion

The present study is a useful beginning in two directions. First, it provides a theoretical framework to examine the interrelationship between investment in fixed capital and inventory investment. Second, the theoretical structures also unravel the channels through which the financial constraints result in an increase in the fixed capital of the firm. Throughout the analysis of the present study it was assumed that there is some flexibility in the use of internal finances (reserves and surpluses). Under these conditions the possibility of cost reduction, when confronted with financial constraints, turns out to be

\(^{24}\) It can also be shown that \(d\theta/dM > 0\), or the firm pays out more if \(d\psi/dM\) is small, indicating that there is no great dependence on borrowings and

\[ h_{11}\theta [g_{1}(1 + \delta) + r] - h_{12} < 0 \]

suggesting that the firm believes that the shareholders assign a greater value to a unit of capital investments compared to a unit increase in \(F\).
the basic motivation for the firm to rearrange its asset structure. There will be an increase in the investment in fixed capital if the firm is operating on the decreasing portion of the long run average cost curve.

It is difficult to assert that the fixed capital decision is primary. Some firms may consider other forms of change in the asset structure as more pertinent. This is likely to happen if the firm expects a continuation of the stagnant markets. It is difficult to justify an increase in fixed capital under these conditions.

The shareholders of the firm may consider the dividends paid and gross capital formation announced by the firm as the basic determinants of the market value because they cannot assess the efficiency of the operations of the firm directly. Under these conditions, there is a possibility of the firm misleading the shareholders by announcing higher dividend payments out of profit generated. They may also jeopardize the profit generating potential of the firm by reducing inventories and channeling resources for fixed capital formation. An increase in fixed capital may then occur even when the decisions of the firm are not related to its efficient operation.

The question about the relative significance of these two effects in empirical practice is then relevant. In order to resolve this the model must be restructured by writing the cost function as \( C(Y,K) \). Similarly, specifying the revenue function as \( R(S,K) \) is a necessity to identify the dominance of the market share considerations. Empirical verification of the precise channels of transmission can be taken up only when such extensions are available.

If equity is the major source of financing the fixed capital assets of the firm the choice of the debt equity ratio becomes important. For, debt financing has implications for cost as well as control and the expansion of productive capacity does not necessarily decrease overall costs. It can be surmised that such extensions will not support an increase in fixed capital.

On the whole, there is a necessity for more theoretical as well as empirical work. There is no necessity for any concern if efficiency considerations turn out to be dominant.
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