Joint Precoder and Receiver Design for Asymmetric Two-way MIMO AFRelaying

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Single-hop cellular architecture

- Two links – wireless access link and **wired backhaul link**.

- Single-hop architecture works well if direct access links are strong.
Cellular scenarios with weak direct links

Two examples: 1) Coverage extension; and 2) Coverage hole.

Installing a BS for few users is costly due to core network & backhaul link.
Relay is a (simplified) BS without backhaul infrastructure.

- Relay functionality can vary from a simple repeater to a full-blown BS.
### Half-duplex relaying

- Half-duplex constraint is imposed on the relay (easy to design).
  - Full-duplex technology is currently being practically evaluated.

- Relay cannot transmit and receive on same spectral resource at same time.

- Two widely studied half-duplex relaying protocols:
  - One-way relay: Needs four time slots to exchange two data units.
  - Spectrally inefficient when compared with direct communication.

![One-way relaying protocol](image)

**Figure**: One-way relaying protocol.
Two-way relaying

- Consists of two phases.

  **MAC phase:** Two source nodes simultaneously transmit to the relay.

  \[
  f(x_1, x_2)
  \]

  - Broadcast phase: Relay broadcasts a function of the sum-signal.

  \[
  g(x_1, x_2)
  \]

  Both nodes can cancel **back-propagating interference** as both know self-data.

- Two slots are required to exchange two data units – **spectrally efficient.**

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Basic assumption in two-way relaying

Two nodes want to exchange data via a relay.

Both nodes should have data to send and receive.

Strong assumption!
Data exchange in cellular systems

- Usually does not happen!\(^2\)

\(\text{BS} \quad \text{RS} \quad \text{TUE} \quad \text{BS} \quad \text{RS} \quad \text{RUE}\)

(1) Transmit-only User (2) Receive-only User

Non-simultaneous traffic scenarios

- E.g., TUE uploading a Youtube video / RUE watching a Netflix movie.

- Users either want to send or receive data.

Two way relaying cannot be used in these scenarios.

Option for BS to serve TUE and RUE

- Use one way relaying.

(1) Transmit-only User
(2) Receive-only User

Non-simultaneous traffic scenarios

- BS will require 4 time slots – spectrally inefficient.
Proposed asymmetric two-way relaying (ATWR)

- **MAC phase:** Both BS and TUE simultaneously transmit to the relay.

- **BC phase:** Relay broadcasts to both BS and RUE.

- BS requires two slots to serve two users.
Back-propagating interference in ATWR

(a) MAC phase

- Relay receive signal: \( y_r = H_u s_u + H_b s_b + n_r \).
- Relay transmit signal: \( s_r = f(y_r) = f(s_u, s_b) \).

(b) BC phase

- BS receive signal: \( y_b = G_b \cdot f(s_u, s_b) \). BS can cancel \( s_b \) and detect \( s_u \).
- RUE receive signal: \( y_u = G_u \cdot f(s_u, s_b) \). RUE cannot cancel \( s_u \).
Relay receive signal: \[ y_r = H_u s_u + H_b s_b + n_r. \]

Relay transmit signal: \[ s_r = f(y_r) = f(s_u, s_b). \]

BS receive signal: \[ y_b = G_b \cdot f(s_u, s_b). \] BS can cancel \( s_b \) and detect \( s_u \).

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Back-propagating interference in ATWR

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- RUE receive signal: \( y_u = G_u \cdot f(s_u, s_b) \). RUE cannot cancel \( s_u \).
State-of-the-art in ATWR

- **Cancel BI using overhearing** - assumes RUE overhears TUE in MAC phase.
  - Consider single antenna nodes.
  - Analysed sum-rate, scheduling and diversity multiplexing trade-off.

- We do not assume overhearing and design a precoder at relay to cancel BI.
  - Consider MIMO nodes.
  - Designed relay precoder, joint precoder and considered multi-user scenario.

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Our work – two different designs

- Design a precoder at relay to cancel back-propagating interference.\(^9\)
  - Assume relay alone has global channel knowledge.
  - Both BS and RUE use SIC to detect data; SIC can lead to errors.
  - Discussed in the first seminar.

- Joint precoder and receiver design.\(^10\)
  - Assume all nodes have global channel knowledge.
  - Diagonalizes MIMO channels. SIC is not required.
  - Realizes beamforming gain over relay precoder.
  - Consider multi-user extension.

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Our work – two different designs

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Classification of relays

- Classified based on relay receive processing.

- **Amplify-forward**: Relay Tx signal: \( x_r = W y_r = WHx + Wn_r \).

  - Amplify-forward relay amplifies noise but introduces minimal latency.

- **Decode-forward**: Relay decodes data. Relay Tx signal: \( x_r = Px_d \).

  - Decode-forward relay removes noise but introduces considerable latency.
System model (1)

- We consider an amplify and forward relay.
- No direct links between the BS and two users.
- Users observe this channel in coverage-extension/coverage-hole scenarios.
TUE and BS transmit precoded signals: \( s_u = B_u x_u \) and \( s_b = B_b x_b \).

Relay signals:
- MAC phase received signal: \( y_r = H_u s_u + H_b s_b + n_r \).
- BC phase transmit signal: \( s_r = W y_r \).

BC phase receive signal:
- RUE: \( y_u = G_u s_r + n_u = G_u W H_u s_u + G_u W H_b s_b + \tilde{n}_u \).
- BS: \( y_b = G_b s_r + n_b = G_b W H_u s_u + G_b W H_b s_b + \tilde{n}_b \).
TUE and BS transmit precoded signals: \( s_u = B_u x_u \) and \( s_b = B_b x_b \).

Relay signals:

- MAC phase received signal: \( y_r = H_u s_u + H_b s_b + n_r \).
- BC phase transmit signal: \( s_r = W y_r \).

BC phase receive signal:

- RUE: \( y_u = G_u s_r + n_u = G_u W H_u s_u + G_u W H_b s_b + \tilde{n}_u \).
- BS: \( y_b = G_b s_r + n_b = G_b W H_u s_u + G_b W H_b s_b + \tilde{n}_b \).
Relay precoder design to cancel back-propagating interference

- Precoder $W$ is decomposed as $W = MDF$.
  - $M = [M_u \ M_b]$ and $F = [F_u; \ F_b]$ are designed to cancel RUE’s BI.
  - $D = \text{anti-diag}(D_u, D_b)$ is used to diagonalize MIMO channels.

- Signal received by RUE and BS after substituting $W = MDF$:
  \[
  y = \begin{bmatrix} y_u; \ y_b \end{bmatrix} = GWHs + \tilde{n} = \left( GM \begin{array}{c} D \end{array} \begin{array}{c} FH \end{array} \right) s + \tilde{n} = \tilde{G}\tilde{D}\tilde{H}s + \tilde{n}
  \]

- Block matrices: $\tilde{G} = \begin{bmatrix} \tilde{G}_u & \tilde{G}_0 \\ \tilde{G}_n & \tilde{G}_b \end{bmatrix}$ and $\tilde{H} = \begin{bmatrix} \tilde{H}_b & \tilde{H}_n \\ \tilde{H}_0 & \tilde{H}_u \end{bmatrix}$.
Design criterion for precoders $\mathbf{M}$ and $\mathbf{F}$

Lemma: To cancel RUE’s BI alone, design $\mathbf{M}$ and $\mathbf{F}$ such that $\mathbf{\tilde{G}}$ and $\mathbf{\tilde{H}}$ are block lower- and upper-triangular matrices, respectively.

Proof: Recall $\mathbf{y} = [\mathbf{y}_u; \mathbf{y}_b] = \mathbf{\tilde{G}}\mathbf{D}\mathbf{H}\mathbf{s} + \mathbf{\tilde{n}}$. With lower/upper-triangular $\mathbf{\tilde{G}}$/\$\mathbf{\tilde{H}}$,

$$
\begin{bmatrix}
\mathbf{y}_u \\
\mathbf{y}_b
\end{bmatrix}
= 
\begin{bmatrix}
(\mathbf{\tilde{G}}_u \mathbf{D}_u \mathbf{\tilde{H}}_u)\mathbf{s}_u \\
(\mathbf{\tilde{G}}_b \mathbf{D}_b \mathbf{\tilde{H}}_b)\mathbf{s}_u + (\mathbf{\tilde{G}}_n \mathbf{D}_u \mathbf{\tilde{H}}_u + \mathbf{\tilde{G}}_b \mathbf{D}_b \mathbf{\tilde{H}}_n)\mathbf{s}_b
\end{bmatrix} + \mathbf{\tilde{n}}. \tag{1}
$$

- RUE receive signal: $\mathbf{y}_u = (\mathbf{\tilde{G}}_u \mathbf{D}_u \mathbf{\tilde{H}}_u)\mathbf{s}_b + \mathbf{\tilde{n}}_u$.

- BS received signal (after BI cancellation): $\mathbf{y}_b = (\mathbf{\tilde{G}}_b \mathbf{D}_b \mathbf{\tilde{H}}_b)\mathbf{s}_u + \mathbf{\tilde{n}}_b$.

- Precoders $\mathbf{M}$ and $\mathbf{F}$ are designed based on null-space projection.
  - Designed in first seminar.
Design criterion for precoders \( \mathbf{M} \) and \( \mathbf{F} \)

Lemma: To cancel RUE’s BI alone, design \( \mathbf{M} \) and \( \mathbf{F} \) such that \( \tilde{\mathbf{G}} \) and \( \tilde{\mathbf{H}} \) are block lower- and upper-triangular matrices, respectively.

Proof: Recall \( \mathbf{y} = [y_u; y_b] = \tilde{\mathbf{G}}\mathbf{D}\tilde{\mathbf{H}}\mathbf{s} + \tilde{\mathbf{n}} \). With lower/upper-triangular \( \tilde{\mathbf{G}}/\tilde{\mathbf{H}} \),

\[
\begin{bmatrix}
  y_u \\
  y_b
\end{bmatrix} = \begin{bmatrix}
  \tilde{\mathbf{G}}_u \mathbf{D}_u \tilde{\mathbf{H}}_u \mathbf{s}_b \\
  (\tilde{\mathbf{G}}_b \mathbf{D}_b \tilde{\mathbf{H}}_b) \mathbf{s}_u + (\tilde{\mathbf{G}}_n \mathbf{D}_u \tilde{\mathbf{H}}_u + \tilde{\mathbf{G}}_b \mathbf{D}_b \tilde{\mathbf{H}}_n) \mathbf{s}_b
\end{bmatrix} + \tilde{\mathbf{n}}. \tag{1}
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- Precoders \( \mathbf{M} \) and \( \mathbf{F} \) are designed based on null-space projection.
  - Designed in first seminar.
Objectives achieved with previous lemma

1. Cancelled RUE’s BI alone.

2. Decoupled a two-way relay channel into two non-interfering one-way relay channels:

   - Downlink (BS→RS→RUE):

     \[
     \begin{align*}
     &B_b &\xrightarrow{H_u} &D_u &\xrightarrow{G_u} &T_u &\xrightarrow{y_u} \\
     &BS & &RS & &RUE & \\
     \end{align*}
     \]

   - Uplink (TUE→RS→BS):

     \[
     \begin{align*}
     &T_b &\xleftarrow{y_b} &D_b &\xleftarrow{H_b} &B_u &\xleftarrow{x_u} \\
     &BS & &RS & &TUE & \\
     \end{align*}
     \]
Design $B_b$, $D_u$ and $T_u$ to diagonalize $BS \rightarrow RS \rightarrow RUE$ MIMO channel.

- Recall, we assume all nodes have global CSI.

- Perform SVD of $\tilde{H}_u = U_h \Sigma_h V_h^H$ and $\tilde{G}_u = U_g \Sigma_g V_g^H$ and choose:

  - $B_h = U_h^H$.  
  - $D_u = V_h \Delta_u U_g^H$.  
  - $T_u = V_g$.

- Diagonalized MIMO channel: $\tilde{y}_u = \Sigma_g \Delta_u \Sigma_h x_b + T_u \tilde{n}_u$.

- Open problem – structure is optimal for one-way relay. Optimal for ATWR?
Design $B_b$, $D_u$ and $T_u$ to diagonalize BS $\rightarrow$ RS $\rightarrow$ RUE MIMO channel.

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Downlink joint transceiver design

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- Perform SVD of $\tilde{H}_u = U_h \Sigma_h V_h^H$ and $\tilde{G}_u = U_g \Sigma_g V_g^H$ and choose:
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- Open problem – structure is optimal for one-way relay. Optimal for ATWR?
Multi-user asymmetric two-way relaying

- **RUE-\(n\)** experiences
  
  - Back-propagating interference from the MAC-phase transmission of TUEs.
  
  - Co-channel interference from data transmitted by BS to other \(N_u - 1\) RUEs.

- RUEs receive interference-free signal by slightly modifying earlier design.
Multi-user asymmetric two-way relaying

Relay signals:

- **MAC phase:** \( y_r = H_b s_b + \sum_{i=1}^{N_u} h_{ui} s_{ui} + n_r \); and **BC phase:** \( s_r = Wy_r \)

RUE-\( n \) receive signal:

- \( y_{un} = g_{un}^T s_r = g_{un}^T W (h_{bi} s_{bi} + \sum_{i \neq n} h_{bi} s_{bi} + \sum_{i=1}^{N_u} h_{ui} s_{ui} + \tilde{n}_{un} ) \)

Relay precoder \( W \) is designed to cancel BI.
Decoupling of channels in single-user scenario

- Relay precoder decouples two-way channel in two one-way relay channels.

- Downlink one-way relay channel:

  - Downlink one-way relay channel:

  \[
  x_b \rightarrow B_b \xrightarrow{\tilde{H}_u} D_u \xrightarrow{\tilde{G}_u} T_u \rightarrow \tilde{y}_u
  \]

  - BS \quad RS \quad RUE

- Uplink one-way relay channel:

  - Uplink one-way relay channel:

  \[
  \tilde{y}_b \leftarrow T_b \xleftarrow{\tilde{G}_u} D_b \xleftarrow{\tilde{H}_b} B_u \leftarrow x_u
  \]

  - BS \quad RS \quad TUE
Decoupling of channels in multi-user scenario

- Relay precoder decouples two-way channel in two one-way relay channels.

- Relay broadcast channel in downlink:

- Relay MAC channel in uplink:
Dirty-paper-coding for point-to-point scalar channels

Consider the following scalar interference channel $y = s + i + n$

- $i$ and $n$ are independent Gaussian random variables.

**Theorem (Dirty Paper Coding)**

*If interference $i$ is known non-casually at the transmitter then the capacity of the Gaussian channel is the same as if $i$ were not present.*
DPC for vector broadcast channels (ZF-DPC)

- Consider the vector broadcast channel:

\[
s = Bx
\]

- Signal received by UE-\(n\):

\[
y_n = g_n^T Bx + n_n = g_n^T b_n x_n + \sum_{j \neq n} g_n^T b_j x_j + n_n
\]

- Stacked signals of UE-\(n\), \(\forall n\): \(y = GBx + n\)
DPC for vector broadcast channels (ZF-DPC)\textsuperscript{11}

- Consider the vector broadcast channel:

- In ZF-DPC, $G$ is decomposed as $G = LQ$. And precoder $B = Q^H$.

- Equivalently $y = GBx + n = Lx + n$:

$$
\begin{bmatrix}
    y_1 \\
    y_2 \\
    y_3 \\
    y_4 
\end{bmatrix} =
\begin{bmatrix}
    \times & 0 & 0 & 0 \\
    \times & \times & 0 & 0 \\
    \times & \times & \times & 0 \\
    \times & \times & \times & \times 
\end{bmatrix} 
\begin{bmatrix}
    x_1 \\
    x_2 \\
    x_3 \\
    x_4 
\end{bmatrix} + n.
$$

Downlink transceiver design

- After BI cancellation, downlink is one-way relay broadcast channel:

\[
\tilde{y}_u = \tilde{G}_u \tilde{D}_u \tilde{H}_u B_b x_b + \tilde{n}_u.
\]

- Signals received by RUE-\(n\), \(\forall n\): \(y_u = \tilde{G}_u \tilde{D}_u \tilde{H}_u B_b x_b + \tilde{n}_u\).

- Design BS and RS precoders such that BS employs DPC to cancel CCI.

  - Perform SVD of \(\tilde{H}_u = U_h \Sigma_h V_h^H\) and LQ dec. of \(\tilde{G}_u = L_g Q_g\).
  - Choose \(B_b = U_h^H\) and \(D_u = Q_g^H \Delta_u V_h\).

- Lower-triangular MIMO channel: \(y_u = L_g \Delta_u \Sigma_h x_b + \tilde{n}_u\).
Downlink transceiver design

- After BI cancellation, downlink is one-way relay broadcast channel:

\[
\begin{align*}
\text{BS} & \quad B_b \quad \tilde{H}_u \quad D_u \\
\text{RS} & \quad RUE-1 \quad \tilde{g}_{u1} \\
& \quad \cdots \\
& \quad \tilde{g}_{uN_u} \quad RUE-N_u
\end{align*}
\]

- Signals received by RUE-\(n\), \(\forall n\): \(y_u = \tilde{G}_u D_u \tilde{H}_u B_b x_b + \tilde{n}_u\).

- Design BS and RS precoders such that BS employs DPC to cancel CCI.
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Downlink transceiver design

- After BI cancellation, downlink is one-way relay broadcast channel:

\[
\begin{align*}
\text{BS} & \quad B_b \quad \tilde{H}_u \quad D_u \\
& \quad \text{RS} \quad \text{RUE-n} \\
& \quad \tilde{g}_{u1} \quad \tilde{g}_{u2} \quad \text{RUE-N_u}
\end{align*}
\]

- Signals received by RUE-n, \( \forall n \): 
  \[ y_u = \tilde{G}_u D_u \tilde{H}_u B_b x_b + \tilde{n}_u. \]

- Design BS and RS precoders such that BS employs DPC to cancel CCI.
  - Perform SVD of \( \tilde{H}_u = U_h \Sigma_h V_h^H \) and LQ dec. of \( \tilde{G}_u = L_g Q_g \).
  - Choose \( B_b = U_h^H \) and \( D_u = Q_g^H \Delta_u V_h \).

- Lower-triangular MIMO channel: 
  \[ y_u = L_g \Delta_u \Sigma_h x_b + \tilde{n}_u. \]
SNR experienced by BS and RUE

SNR observed by the $m$th stream received by the RUE and BS:

\[ \text{SNR}^m_i = \frac{a^m_i \delta_{i,m}}{\sigma^2_r \left( \sum_{k=1}^{M} b^m_{i,k} \delta_{u,k} + c^m_{i,k} \delta_{b,k} \right) + \sigma^2}. \]

Here \( \{a^m_i, b^m_{i,k}, c^m_{i,k}\} \geq 0. \)

Note that coefficients of power allocation variables $\delta_{u,k}$ and $\delta_{b,k}$ are positive.

Fact will be used to prove the convexity of sum-rate optimization program.

\[ R_{\text{sum}}(\delta) = \frac{1}{2} \sum_{m=1}^{M} \log (1 + \text{SNR}^m_u(\delta)) + \log (1 + \text{SNR}^m_b(\delta)) \]
Sum-rate maximization

- Will cast sum-rate max. as a geometric program; we explain its terminology.

- **A monomial** is a function $f : \mathbb{R}^n_{++} \rightarrow \mathbb{R}$ of the form
  $$f(x) = cx_1^{a_1}x_2^{a_2} \cdots x_n^{a_n}, \text{ where } c > 0 \text{ and } a_j \in \mathbb{R}.$$

- A positive sum of monomials is called a **posynomial**:
  $$f(x) = \sum_{k=1}^{K} c_k x_1^{a_{1k}}x_2^{a_{2k}} \cdots x_n^{a_{nk}}, \text{ where } c_k > 0$$

- Posynomials are closed under addition & multiplication, not under division.

- In a GP, the objective function and inequality constraints are posynomials.
Sum-rate maximization subject to relay power constraint

- Optimization problem can be cast as:

$$\begin{align*}
\max_{\delta : \delta \geq 0} & \quad R_{\text{sum}}(\delta) \\
\text{s.t.} & \quad P(\delta) \leq P_r
\end{align*}$$

(3)

- To cast (3) as a GP, both $R_{\text{sum}}(\delta)$ and $p_r(\delta)$ must be posynomials.

- Relay power $p_r(\delta)$ is a posynomial.\(^\text{12}\) We show that $R_{\text{sum}}$ is not a posynomial.

$$R_{\text{sum}}(\delta) = \frac{1}{2} \sum_{m=1}^{M} \log (1 + \text{SNR}_u^m(\delta)) + \log (1 + \text{SNR}_b^m(\delta))$$

$$= \frac{1}{2} \log \left[ \prod_{m} (1 + \text{SNR}_u^m(\delta))(1 + \text{SNR}_b^m(\delta)) \right].$$

Sum-rate maximization

- Recall SNR observed by the $m$th stream received by the RUE and BS:

$$\text{SNR}_i^m = \frac{a_i^m \delta_{i,m}}{\sigma_r^2 \left( \sum_{k=1}^{M} b_{i,k}^m \delta_{u,k} + c_{i,k}^m \delta_{b,k} \right) + \sigma^2}.$$ 

- Here $\{a_i^m, b_{i,k}^m, c_{i,k}^m\} \geq 0$. Equivalently $\text{SNR}_i^m = \frac{\text{monomial}}{\text{posynomial}}$.

- Sum-rate: $R_{\text{sum}}(\delta) = \frac{1}{2} \log \prod (1 + \text{SNR}_u^m(\delta))(1 + \text{SNR}_b^m(\delta))$.

- $R_{\text{sum}}$ is a ratio of two posynomials – not a posynomial.

  - Sum-rate maximization is not a GP.
Sum-rate maximization

- Use high SNR approximation: \( \log(1 + \text{SNR}) \simeq \log(\text{SNR}) \).

- Therefore

  \[
  R_{\text{sum}} \simeq \frac{1}{2} \log \left[ \prod_m \text{SNR}_u^m(\delta) \text{SNR}_b^m(\delta) \right] = -\frac{1}{2} \log \left[ \prod_m \text{ISNR}_u^m(\delta) \text{ISNR}_b^m(\delta) \right].
  \]

  - Here \( \text{ISNR}_u^m = 1/\text{SNR}_u^m \). Note that \( \text{ISNR} = \frac{\text{posynomial}}{\text{monomial}} \) is a posynomial.

- Sum-rate maximization can now be cast as a GP:

  \[
  \begin{align*}
  \text{Min.} & \quad \delta \preceq 0 \quad \prod_m \text{ISNR}_u^m(\delta) \text{ISNR}_b^m(\delta) \\
  \text{s.t.} & \quad p_r(\delta) \leq P_r.
  \end{align*}
  \]
Sum-rate comparison of different precoders

(a) $N_r = 4$ and $N_u = N_b = 2$.  
(b) $N_r = 8$ and $N_u = N_b = 4$.

- Precoders – BICD, BICT and ZF/MMSE.

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Sum-rate comparison for multi-user ATWR

(c) With $\eta_b = 15$ dB.  
(d) With $\eta_b = 20$ dB.

- Precoders – BIDPC,$^{16}$ e-DPC1,$^{17}$ e-DPC2,$^{18}$ and ZF/MMSE.$^{19}$

Conclusions

- Considered problem of asymmetric data-flow in two-way relaying.

- Designed a novel precoder to selectively cancel back-propagating interference.

- Maximized sum-rate using geometric programming.

- Proposed precoder outperforms conventional precoders.