Precoder Design for Asymmetric Two-way AF Relay

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ICC-2013
**Background**

- Cooperative communication can lead to significant performance improvements in wireless systems.

- Conventional one-way relaying is an example.

- Half-duplex signaling in one-way relaying leads to loss of \( \frac{1}{2} \) of spectral resources.

- Four channel uses are required for bidirectional data exchange.

- Two-way relaying requires two channel uses instead of four.\(^1\)

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Two-way relaying

- Two source nodes simultaneously transmit to the relay during first phase.

\[ f(x_1, x_2) \]

**Figure:** First phase of two-way relaying

- Relay broadcasts a function of the sum-signal during second phase.

\[ g(x_1, x_2) \]

**Figure:** Broadcast phase of two-way relaying
Two-way relaying is most appropriate when two nodes exchange data simultaneously.

- Simultaneous two-way data exchange need not happen in cellular systems.

- User might have uplink data to transmit but no downlink data to receive.

- Two-way relaying will reduce to conventional one-way relaying.
Asymmetric Two-way relaying

- Consider an infrastructure relay where multiple UEs are served via relay.\(^2\)
  - Used for coverage extension and filling coverage holes.

- A user UE\(_1\) wants to send UL data to BS through the relay. Has no DL data to receive.

- Consider another UE (UE\(_2\) ), which wants to receive DL data from the BS.
  - Can be found with a high probability in a multi-user system.

Asymmetric Two-way relaying (Contd…)

**Figure:** Asymmetric two-way relaying.

Propose a new two-way relaying protocol

1. BS and UE₁ transmit to the relay during MAC phase.
2. Relay transmits to the BS and UE₂ during BC phase.

Two way relaying becomes asymmetric. Helps in recovering the spectral efficiency loss.
Asymmetric Two-way relaying (Contd...)
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- But leads to problem of asymmetric back-propagating interference (BPI).

- BS can cancel the BPI, while downlink single-antenna UE (UE₂) cannot.
  - Downlink UE cannot overhear uplink UE transmission.

Problem Description

Precoder design at the relay to cancel the BPI for downlink UE
System model

- BSs and UEs have one antenna each while relay has multiple antennas.

\[ y_r = Hx + n_r: \] Sum-signal received by the relay during MAC phase.

- Here \( H = \begin{bmatrix} h_1 & h_2 \end{bmatrix} \) and \( x = [x_1 \ x_2]^T \).

- Signal transmitted by the relay: \( x_r = Wy_r \).

\( W \) is the precoder matrix to be designed.
System model (Contd...)

- Signals received by UE2 and BS, $y_1$ and $y_2$ during BC phase
  \[ y_i = (g_i)^T x_r + n_i, \quad i = 1, 2. \]

- Maximum rates observed by BS and UE2 are given respectively as:
  \[ R_b = \log(1 + \text{SNR}_b), \quad R_u = \log(1 + \text{SNR}_u). \]

- $R_{sum} = \frac{1}{2} (R_b + R_u)$ is the system sum-rate.
  - Used for performance comparison.
Precoder design

- Precoder structure: \( W = MDF \), where
  - \( F \): Uplink precoder, \( M \): Downlink precoder and
  - \( D \): Permutation and power-normalization matrix

\[
D = \begin{bmatrix}
  0 & \beta \\
  \beta & 0
\end{bmatrix}
\]

\( \beta \) is used to normalize the relay power.

Uplink and downlink precoder matrix design

ZF-based solution

- \( F = H^\dagger = (H^H H)^{-1} H^H \),
- \( M = G^\dagger = G^H (G G^H)^{-1} \).

\( F \) and \( M \) can also be designed using MMSE criterion.
Precoder design

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- \( \mathbf{F} \) and \( \mathbf{M} \) can also be designed using MMSE criterion.
Precoder design

- ZF/MMSE precoders mitigate the interference for UE\(_2\) and BS.

- ZF/MMSE precoders are sub-optimal as BS can perform self-interference cancellation.

Precoder design based on channel decomposition proposed

- Cancels the interference for downlink UE alone.
- Sum-rate performance is better than ZF/MMSE precoders.

Rewrite the signals received by UE\(_2\) and BS during the BC phase:

\[
y = GW (Hx + n_r) + n
= \underbrace{GM D FH}_Gt x + \underbrace{GWn_r + n}_n
= G_t DH_t x + n_t
\]
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G_t \end{array}FHx + GWn_r + n
\]

\[
= G_tDH_tx + n_t
\]
Precoder design - Based on Channel decomposition

Lemma

Precoders $\mathbf{M}$ and $\mathbf{F}$ should be designed such that $\mathbf{G}_t$ and $\mathbf{H}_t$ are lower-triangular and upper-triangular matrices respectively.

Proof: With the lower- and upper-triangular matrices $\mathbf{G}_t$ and $\mathbf{H}_t$ will become

$$
\begin{bmatrix}
y_1 \\
y_2
\end{bmatrix} =
\begin{bmatrix}
l_1 & 0 \\
l_2 & l_3
\end{bmatrix}
\begin{bmatrix}
0 & \beta \\
\beta & 0
\end{bmatrix}
\begin{bmatrix}
u_1 & u_2 \\
0 & u_3
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix}
+ \tilde{\mathbf{n}}

= \begin{bmatrix}
\beta(l_1 u_3) x_2 \\
\beta(l_3 u_1)x_1 + \beta(l_3 u_2 + l_2 u_3)x_2
\end{bmatrix}
+ \tilde{\mathbf{n}}.
$$

- UE$_2$ receives its desired data $x_2$ without any interference.
Design of uplink and downlink precoder matrices

- Decompose $H$ into a unitary matrix and an upper-triangular matrix using QR decomposition.

\[
H = Q_H R_H = \begin{bmatrix} Q_H^{(1)} & Q_H^{(0)} \end{bmatrix} \begin{bmatrix} R_H^{(1)} \\ 0 \end{bmatrix}
\]

- Decompose $G$ into a lower-triangular matrix and unitary matrix using LQ decomposition.

\[
G = L_G Q_G = \begin{bmatrix} L_G^{(1)} & 0 \end{bmatrix} \begin{bmatrix} Q_G^{(1)} \\ Q_G^{(0)} \end{bmatrix}
\]

- Precoder $W$ is therefore given as

\[
W = Q_G^{(1)H} D Q_H^{(1)H} = \left( Q_H^{(1)} D Q_G^{(1)} \right)^H.
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Sum-rate comparison of proposed solutions

Figure: Average sum-rate comparison for balanced links. Number of relay antennas = 2.
Sum-rate comparison of proposed solutions

Figure: Avg sum-rate comparison for unbalanced links. Number of relay antennas = 2.
Conclusions

- Simultaneous exchange of two-way data traffic, assumed in two-way relaying, normally does not happen in the cellular systems.

- Problem of data-flow asymmetry in two-way AF relaying is considered.

- Novel precoder to cancel the back-propagating interference is designed.

- Sum-rate performance of proposed precoder is significantly better than OWR.

- Proposed precoder also performs better than the conventional precoders.

- Can help in the integration of TWR in the existing cellular systems.
Further work

- Designed precoder with multiple antennas at all the nodes.

- Studied power allocation by the relay to BS and downlink UE to maximize the weighted sum-rate.
  - Can be formulated as a convex optimization program