Precoder Design for Asymmetric Two-way AF Relay

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Background

- Cooperative communication can lead to significant performance improvements in wireless systems.
- Conventional one-way relaying is an example.
- Half-duplex signaling in one-way relaying leads to loss of $\frac{1}{2}$ of spectral resources.
- Four channel uses are required for bidirectional data exchange.
- Two-way relaying requires two channel uses instead of four.[1]

[1] Rankov, B and Wittneben, A, "Spectral Efficient Protocols for Half-Duplex Fading Relay Channels", IEEE J. Sel. Areas Commun, vol. 25, no. 2, pp. 375–385, 2007.

Two-way relaying

- Two source nodes simultaneously transmit to the relay during first phase.

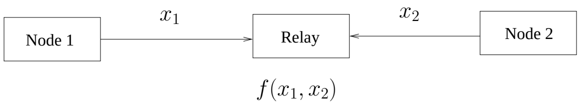


Figure: First phase of two-way relaying

- Relay broadcasts a function of the sum-signal during second phase.

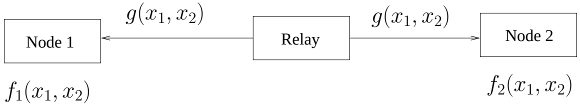


Figure: Broadcast phase of two-way relaying

Two-way relaying (Contd...)

Two-way relaying is most appropriate when two nodes exchange data simultaneously.

- Simultaneous two-way data exchange need not happen in cellular systems.
- User might have uplink data to transmit but no downlink data to receive.
- Two-way relaying will reduce to conventional one-way relaying.

Asymmetric Two-way relaying

- Consider an infrastructure relay where multiple UEs are served via relay.[2]
  - Used for coverage extension and filling coverage holes.
- A user UE_1 wants to send UL data to BS through the relay. Has no DL data to receive.
- Consider another UE (UE_2), which wants to receive DL data from the BS.
  - Can be found with a high probability in a multi-user system.

[2] S.W. Peters, A.Y. Panah, K.T. Truong and R.W Heath "Relay architectures for 3GPP LTE-Advanced", EURASIP J. Wireless Commun. and Netw.

Asymmetric Two-way relaying (Contd...)

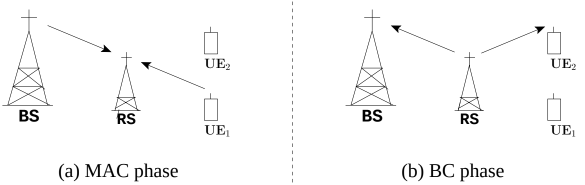


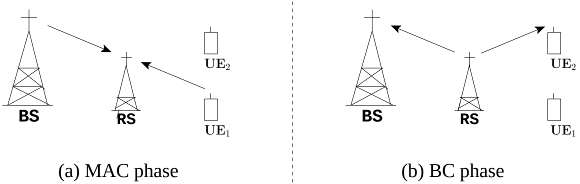
Figure: Asymmetric two-way relaying.

Propose a new two-way relaying protocol

1. BS and UE_1 transmit to the relay during MAC phase.
2. Relay transmits to the BS and UE_2 during BC phase.

Two way relaying becomes asymmetric. Helps in recovering the spectral efficiency loss.

Asymmetric Two-way relaying (Contd...)



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Propose a new two-way relaying protocol

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Asymmetric Two-way relaying (Contd...)

- But leads to problem of asymmetric back-propagating interference (BPI).

- BS can cancel the BPI, while downlink single-antenna UE (UE_2) cannot.
  - Downlink UE cannot overhear uplink UE transmission.

Problem Description

Precoder design at the relay to cancel the BPI for downlink UE

System model

- BSs and UEs have one antenna each while relay has multiple antennas.
- $\mathbf{y}_r = \mathbf{H}\mathbf{x} + \mathbf{n}_r$: Sum-signal received by the relay during MAC phase.
- Here $\mathbf{H} = [\mathbf{h}_1 \ \mathbf{h}_2]$ and $\mathbf{x} = [x_1 \ x_2]^T$.
- Signal transmited by the relay: $\mathbf{x}_r = \mathbf{W}\mathbf{y}_r$.

Here $\mathbf{W}$ is the precoder matrix to be designed.

System model (Contd...)

- Signals received by UE_2 and BS, y_1 and y_2 during BC phase

$$y_i = (\mathbf{g}_i)^T \mathbf{x}_r + n_i, \qquad i = 1, 2.$$

- Maximum rates observed by BS and UE_2 are given respectively as:

$$R_b = \log(1 + \mathrm{SNR}_b), R_u = \log(1 + \mathrm{SNR}_u).$$

- $R_{sum} = \frac{1}{2}(R_b + R_u)$ is the system sum-rate.
  - Used for performance comparison.

Precoder design

- Precoder structure: $\mathbf{W} = \mathbf{MDF}$, where
  - $\mathbf{F}$: Uplink precoder, $\mathbf{M}$: Downlink precoder and
  - $\mathbf{D}$: Permutation and power-normalization matrix

$$\mathbf{D} = \begin{bmatrix} 0 & \beta \\ \beta & 0 \end{bmatrix}. \; \beta \text{ is used to normalize the relay power.}$$

Uplink and downlink precoder matrix design

ZF-based solution

- $\mathbf{F} = \mathbf{H}^{\dagger} = \left(\mathbf{H}^H\mathbf{H}\right)^{-1}\mathbf{H}^H$,
- $\mathbf{M} = \mathbf{G}^{\dagger} = \mathbf{G}^H\left(\mathbf{G}\mathbf{G}^H\right)^{-1}$.

- $\mathbf{F}$ and $\mathbf{M}$ can also be designed using MMSE criterion.

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Precoder design

- ZF/MMSE precoders mitigate the interference for UE_2 and BS.
- ZF/MMSE precoders are sub-optimal as BS can perform self-interference cancellation.

Precoder design based on channel decomposition proposed

- Cancels the interference for downlink UE alone.
- Sum-rate performance is better than ZF/MMSE precoders.

- Rewrite the signals received by UE_2 and BS during the BC phase:

$$
\begin{aligned}
\mathbf{y} &= \mathbf{GW}\left(\mathbf{Hx} + \mathbf{n}_r\right) + \mathbf{n} \\
&= \underbrace{\mathbf{GM}}_{\mathbf{G}_t}\mathbf{D}\underbrace{\mathbf{FH}}_{\mathbf{H}_t}\mathbf{x} + \underbrace{\mathbf{GWn}_r + \mathbf{n}}_{\mathbf{n}_t} \\
&= \mathbf{G}_t\mathbf{DH}_t\mathbf{x} + \mathbf{n}_t
\end{aligned}
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Precoder design - Based on Channel decomposition

Lemma

Precoders **M** *and* **F** *should be designed such that* $\mathbf{G}_t$ *and* $\mathbf{H}_t$ *are lower-triangular and upper-triangular matrices respectively.*

Proof: With the lower- and upper-triangular matrices $\mathbf{G}_t$ and $\mathbf{H}_t$ will become

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} l_1 & 0 \\ l_2 & l_3 \end{bmatrix} \begin{bmatrix} 0 & \beta \\ \beta & 0 \end{bmatrix} \begin{bmatrix} u_1 & u_2 \\ 0 & u_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \widetilde{\mathbf{n}}$$

$$= \begin{bmatrix} \beta\,(l_1 u_3)\,x_2 \\ \beta(l_3 u_1)x_1 + \beta(l_3 u_2 + l_2 u_3)x_2 \end{bmatrix} + \widetilde{\mathbf{n}}.$$

- UE_2 receives its desired data x_2 without any interference.

Design of uplink and downlink precoder matrices

- Decompose $\mathbf{H}$ into a unitary matrix and an upper-triangular matrix using QR decomposition.

$$\mathbf{H} = \mathbf{Q_H R_H} = \left[\begin{array}{cc} \mathbf{Q_H^{(1)}} & \mathbf{Q_H^{(0)}} \end{array} \right] \left[\begin{array}{c} \mathbf{R_H^{(1)}} \\ \mathbf{0} \end{array} \right]$$

- Decompose $\mathbf{G}$ into a lower-triangular matrix and unitary matrix using LQ decomposition.

$$\mathbf{G} = \mathbf{L_G Q_G} = \left[\begin{array}{cc} \mathbf{L_G^{(1)}} & \mathbf{0} \end{array} \right] \left[\begin{array}{c} \mathbf{Q_G^{(1)}} \\ \mathbf{Q_G^{(0)}} \end{array} \right]$$

- Precoder $\mathbf{W}$ is therefore given as

$$\mathbf{W} = \mathbf{Q_G^{(1)H} D Q_H^{(1)H}} = \left(\mathbf{Q_H^{(1)} D Q_G^{(1)}} \right)^H .$$

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Sum-rate comparison of proposed solutions

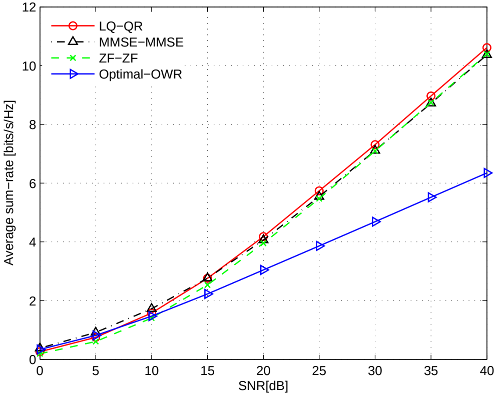

Figure: Average sum-rate comparison for balanced links. Number of relay antennas = 2.

Sum-rate comparison of proposed solutions

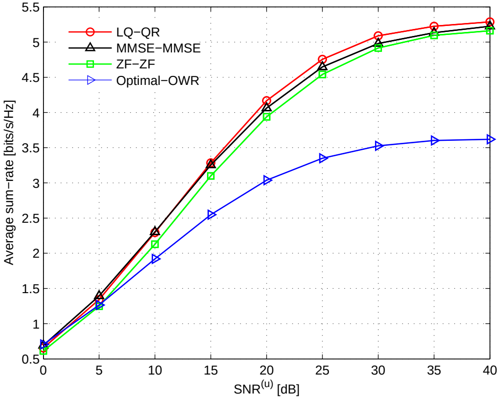

Figure: Avg sum-rate comparison for unbalanced links. Number of relay antennas = 2.

Conclusions

- Simultaneous exchange of two-way data traffic, assumed in two-way relaying, normally does not happen in the cellular systems.
- Problem of data-flow asymmetry in two-way AF relaying is considered.
- Novel precoder to cancel the back-propagating interference is designed.
- Sum-rate performance of proposed precoder is significantly better than OWR
- Proposed precoder also performs better than the conventional precoders.
- Can help in the integration of TWR in the existing cellular systems

Further work

- Designed precoder with multiple antennas at all the nodes.

- Studied power allocation by the relay to BS and downlink UE to maximize the weighted sum-rate.
  - Can be formulated as a convex optimization program