Precoder Design for Asymmetric Two-way AF Shared Relay

Rohit Budhiraja
Prof. Bhaskar Ramamurthi

Department of Electrical Engineering
Indian Institute of Technology Madras
May 29, 2013
Background

- Cooperative communication can lead to significant performance improvements in wireless systems.

- Conventional one-way relaying is an example.
Cooperative communication can lead to significant performance improvements in wireless systems.

Conventional one-way relaying is an example.

Half-duplex signaling in one-way relaying leads to loss of $\frac{1}{2}$ of spectral resources.

Four channel uses are required for bidirectional data exchange.
Background

- Cooperative communication can lead to significant performance improvements in wireless systems.

- Conventional one-way relaying is an example.

- Half-duplex signaling in one-way relaying leads to loss of $\frac{1}{2}$ of spectral resources.

- Four channel uses are required for bidirectional data exchange.

- Two-way relaying requires two channel uses instead of four.\(^1\)
Cooperative communication can lead to significant performance improvements in wireless systems.

Conventional one-way relaying is an example.

Half-duplex signaling in one-way relaying leads to loss of $\frac{1}{2}$ of spectral resources.

Four channel uses are required for bidirectional data exchange.

Two-way relaying requires two channel uses instead of four.\(^1\)
Background

- Cooperative communication can lead to significant performance improvements in wireless systems.

- Conventional one-way relaying is an example.

- Half-duplex signaling in one-way relaying leads to loss of $\frac{1}{2}$ of spectral resources.

- Four channel uses are required for bidirectional data exchange.

- Two-way relaying requires two channel uses instead of four.\(^1\)

---

Two-way relaying

- Two source nodes simultaneously transmit to the relay during first phase.

\[ f(x_1, x_2) \]

**Figure:** First phase of two-way relaying
Two-way relaying

- Two source nodes simultaneously transmit to the relay during first phase.

![First phase of two-way relaying](image)

**Figure:** First phase of two-way relaying

- Relay broadcasts a function of the sum-signal during second phase.

![Broadcast phase of two-way relaying](image)

**Figure:** Broadcast phase of two-way relaying
Two-way relaying (Contd…)

- Two-way relaying is most appropriate when two nodes exchange data simultaneously.
Two-way relaying (Contd...)

- Two-way relaying is most appropriate when two nodes exchange data simultaneously.

- Simultaneous two-way data exchange need not happen in cellular systems.
Two-way relaying (Contd…)

- Two-way relaying is most appropriate when two nodes exchange data simultaneously.

- Simultaneous two-way data exchange need not happen in cellular systems.

- User might have uplink data to transmit but no downlink data to receive.
Two-way relaying (Contd...)

- Two-way relaying is most appropriate when two nodes exchange data simultaneously.

- Simultaneous two-way data exchange need not happen in cellular systems.

- User might have uplink data to transmit but no downlink data to receive.

- Two-way relaying will reduce to conventional one-way relaying.
Asymmetric Two-way relaying

- Consider infrastructure relay scenario, where multiple UEs are served through a relay.

**Figure:** Asymmetric two-way relaying.
Asymmetric Two-way relaying

- Consider infrastructure relay scenario, where multiple UEs are served through a relay.

**Figure:** Asymmetric two-way relaying.

- Two single-antenna UEs want to communicate with a BS.
Asymmetric Two-way relaying

- Consider infrastructure relay scenario, where multiple UEs are served through a relay.

![Asymmetric two-way relaying diagram](image)

**Figure:** Asymmetric two-way relaying.

- Two single-antenna UEs want to communicate with a BS.

- UE$_1$ has data to be sent to the BS. UE$_2$ wants to receive data from BS.
Asymmetric Two-way relaying (Contd...)

- Leads to problem of asymmetric back-propagating interference (BPI).
Asymmetric Two-way relaying (Contd...) 

- Leads to problem of asymmetric back-propagating interference (BPI).

- BS can cancel the BPI, while **Downlink single-antenna UE (UE₂) cannot.**
Leads to problem of asymmetric back-propagating interference (BPI).

BS can cancel the BPI, while Downlink single-antenna UE (UE\textsubscript{2}) cannot.

We have proposed a precoder to cancel the asymmetric back-propagating interference for UE\textsubscript{2}.\textsuperscript{2}

Asymmetric Two-way relaying (Contd...)

- Leads to problem of asymmetric back-propagating interference (BPI).

- BS can cancel the BPI, while Downlink single-antenna UE (UE₂) cannot.

- We have proposed a precoder to cancel the asymmetric back-propagating interference for UE₂.²

- Proposed precoder is shown to be better than the conventional ZF and MMSE precoders.

Asymmetric Two-way relaying for a shared relay

- Data-flow asymmetry problem is extended to shared relay in this work.\(^3\)

Figure: Illustration of asymmetric two-way shared relaying.

Asymmetric Two-way relaying for a shared relay

- Data-flow asymmetry problem is extended to shared relay in this work.\(^3\)

\[\text{Figure: Illustration of asymmetric two-way shared relaying.}\]

- BSs can cancel the back-propagating interference (BPI), but not IUI.

Asymmetric Two-way relaying for a shared relay

- Data-flow asymmetry problem is extended to shared relay in this work.\(^3\)

![Diagram](image.png)

**Figure:** Illustration of asymmetric two-way shared relaying.

- BSs can cancel the back-propagating interference (BPI), but not IUI.

- Downlink UEs cannot cancel both BPI and IUI.

---

System model

- BSs and UEs have one antenna each.
System model

- BSs and UEs have one antenna each.

- $x_1^k$ and $x_2^k$: Data transmitted by UE$_1^{(k)}$ and BS$_{(k)}$ to the relay during phase-1.
System model

- BSs and UEs have one antenna each.
- $x_1^k$ and $x_2^k$: Data transmitted by UE$_1^k$ and BS$_k$ to the relay during phase-1.
- $h_1^k$ and $h_2^k$: Uplink channels observed by the relay from UE$_1^k$ and BS$_k$. 
System model

- BSs and UEs have one antenna each.
- $x_1^k$ and $x_2^k$: Data transmitted by UE$_1^{(k)}$ and BS$_1^{(k)}$ to the relay during phase-1.
- $h_1^k$ and $h_2^k$: Uplink channels observed by the relay from UE$_1^{(k)}$ and BS$_2^{(k)}$.
- $y_r = \sum_{k=1}^{K} H_k x_k + n_r$: Sum-signal received by the relay during phase-1.
System model

- BSs and UEs have one antenna each.

- $x^k_1$ and $x^k_2$: Data transmitted by UE$_1^{(k)}$ and BS$_1^{(k)}$ to the relay during phase-1.

- $h^k_1$ and $h^k_2$: Uplink channels observed by the relay from UE$_1^{(k)}$ and BS$_1^{(k)}$.

- $y_r = \sum_{k=1}^K H_k x_k + n_r$: Sum-signal received by the relay during phase-1.

- Here $H_k = \begin{bmatrix} h^k_1 & h^k_2 \end{bmatrix}$ and $x_k = \begin{bmatrix} x^k_1 & x^k_2 \end{bmatrix}^T$. 
System model

- BSs and UEs have one antenna each.

- $x_1^k$ and $x_2^k$: Data transmitted by UE$_1^k$ and BS$_1^k$ to the relay during phase-1.

- $h_1^k$ and $h_2^k$: Uplink channels observed by the relay from UE$_1^k$ and BS$_1^k$.

- $y_r = \sum_{k=1}^{K} H_k x_k + n_r$: Sum-signal received by the relay during phase-1.

- Here $H_k = [h_1^k \ h_2^k]$ and $x_k = [x_1^k \ x_2^k]^T$.

- Signal transmit by the relay: $x_r = W y_r$. 
System model (Contd...)

- $g_1^k$ and $g_2^k$: Downlink channels seen by $\text{UE}_2^{(k)}$ and $\text{BS}^{(k)}$ respectively.
System model (Contd…)

- $g_1^k$ and $g_2^k$: Downlink channels seen by $\text{UE}_2^{(k)}$ and $\text{BS}^{(k)}$ respectively.

- Signals received by $\text{UE}_2^{(k)}$ and $\text{BS}^{(k)}$, $y_1^k$ and $y_2^k$ during phase-2

$$y_i^k = (g_i^k)^T x_r + n_i^k, \quad i = 1, 2. \quad (1)$$
System model (Contd...) 

- \( \mathbf{g}_1^k \) and \( \mathbf{g}_2^k \): Downlink channels seen by UE\(_2^k\) and BS\(_k\) respectively.

- Signals received by UE\(_2^k\) and BS\(_k\), \( y_1^k \) and \( y_2^k \) during phase-2

\[
y_i^k = (\mathbf{g}_i^k)^T \mathbf{x}_r + n_i^k, \quad i = 1, 2. \tag{1}
\]

- Maximum rates observed by BS\(_k\) and UE\(_2^k\) are given respectively as:

\[
R_b^k = \log \left( 1 + \text{SNR}_b^k \right), \quad R_u^k = \log \left( 1 + \text{SNR}_u^k \right). \tag{2}
\]
System model (Contd...)

- $g_1^k$ and $g_2^k$: Downlink channels seen by UE$_2^k$ and BS$_k$ respectively.

- Signals received by UE$_2^k$ and BS$_k$, $y_1^k$ and $y_2^k$ during phase-2
  \[
y_i^k = (g_i^k)^T x_r + n_i^k, \quad i = 1, 2.
\]

- Maximum rates observed by BS$_k$ and UE$_2^k$ are given respectively as:
  \[
  R_b^k = \log \left( 1 + \text{SNR}_b^k \right), \quad R_u^k = \log \left( 1 + \text{SNR}_u^k \right).
  \]

- $R_{sum} = \frac{1}{2} \sum_{k=1}^K R_b^k + R_u^k$ is the system sum-rate.
System model (Contd...)

- $g_1^k$ and $g_2^k$: Downlink channels seen by UE$_2^k$ and BS$_k$ respectively.

- Signals received by UE$_2^k$ and BS$_k$, $y_1^k$ and $y_2^k$ during phase-2

\[ y_i^k = (g_i^k)^T x_r + n_i^k, \quad i = 1, 2. \] (1)

- Maximum rates observed by BS$_k$ and UE$_2^k$ are given respectively as:

\[ R_b^k = \log \left( 1 + \text{SNR}_b^k \right), \quad R_u^k = \log \left( 1 + \text{SNR}_u^k \right). \] (2)

- $R_{sum} = \frac{1}{2} \sum_{k=1}^{K} R_b^k + R_u^k$ is the system sum-rate.

  - Used for performance comparison.
Problem Description

- Design of precoder to cancel the BPI and IUI for downlink UEs and IUI for the BS.
Problem Description

- Design of precoder to cancel the BPI and IUI for downlink UEs and IUI for the BS.

- Precoder structure: $W = \text{MDF}$, where
Problem Description

- Design of precoder to cancel the BPI and IUI for downlink UEs and IUI for the BS.

- Precoder structure: \( W = MDF \), where
  - \( F \): Uplink precoder matrix
Problem Description

- Design of precoder to cancel the BPI and IUI for downlink UEs and IUI for the BS.

- Precoder structure: \( W = MDF \), where
  - \( F \): Uplink precoder matrix
  - \( D \): Permutation and power-normalization matrix, and
Problem Description

- Design of precoder to cancel the BPI and IUI for downlink UEs and IUI for the BS.

- Precoder structure: $W = MDF$, where
  - $F$: Uplink precoder matrix
  - $D$: Permutation and power-normalization matrix, and
  - $M$: Downlink precoder matrix.
Problem Description

- Design of precoder to cancel the BPI and IUI for downlink UEs and IUI for the BS.

- Precoder structure: \( W = MDF \), where
  - \( F \): Uplink precoder matrix
  - \( D \): Permutation and power-normalization matrix, and
  - \( M \): Downlink precoder matrix.

- \( F = [ F_1^T \  F_2^T \  \cdots \  F_K^T ]^T \) and \( M = [ M_1 \  M_2 \  \cdots \  M_K ] \) are individual uplink and downlink precoders.
Problem Description

- Design of precoder to cancel the BPI and IUI for downlink UEs and IUI for the BS.

- Precoder structure: $W = MDF$, where
  - $F$: Uplink precoder matrix
  - $D$: Permutation and power-normalization matrix, and
  - $M$: Downlink precoder matrix.

- $F = [F_1^T, F_2^T, \cdots, F_K^T]^T$ and $M = [M_1, M_2, \cdots, M_K]$ are individual uplink and downlink precoders.

- $D = \text{diag}\{D_1, D_2, \cdots, D_K\}$. Permutation matrix $D_k$ is given as:
  $$D_k = \begin{bmatrix} 0 & \beta \\ \beta & 0 \end{bmatrix}.$$  $eta$ is used to normalize the relay power.
Precoder design

- Two-step precoder design
Precoder design

- Two-step precoder design
  - Precoder to block-diagonalize the channel matrix to cancel the IUI\(^4\).

---

Precoder design

- Two-step precoder design
  1. Precoder to block-diagonalize the channel matrix to cancel the IUI\(^4\).
  2. Precoder to cancel the BPI for UE\(_k\) from the IUI-free channel

---

Precoder design

- Two-step precoder design
  1. Precoder to block-diagonalize the channel matrix to cancel the IUI\(^4\).
  2. Precoder to cancel the BPI for UE\(_k\) from the IUI-free channel.

To cancel the UL IUI, \(\mathbf{F}\(_{k}^{(1)}\) should be in the null space of

\[\tilde{\mathbf{H}}_k = [\mathbf{H}_1 \; \cdots \; \mathbf{H}_{k-1} \; \mathbf{H}_{k+1} \; \cdots \; \mathbf{H}_K].\]

---

Precoder design

- Two-step precoder design
  1. Precoder to block-diagonalize the channel matrix to cancel the IUI\(^4\).
  2. Precoder to cancel the BPI for UE\(_k\) from the IUI-free channel

To cancel the UL IUI, \(\mathbf{F}_k^{(1)}\) should be in the null space of 
\[
\tilde{\mathbf{H}}_k = \begin{bmatrix}
\mathbf{H}_1 & \cdots & \mathbf{H}_{k-1} & \mathbf{H}_{k+1} & \cdots & \mathbf{H}_K
\end{bmatrix}.
\]

To cancel the DL IUI, \(\mathbf{M}_k^{(1)}\) should be in the null space of 
\[
\tilde{\mathbf{G}}_k = \begin{bmatrix}
\mathbf{G}_1 & \cdots & \mathbf{G}_{k-1} & \mathbf{G}_{k+1} & \cdots & \mathbf{G}_K
\end{bmatrix}.
\]

---

Precoder design

- Two-step precoder design
  1. Precoder to block-diagonalize the channel matrix to cancel the IUI\(^4\).
  2. Precoder to cancel the BPI for UE\(_k\) from the IUI-free channel.

To cancel the UL IUI, \( \mathbf{F}^{(1)}_k \) should be in the null space of\(^5\)
\[
\tilde{\mathbf{H}}_k = \begin{bmatrix} \mathbf{H}_1 & \cdots & \mathbf{H}_{k-1} & \mathbf{H}_{k+1} & \cdots & \mathbf{H}_K \end{bmatrix}.
\]

To cancel the DL IUI, \( \mathbf{M}^{(1)}_k \) should be in the null space of\(^5\)
\[
\tilde{\mathbf{G}}_k = \begin{bmatrix} \mathbf{G}_1 & \cdots & \mathbf{G}_{k-1} & \mathbf{G}_{k+1} & \cdots & \mathbf{G}_K \end{bmatrix}.
\]

\( \mathbb{N}(\tilde{\mathbf{H}}_k) \) and \( \mathbb{N}(\tilde{\mathbf{G}}_k) \) are found using singular-value-decomposition.

---

Precoder design

- Two-step precoder design
  1. Precoder to block-diagonalize the channel matrix to cancel the IUI\(^4\).
  2. Precoder to cancel the BPI for UE\(^k\) from the IUI-free channel

- To cancel the UL IUI, \(\mathbf{F}_{\!k}(1)\) should be in the null space of
  \[
  \tilde{\mathbf{H}}_k = [\mathbf{H}_1 \cdots \mathbf{H}_{k-1} \mathbf{H}_{k+1} \cdots \mathbf{H}_K].
  \]

- To cancel the DL IUI, \(\mathbf{M}_{\!k}(1)\) should be in the null space of
  \[
  \tilde{\mathbf{G}}_k = [\mathbf{G}_1 \cdots \mathbf{G}_{k-1} \mathbf{G}_{k+1} \cdots \mathbf{G}_K].
  \]

- \(\mathbb{N}(\tilde{\mathbf{H}}_k)\) and \(\mathbb{N}(\tilde{\mathbf{G}}_k)\) are found using singular-value-decomposition.

- IUI-free channel can be viewed as multiple single-user-pair systems.

---

Precoder design (Contd...) 

- Precoder designed to cancel the BPI for single-user-pair can be used\(^5\).

---

Precoder design (Contd…)

- Precoder designed to cancel the BPI for single-user-pair can be used$^5$.

- Signal received by UE$_2$ and BS during the BC phase are re-written as:

$$y = GW (Hx + n_r) + n$$

$$= \underbrace{GM}_{G_t} \underbrace{D}_{H_t} \underbrace{FH}_{X} + \underbrace{GWn_r}_{n_t} + n$$

$$= G_t DH_t x + n_t$$

---

Precoder design (Contd...)

- Precoder designed to cancel the BPI for single-user-pair can be used\(^5\).

- Signal received by UE\(_2\) and BS during the BC phase are re-written as:

\[
y = GW (Hx + n_r) + n = GM \underbrace{D \, FH}^{G_t} x + \underbrace{GWn_r + n}_{n_t} = G_t DH_t x + n_t
\]

- Precoders \(M_k^{(2)}\) and \(F_k^{(2)}\) should be designed such that \(G_t\) and \(H_t\) are lower-triangular and upper-triangular.

---

Precoder designed (Contd…)

- Precoder designed to cancel the BPI for single-user-pair can be used\(^5\).

- Signal received by UE\(_2\) and BS during the BC phase are re-written as:

\[
y = GW (Hx + n_r) + n = GM_{G_t}D_{H_t}FH x + GW_{n_t} + n
\]

\[
= G_t D_{H_t x} + n_t
\]

- Precoders \(M^{(2)}_k\) and \(F^{(2)}_k\) should be designed such that \(G_t\) and \(H_t\) are lower-triangular and upper-triangular.
  - LQ and QR factorizations are used.

---

Precoder design (Contd...) 

- Precoder designed to cancel the BPI for single-user-pair can be used\(^5\).

- Signal received by UE\(_2\) and BS during the BC phase are re-written as:

\[
y = GW (Hx + n_r) + n = GM \underbrace{D}_{G_t} \underbrace{FH}_{H_t} x + GWn_r + n
\]

\[
= G_t DH_t x + n_t
\]

- Precoders \(M_k^{(2)}\) and \(F_k^{(2)}\) should be designed such that \(G_t\) and \(H_t\) are lower-triangular and upper-triangular.

  - LQ and QR factorizations are used.

- \(M_k = M_k^{(2)} M_k^{(1)}\) and \(F_k = F_k^{(2)} F_k^{(1)}\)

Sum-rate comparison between proposed and ZF precoder

Figure: Average sum-rate comparison with two BSs and number of relay antennas = 4.
Conclusions

- Simultaneous exchange of two-way data traffic, assumed in two-way relaying, normally does not happen in the cellular systems.
Conclusions

- Simultaneous exchange of two-way data traffic, assumed in two-way relaying, normally does not happen in the cellular systems.

- Problem of data-flow asymmetry in shared two-way AF relaying is considered.
Conclusions

- Simultaneous exchange of two-way data traffic, assumed in two-way relaying, normally does not happen in the cellular systems.

- Problem of data-flow asymmetry in shared two-way AF relaying is considered.

- Novel precoder to jointly cancel the IUI and back-propagating interference is designed.
Conclusions

- Simultaneous exchange of two-way data traffic, assumed in two-way relaying, normally does not happen in the cellular systems.

- Problem of data-flow asymmetry in shared two-way AF relaying is considered.

- Novel precoder to jointly cancel the IUI and back-propagating interference is designed.

- Sum-rate performance is shown to be better for the proposed precoder than for the conventional ZF precoder.