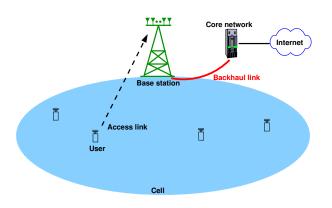
# Joint Transceiver Design for QoS-Constrained MIMO Two-Way Non-Regenerative Relaying

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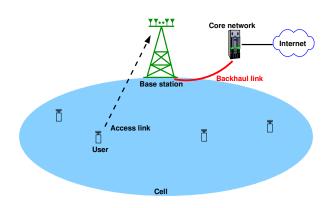
### Brief architecture of cellular systems



#### Three different nodes:

- Users who communicate via base station with wireless access links.
- ► Core n/w e.g., billing; connected to base station with wired backhaul links.

# Brief architecture of cellular systems

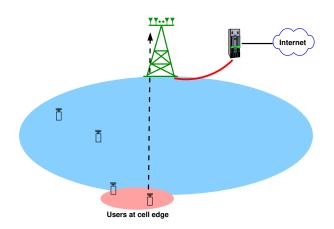


#### Three different nodes:

- Users who communicate via base station with wireless access links.
- ightharpoonup Core n/w e.g., billing; connected to base station with wired backhaul links.

Architecture works well if wireless access links are strong.

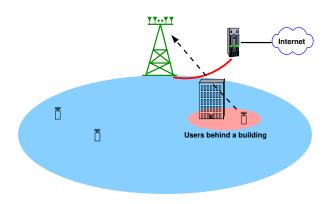
### Cellular scenarios with weak direct links



High attenuation due to large distance between base station and users.

Scenario usually observed in rural areas with large cell sizes.

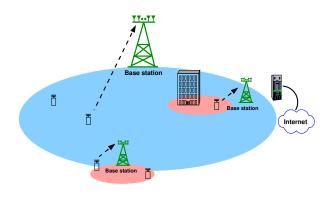
### Cellular scenarios with weak direct links



High attenuation due to multiple walls between base station and users.

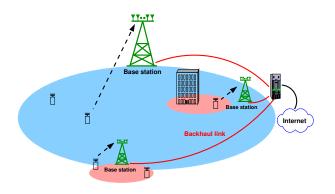
► Scenario usually observed in urban areas with high-rise apartments.

### Serve weak users with proximate base stations



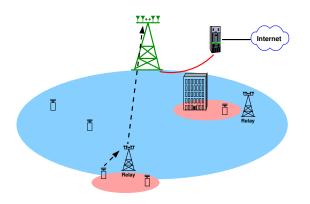
Advantage: Users will now observe strong signals from the base station.

### Serve weak users with proximate base stations



Limitation: Backhaul links make base station installation for few users costly.

### Serve weak users with relays



Relay is a base station with wireless backhaul – cheaper than base station.

Relay amplifies signal before retransmitting – improves signal strength.

Two different relaying protocols.

Half-duplex relays – cannot receive and transmit at same time on same frequency.

#### Time slot 1:



Half-duplex relays – cannot receive and transmit at same time on same frequency.

#### Time slot 1:



#### Time slot 2:



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### Time slot 2:



User needs two orthogonal time slots to send one data packet to base station.

► Twice the number of slots when user and base station communicate directly.

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### Time slot 2:



User needs two orthogonal time slots to send one data packet to base station.

► Twice the number of slots when user and base station communicate directly.

Requires two orthogonal time slots to transmit two data units.

Assumes a user wants to simultaneously exchange data with base station.

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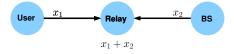
#### Time slot 2:



Requires two orthogonal time slots to transmit two data units.

Assume a user wants to simultaneously exchange data with base station.

#### Time slot 1:



#### Time slot 2:



Requires two orthogonal time slots to transmit two data units.

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#### Time slot 1:



#### Time slot 2:



[Rankov et. al, 2007]: Twice spectrally-efficient than one-way relaying.

### Basic assumption in two-way relaying

Assumption: user wants to simultaneously exchange data with base station.

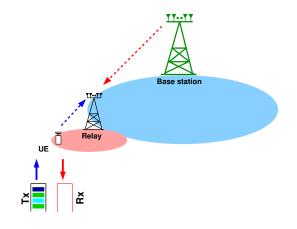


First time slot of two-way relaying

Strong assumption!

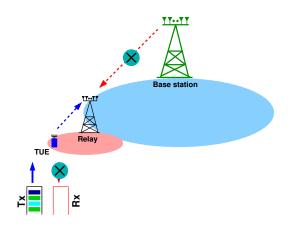
### Two-way relaying in cellular systems

User simultaneously sends and receives data from base station.



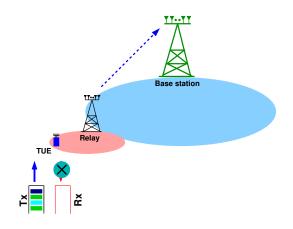
First time slot of two-way relaying

Usually does not happen. E.g., a user TUE uploading a video on facebook.



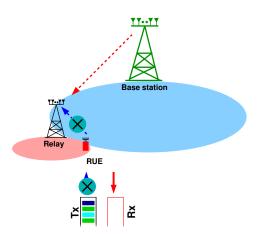
First time slot of two-way relaying

Two-way relaying reduces to one way relaying.

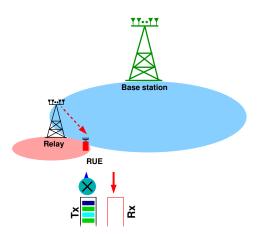


Second time slot of two-way relaying

Another example: a receive-only user RUE watching a youtube video.

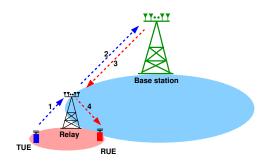


Two-way relaying again reduces to one-way relaying.



# To summarize: two-way relaying for asymmetric traffic

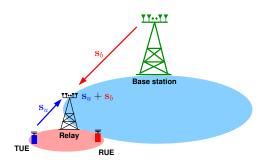
Is equivalent to one-way relaying.



Requires four orthogonal time slots. Two end-to-end non-interfering links.

# Proposed asymmetric two-way relaying (1)

Protocol solves considered problem using only two time slots.

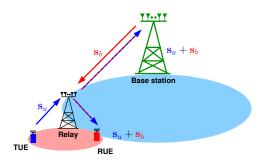


Time slot 1: Both base station and TUE transmit to relay.

Remember – links/data are color-coded.

# Proposed asymmetric two-way relaying (2)

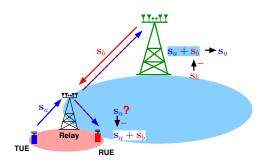
Protocol solves considered problem using only two time slots.



Time slot 2: Both base station and RUE receive the sum-signals.

# Proposed asymmetric two-way relaying (2)

Protocol solves considered problem using only two time slots.



Base station alone can cancel the interference while RUE cannot!

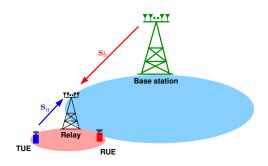
### Objective of the work

To establish two interference-free channels as in one-way relaying by using only two time slots.

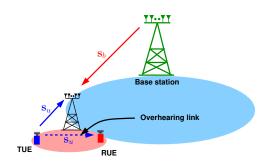
- ▶ Performs better than one-way relaying due to reduced number of time slots.
- Requires additional antennas at relay.

Optimal power allocation to maximize the system sum rate.

▶ Problem is hard-to-solve non-convex signomial program.

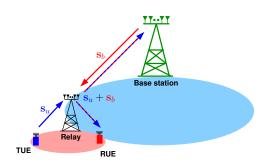


[Carvalho 2013] assumes RUE overhears TUE in first time slot.



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► Time slot 1 rx signal:  $y_1 = s_u$ 

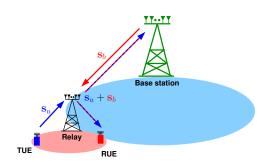


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► Time slot 1 rx signal:  $y_1 = s_u$ 

► Time slot 2 rx signal:  $y_2 = s_u + s_b$ .

[Sun 2013], [Paulraj 2014] – diversity-multiplexing tradeoff and sum rate.



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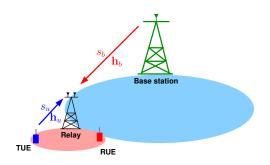
### Limitations of overhearing approach

- ▶ Practical in a cellular system, two users do not overhear each other.
- Analytical consider single-antenna nodes.
  - \* Difficult to extend optimization techniques for MIMO systems.

#### Our approach:

- Do not assume overhearing.
- Consider MIMO nodes.
  - **★** Geometric programming framework for sum rate optimization.

### System model and solution – first time slot

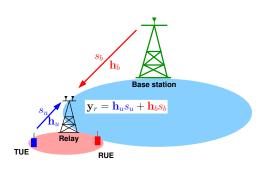


We limit discussion to single-antenna base station and users.

▶ Relay has two antennas; requires twice the number of TUE antennas.

Papers extend the work to multi-antenna base station and users.

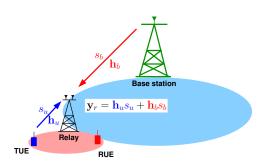
# System model and solution – first time slot



Relay receive signal:  $y_r = h_u s_u + h_b s_b$ .

Relay spatially separates  $s_u$  and  $s_b$  using receiver matrix  $\mathbf{H}^{-1}$ :

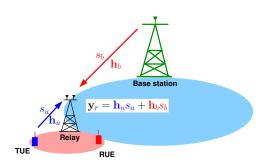
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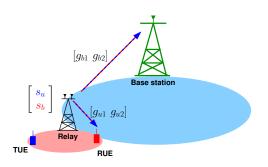
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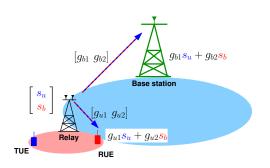


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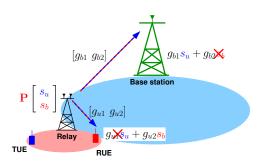


Relay has to broadcast  $\begin{bmatrix} s_u \\ s_b \end{bmatrix}$  to RUE and BS in second time slot.



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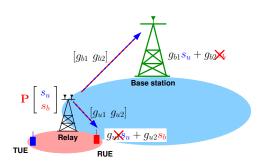
RUE still experiences interference from  $s_u$ .



Before broadcasting, relay multiplies  $\begin{bmatrix} s_u \\ s_b \end{bmatrix}$  with **P** to cancel  $s_u$  at RUE.

Signals received by RUE and base statior

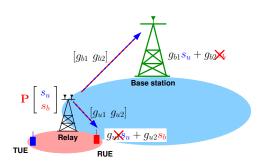
Set  $P = G^{-1}\Delta$ ; where  $\Delta$  is anti-diagonal power allocation matrix.



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Set  $\mathbf{P}=\mathbf{G}^{-1}\mathbf{\Delta};$  where  $\mathbf{\Delta}$  is anti-diagonal power allocation matrix.

RUE and BS receive signals become

$$\blacktriangleright \left[ \begin{array}{c} y_u \\ y_b \end{array} \right] = \mathbf{G}\mathbf{G}^{-1}\mathbf{\Delta} \left[ \begin{array}{c} \mathbf{s}_u \\ \mathbf{s}_b \end{array} \right] = \left[ \begin{array}{cc} \mathbf{0} & \delta_u \\ \delta_b & \mathbf{0} \end{array} \right] \left[ \begin{array}{c} \mathbf{s}_u \\ \mathbf{s}_b \end{array} \right] = \left[ \begin{array}{c} \delta_u \mathbf{s}_b \\ \delta_b \mathbf{s}_u \end{array} \right].$$

Sub-optimal approach as it cancels interference for base station also.

End-to-end input-output system

Design constructs  $\textbf{P}_2$  and  $\textbf{P}_1$  such that  $\textbf{GP}_2=\textbf{P}_1\textbf{H}=\left[\begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array}\right]$ 

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## Approach to cancel interference for RUE alone

#### **Theorem**

Design 
$$\mathbf{P}_2$$
 and  $\mathbf{P}_1$  such that  $\mathbf{GP}_2 = \left[ \begin{array}{cc} \times & 0 \\ \times & \times \end{array} \right]$  and  $\mathbf{P}_1\mathbf{H} = \left[ \begin{array}{cc} \times & \times \\ 0 & \times \end{array} \right]$ .

Receive signals in second time slot

- ▶ RUE:  $y_u = ()\delta_u s_b$ .
- $BS: y_b = ()\delta_b s_u + ()s_b.$

 $P_1$  and  $P_2$  are designed using LQ and QR decompositions of G and H.

SNR observed by RUE and BS

$$\mathsf{SNR}_i = rac{a_i \delta_i^2}{\sigma_r^2 (b_i \delta_u^2 + c_i \delta_b^2) + \sigma^2}$$
, where  $a_i, b_i, c_i \geq 0$ .

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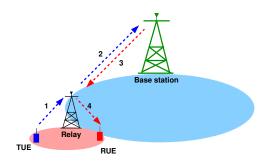
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► This fact will be used to prove convexity of optimization programs.

## Comparison with one-way relaying

Proposed protocol and one-way relaying establish two interference-free channels.

- ▶ Proposed protocol requires only two time slots and two relay antennas.
- ▶ In contrast, one-way relaying requires four time slots and single relay antenna.



## Optimal power allocation

Maximize system sum rate; cast as a geometric program.

#### Geometric program terminology

▶ A monomial is a function  $f : \mathbf{R}_{++}^n : \to \mathbf{R}$  of the form

$$f(\mathbf{x}) = c x_1^{a_1} x_2^{a_2} \cdots x_n^{a_n}, \text{ where } c > 0 \text{ and } a_j \in \mathbf{R}.$$

- A posynomial is a positive sum of monomials; not closed under division.
- Objective in a geometric program is a posynomial.
- Inequality constraints are upper-bounded posynomials.

## Sum rate maximization

$$\begin{array}{ll} \max_{\pmb{\delta}\succeq 0} & \log\{1+\mathsf{SNR}_u(\pmb{\delta})\} + \log\{1+\mathsf{SNR}_b(\pmb{\delta})\} \\ \mathsf{subject to} & P_{\mathsf{relay}}(\pmb{\delta}) \leq p_r \end{array}$$

 $P_{
m relay}(\delta)$  is a posynomial [Budhiraja 2014] while objective isn't. We show why.

Recall SNR<sub>i</sub> = 
$$\frac{a_i \delta_i^2}{\sigma_r^2(b_i \delta_b^2 + c_i \delta_b^2) + \sigma^2}$$
; where  $a_i, b_i, c_i \geq 0$ .

- $ightharpoonup \{1 + \mathsf{SNR}_i(\pmb{\delta})\}$  is a ratio of two posynomials due to term 1.
- Sum rate maximization is a non-convex signomial program
- Note that ISNR<sub>i</sub> = 1/SNR<sub>i</sub> is a posynomial.



#### Sum rate maximization

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# Sum rate maximization at high SNR

Use high SNR approximation:  $log(1 + SNR) \simeq log(SNR)$ .

- $\qquad \qquad \mathsf{R}_{\mathsf{sum}}^{\mathsf{approx}} \simeq \tfrac{1}{2} \log \{\mathsf{SNR}_u(\boldsymbol{\delta}) \cdot \mathsf{SNR}_b(\boldsymbol{\delta})\} = -\log \{\mathsf{ISNR}_u(\boldsymbol{\delta}) \cdot \mathsf{ISNR}_b(\boldsymbol{\delta})\}.$
- ▶ Note that  $ISNR_u(\delta) \cdot ISNR_b(\delta)$  is a posynomial.

Sum rate can now be maximized as a geometric program:

$$\begin{array}{ll} \mathop{\sf Min.}_{\delta\succeq 0} & \mathop{\sf ISNR}_u(\delta)\cdot\mathop{\sf ISNR}_b(\delta) \\ \\ \mathop{\sf subject\ to} & P_{\mathsf{relay}}(\delta)\leq p_r. \end{array}$$

At low SNR, sum rate is maximized using successive convex approximation.

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## Sum rate maximization with user rate constraints

$$\begin{array}{ll} \underset{\delta \succeq 0}{\mathsf{Min.}} & \mathsf{ISNR}_u(\delta) \cdot \mathsf{ISNR}_b(\delta) \\ \mathsf{subject to} & P_{\mathsf{relay}}(\delta) \leq p_r \\ & \mathsf{log}(1 + \mathsf{SNR}_u(\delta)) \geq r_u \\ & \mathsf{log}(1 + \mathsf{SNR}_b(\delta)) \geq r_b \end{array}$$

Cast as a geometric program by re-stating the rate constraints:

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Used to analyze the effect of rate constraints on sum rate.



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## Relay power minimization with user rate constraints

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Decides relay transmit power to support user rates.



# Maximize rate of the user with minimum SNR (max-min)

$$\begin{array}{ll} \mathsf{Max}. & \mathsf{Min.} \; \; (\mathsf{SNR}_u(\boldsymbol{\delta}), \mathsf{SNR}_b(\boldsymbol{\delta})) \\ \mathsf{subject} \; \mathsf{to} & P_{\mathsf{relay}}(\boldsymbol{\delta}) \leq p_r \end{array}$$

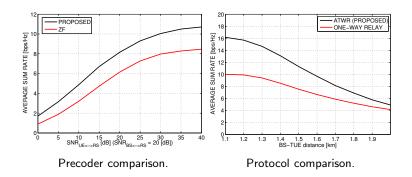
Cast as a geometric program by using epigraph form

$$\begin{array}{ll} \underset{\delta,t}{\mathsf{Min.}} & 1/t \\ \text{subject to} & t \cdot \mathsf{ISNR}_u(\delta) \leq 1, \ t \cdot \mathsf{ISNR}_b(\delta) \leq 1 \\ & P_{\mathsf{relay}}(\delta) \leq p_r \end{array}$$

Enforces fairness among users.

Optimal power allocation for energy-efficient design, multi-user transmission.

## Sum rate comparison of proposed precoders/protocols



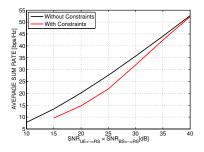
Assume 4 relay antennas and 2 antennas at the base station and users.

Antenna configuration is same for all precoders and protocols.

Proposed precoder works better as it avoids matrix inversion.

▶ Both precoders have same computational complexity.

#### Sum rate maximization with user rate constraints



Sum-rate with rate constraints.

Assume 8 relay antennas and 4 antennas at the base station and users.

Rate-constrained sum rate is inferior.

# Thank you

#### References

[1] Rohit Budhiraja and Ajit Chaturvedi "Common Transceiver Design for asymmetric and symmetric tw-way relaying", IEEE Trans. Vehicular Tech., accepted Jan. 2017.

[2] Rohit Budhiraja and Bhaskar Ramamurthi "Joint Precoder and Receiver Design for AF Non-Simultaneous Two-way MIMO Relaying", IEEE Trans. Wireless Commun., Jun. 2015.