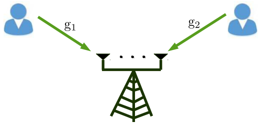
# Energy-Efficient Full-Duplex Massive MIMO Relays

Rohit Budhiraja

July 17, 2018

# Single-hop massive MIMO

- Consider a single-hop multiple-access system with two users



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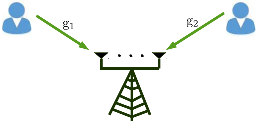


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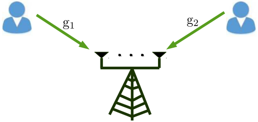


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- Received signal $\mathbf{y} = \mathbf{g}_1 x_1 + \mathbf{g}_2 x_2 + \mathbf{n}$
- Using maximal ratio combiner $\mathbf{w}_1^H = \frac{1}{N}\mathbf{g}_1$, we have

$$y_1 = \mathbf{w}_1^H \mathbf{y} = \mathbf{w}_1^H \mathbf{g}_1 x_1 + \mathbf{w}_1^H \mathbf{g}_2 x_2 + \mathbf{w}_1^H \mathbf{n}$$

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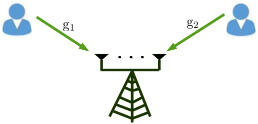


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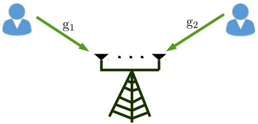


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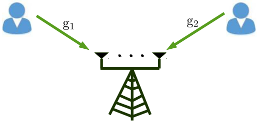


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y_1 \xrightarrow[N \to \infty]{\text{a.s.}} x_1
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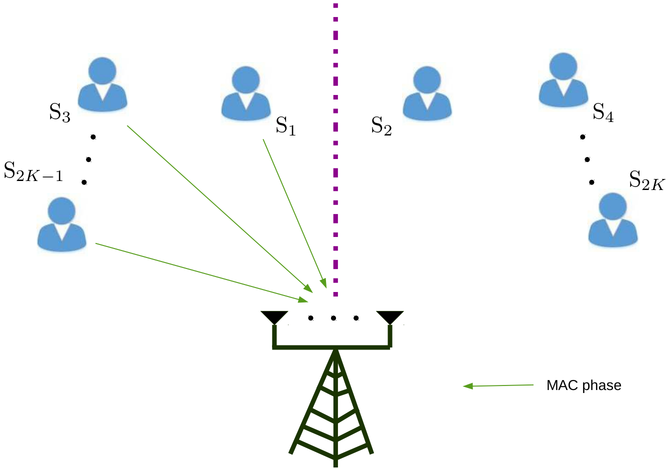
- Both interference and noise asymptotically vanish

# Half-duplex: One-way relay (MAC phase)



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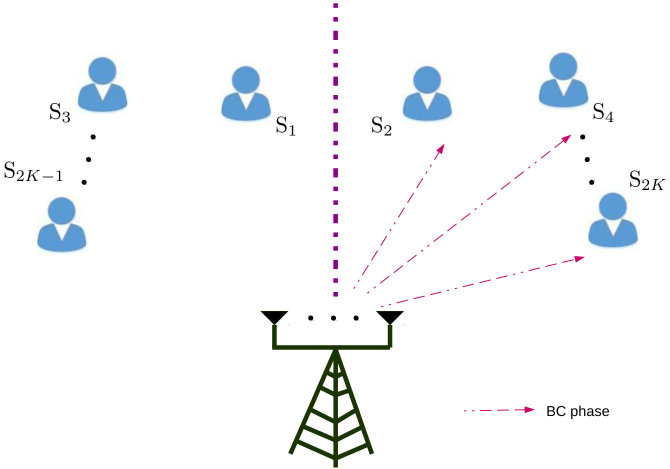

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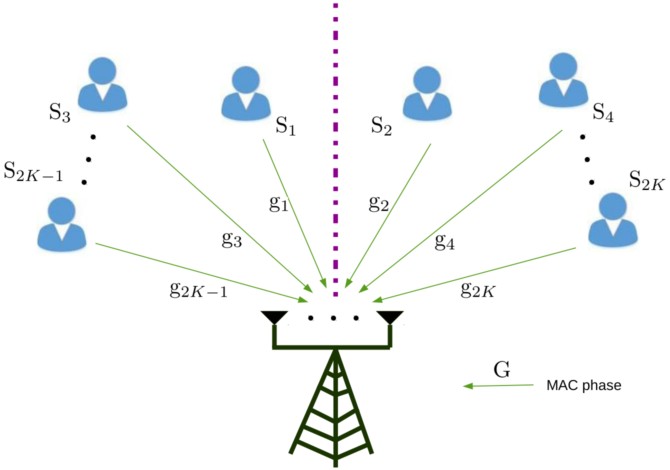

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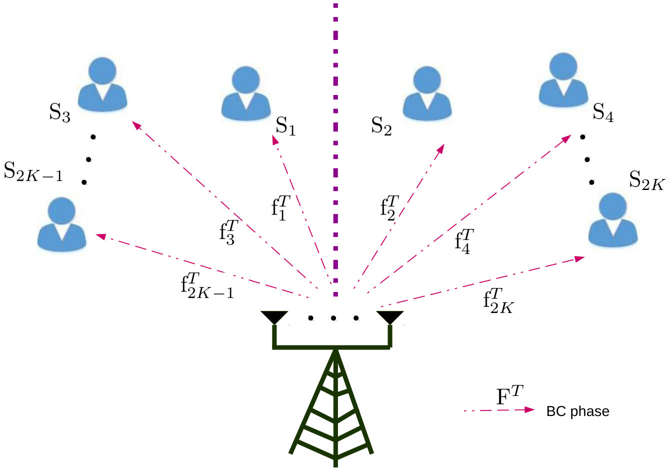
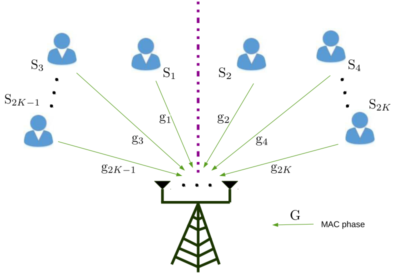


# Half-duplex: mathematical model for MAC phase




- Received signal at the relay, $\mathbf{y}_R = \sum_{k=1}^{2K} \sqrt{p_k}\mathbf{g}_k x_k + \mathbf{z}_R$

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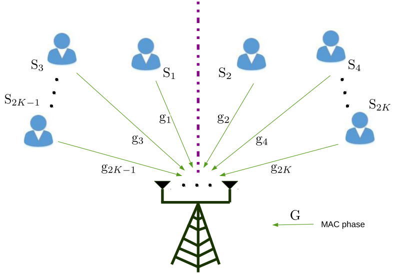


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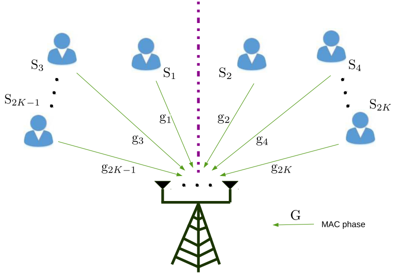


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$$\mathbf{x}_R = \alpha \mathbf{W}\mathbf{y}_R = \alpha \mathbf{W} \sum_{k=1}^{2K} \sqrt{p_k}\mathbf{g}_k x_k + \alpha \mathbf{W}\mathbf{z}_R$$

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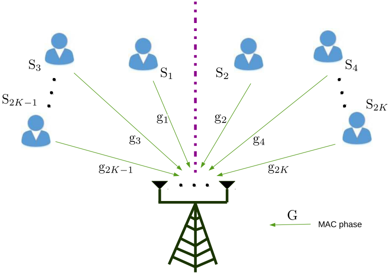


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- Relay MRC/MRT precoder is designed as $\mathbf{W} = \mathbf{F}^*\mathbf{G}^H$

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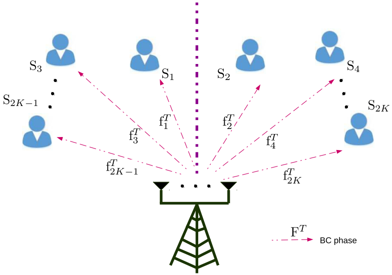


- Received signal at $k^{'}$th user is

$$y_{k'} = \mathbf{f}_{k'}^T \mathbf{x}_R + z_{k'}$$

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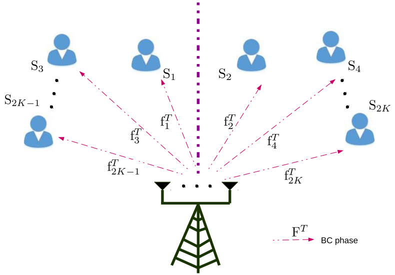


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$$y_{k'} \quad = \quad \mathbf{f}_{k'}^T \mathbf{x}_R + z_{k'} = \alpha \mathbf{f}_{k'}^T \mathbf{W} \sum_{k=1}^{2K} \sqrt{p_k} \mathbf{g}_k x_k + \alpha \mathbf{f}_{k'}^T \mathbf{W} \mathbf{z}_R + z_{k'}$$

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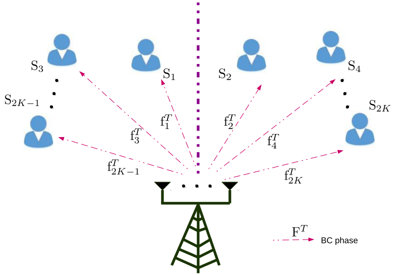


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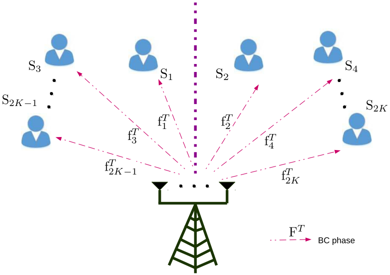


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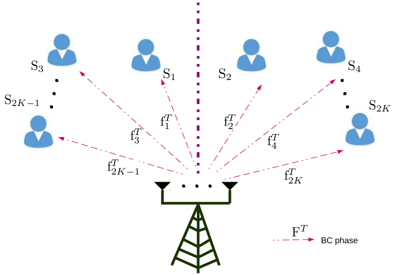


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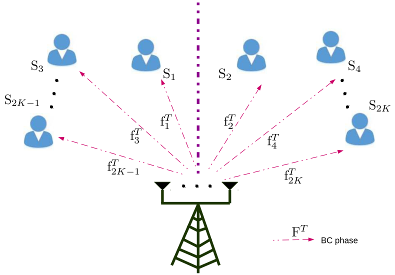


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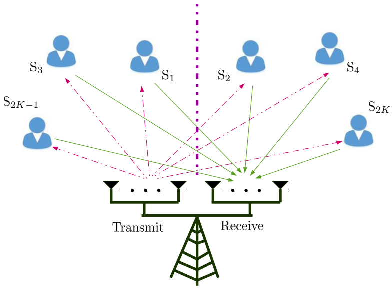
\end{aligned}
$$

# Full-duplex: Two-way relay



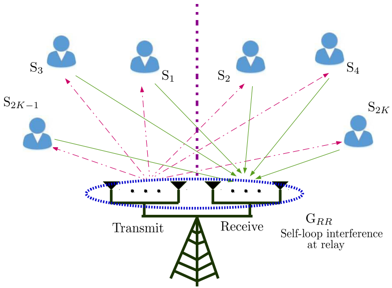

- Relay receive signal

$$\mathbf{y}_R(n) = \sum_{k=1}^{2K} \sqrt{p_k}\mathbf{g}_k x_k(n) + \mathbf{z}_R(n)$$

- Signal received by $k$th user

$$y_k(n) = \mathbf{f}_k^T \mathbf{x}_R(n) + z_k(n)$$

# Full-duplex: Two-way relay with loop interference at relay

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$$\mathbf{y}_R(n) = \sum_{k=1}^{2K} \sqrt{p_k}\mathbf{g}_k x_k(n) + \mathbf{G}_{RR}\mathbf{x}_R(n) + \mathbf{z}_R(n)$$

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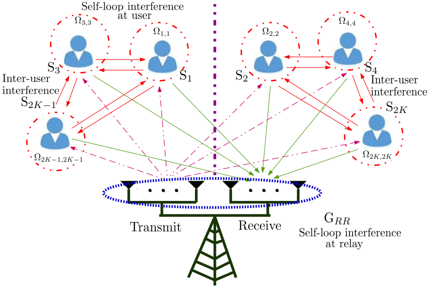
# Full-duplex: Two-way relay with loop and inter-user interference

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- Signal received by $k$th user

$$y_k(n) = \mathbf{f}_k^T \mathbf{x}_R(n) + \sum_{i,k \in U_k} \Omega_{k,i}\sqrt{p_i}x_i(n) + z_k(n)$$

# Full-duplex: Two-way relay with self-interference suppression

- At instant $n$, the relay transmit signal is $\mathbf{x}_R(n) = \alpha \mathbf{W} \mathbf{y}_R(n-1)$

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$$\mathbf{x}_R(n) = f[\mathbf{x}(n-1) + \mathbf{x}(n-2) + \cdots + \mathbf{z}_R(n-1) + \mathbf{z}_R(n-2) + \cdots]$$

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- With loop interference suppression, $\mathbf{x}_R(n)$ is modelled as a Gaussian noise source $\bar{\mathbf{x}}_R(n)$

$$\bar{\mathbf{y}}_R(n) = \sum_{k=1}^{2K} \sqrt{p_k} \mathbf{g}_k x_k(n) + \mathbf{G}_{RR} \bar{\mathbf{x}}_R(n) + \mathbf{z}_R(n), \text{ and}$$

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# Full-duplex two-way relay: signal detection

- Receive signal after self-interference cancellation at the user $S_k$ as

$$\tilde{y}_k = \underbrace{\alpha \mathbf{f}_k^T \mathbf{W} \sqrt{p_{k'}} \mathbf{g}_{k'} x_{k'}}_{\text{desired signal}}$$

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$$+ \underbrace{\alpha \mathbf{f}_k^T \mathbf{W} \mathbf{z}_R}_{\text{amplified noise from relay}} + \underbrace{z_k}_{\text{AWGN at } S_k}$$

- We only exploit the knowledge of the $\mathbb{E}\left[\mathbf{f}_k^T \mathbf{W} \mathbf{g}_{k'}\right]$ in the detection

$$\tilde{y}_k = \underbrace{\alpha \sqrt{p_{k'}}\, \mathbb{E}\left[\mathbf{f}_k^T \mathbf{W} \mathbf{g}_{k'}\right] x_{k'}}_{\text{desired signal}} + \underbrace{\tilde{n}_k}_{\text{effective noise}}, \text{ where}$$

# Full-duplex two-way relay: spectral efficiency lower bound

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$$R_{\text{lower}} = \sum_{k=1}^{2K} \log_2\left(1 + \text{SNR}_k\right), \text{ where}$$

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$$\text{SNR}_k = \frac{\alpha^2 p_{k'} \left|\mathbb{E}\left[\mathbf{f}_k^T \mathbf{W} \mathbf{g}_{k'}\right]\right|^2}{\mathbb{E}\left[|\tilde{n}_k|^2\right]} = \frac{a_k p_{k'}}{\sum_{i=1}^{2K}\left(b_{k,i}^{(1)} + b_{k,i}^{(2)} P_R^{-1} + \sum_{i,k\in U_k} p_i P_R^{-1} b_{k,i}^{(3)}\right) p_i + \left(d_k^{(1)} + d_k^{(2)} P_R + d_k^{(3)} P_R^{-1}\right)}$$

# Energy efficiency metrics and insights (1)

- Energy efficiency of $k - k^{'}$ pair is defined as

$$\frac{\log_2\left(1 + \mathrm{SNR}_k(p_k, P_R)\right)}{\mu_k p_k + P_R/2K + P_c}$$

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- Energy efficiency is a link-centric (or user-centric) performance metric
  - Global energy efficiency metric combines the individual EEs of different links

# Energy efficiency metrics and insights (2)

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# Global energy efficiency when SE (numerator) is optimized

$$\mathrm{GEE} = \frac{\sum_{k=1}^{2K} \log_2\left(1 + \mathrm{SNR}_k(p_k, P_R)\right)}{\sum_{k=1}^{2K} p_k + P_R + P_c}$$

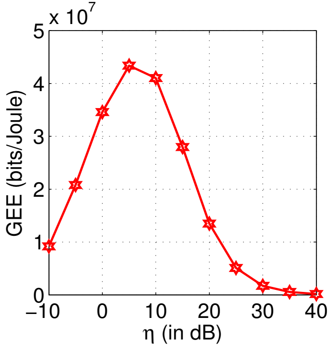


Figure: When SE (numerator of GEE) is optimized

# Global energy efficiency optimization

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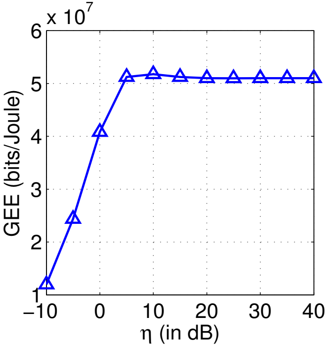


Figure: When GEE is optimized

# GEE Maximization

- GEE maximization problem is formulated as

$$\mathbf{P1}\ \text{:}\underset{p_k, P_R}{\text{Maximize}} \frac{\sum_{k=1}^{2K} \log_2 \left(1 + \mathrm{SNR}_k(p_k, P_R)\right)}{\sum_{k=1}^{2K} p_k + P_R + P_c}$$

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  - Approximate it as concave – problem becomes concave-convex fractional program

# Solution of concave-convex fractional program

**Proposition**

*Consider a concave-convex fractional program (CCFP) $g(x) = u(x)/v(x)$, with $u$ being non-negative, differentiable and concave,*

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# WSEE maximization

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$$\mathbf{P2}: \underset{\mathbf{p}}{\text{Maximize}} \sum_{k=1}^{2K} w_k \mathrm{EE}_k = \sum_{k=1}^{2K} w_k \frac{\log_2(1+\mathrm{SNR}(p_k))}{p_k + P_{c,k}} \tag{2a}$$

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- WSEE maximization problem for full duplex relays is an open problem

# WSEE optimization for half-duplex relays

- Epigraph form of the problem **P2** is as follows

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where $\mathrm{SNR}_k(\mathbf{p}) = \dfrac{N\beta_k^2\beta_{k'} p_k}{\sum_{i\ne k'}^{2K}(\beta_i\beta_{k'}\beta_k + \beta_i^2\beta_{i'})p_i + \sigma_{nr}^2\beta_{k'}\beta_k}$

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- Constraint in (3b) is non-convex; linearly approximate it using Taylor series

# Global energy efficiency results

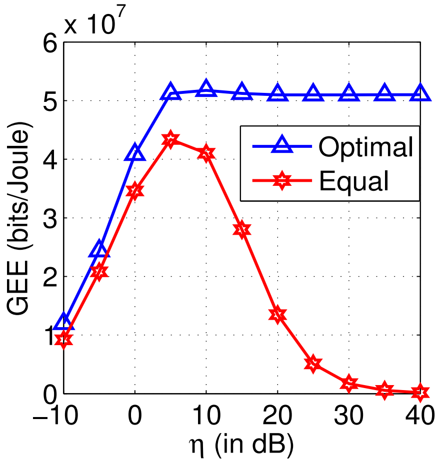



Figure: GEE versus $\eta = P_t^{\max}/\sigma^2$

# Effect of weights on weighted sum energy efficiency (1)

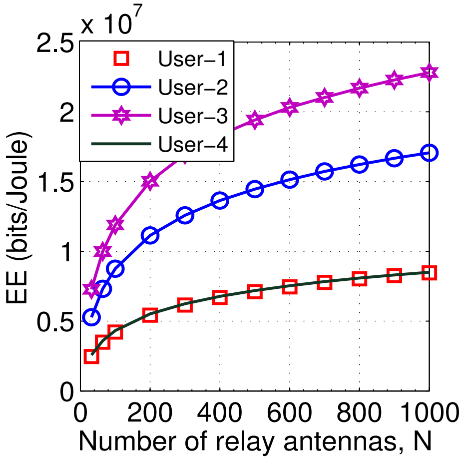
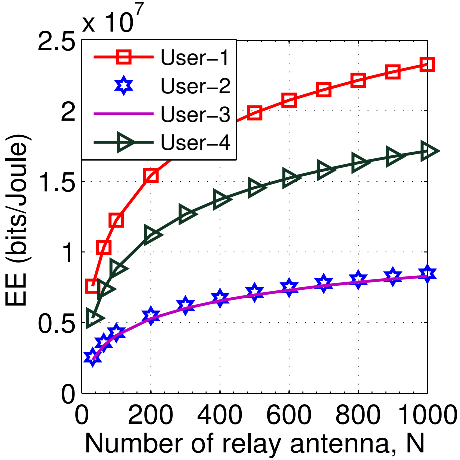



Figure: EE of each user versus $N$, with different weights: $\Lambda_1 : \{w_1 = 0.15, w_2 = 0.30, w_3 = 0.40, w_4 = 0.15\}$

# Effect of weights on weighted sum energy efficiency (2)

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# Thank you

## Relevant references


**[1] Ekant Sharma, Rohit Budhiraja et. al. "Full-Duplex Massive MIMO Multi-Pair Two-Way AF Relaying: Energy Efficiency Optimization ", IEEE Trans. Communications, accepted, to appear, 2018.**

**[2] Ekant Sharma, Swadha Siddhi Chauhan, and Rohit Budhiraja "Weighted Sum Energy Efficiency Optimization for Massive MIMO Two-Way Half-Duplex AF Relaying", IEEE Wireless Communications Letters, accepted, to appear, 2018**.