Joint Transceiver Design for QoS-Constrained MIMO Two-Way Non-Regenerative Relaying

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***Abstract*—Transceiver designs for multiple-input multiple-output (MIMO) two-way relaying are being actively explored. Most of the state-of-the-art studies optimize a system-wide objective function subject to the transmit power constraints of the two source nodes and the relay. Transceiver designs with quality-of-service (QoS) constraints have lacked attention in two way relaying literature. In this work, we study a MIMO transceiver design, based on the generalized singular value decomposition, that incorporates per-stream QoS constraints specified as the rate required by the individual transmit streams of two source nodes. With these QoS constraints, we use geometric programming to jointly allocate power at the source and relay nodes to optimize the sum-rate. Through extensive numerical evaluations, we show that the proposed design outperforms the existing solutions not only with the QoS constraints but also without them.**

***Index Terms*—Geometric program (GP), generalized singular value decomposition (GSVD), quality-of-service (QoS).**

I. Introduction

Relaying is a well known cooperative communication technique that is used to enhance the throughput and coverage of wireless systems [1]–[3]. Two relaying protocols are widely studied in the literature – one-way [1] and two-way [2], [3]. A relay is usually assumed to be half-duplex as it is easy to implement; a half-duplex relay, however cannot simultaneously transmit and receive on the same spectral resource. This reduces the spectral efficiency of one-way relaying, as two spectral resources are now required to transmit a single data unit. This is twice the number of channel uses when two nodes communicate directly without a relay. Two-way relaying (TWR) requires two spectral resources to transmit two data units and recovers the loss in spectral efficiency.

The relay in TWR protocol is designed commonly using two different technologies – regenerative decode-and-forward [2] and non-regenerative amplify-and-forward [3]–[5]. A non-regenerative two-way relay is of special interest as it is easy to implement and has lower delay, when compared with a regenerative two-way relay. To further enhance the spectral-efficiency of non-regenerative TWR, multiple antennas are also employed in all the nodes. Many studies have investigated transceiver designs for non-regenerative TWR to explore the potential of multiple antennas [3]–[13].

The authors in [3], [4] investigated space-time coding designs for multiple-input multiple-output (MIMO) TWR. Lee *et al.* in [5] designed source and relay precoders to maximize sum-rate. In [6], the source and relay precoders are jointly constructed to minimize sum-MSE. Reference [7] developed a unified framework to optimize a broad class of Schur-convex and Schur-concave functions for MIMO TWR. Transceiver designs to minimize the MSE and maximize the sum-rate using MSE duality are recently investigated in [8] and [9], respectively. Park *et al.* in [10] proposed a transceiver based on the principles of signal alignment and uses generalized singular value decomposition (GSVD). The channel orthogonalization involves zero-forcing operation at the relay, which degrades the sum-rate for ill-conditioned channels due to noise enhancement. The design in [11] avoids zero-forcing operation at the relay but only semi-orthogonalizes the end-to-end MIMO channels for small number of antennas. This design is shown to have better sum-rate than the [10], if the residual inter-stream interference due to semi-orthogonalization of MIMO channels is ignored.

All the aforementioned references optimize a given objective function e.g., sum-rate or MSE, subject to the constraints on the transmit power of two source nodes and the relay. They, however, ignore quality-of-service (QoS) constraints which affect the user experience and are therefore crucial in practical systems. A few recent efforts have considered QoS-constrained transceiver designs for TWR [12], [13]. Leow *et al.* in [12] designed source and relay precoders to minimize the total network transmit power subject to per-stream QoS constraints expressed as the transmit rate required by the individual streams of the two source nodes. Reference [13] also jointly optimized source and relay precoders to minimize total network transmit power subject to the QoS constraints which are expressed as upper-bounds on the mean-squared error (MSE) of the signal waveform estimated at the destination nodes.

In the present work, motivated by [10], [11], we first construct a MIMO transceiver using GSVD for TWR. We then maximize sum-rate subject to the per-stream QoS constraints that similar to [12], are expressed as the transmit rates required by the individual streams of two source nodes. Optimization of these objectives, for the GSVD-based transceiver design considered herein, reduces to joint power allocation at the source and relay nodes.

We next list the **main contributions** of this paper when compared with [10]–[13].

1) We propose a novel modification to the existing GSVD-based transceiver designs in [10], [11] that overcomes two limitations: i) noise enhancement in ill-conditioned channels in [10]; and ii) residual inter-stream interference for small number of antennas in [11]. Though in this work we concentrate on

optimizing objectives with per-stream rate constraints, we also show that the proposed design yields better sum-rate than the designs in [10], [11] without the per-stream rate constraints.

2) We maximize the sum-rate, via joint power allocation at the source and relay nodes, subject to the per-stream transmit rate required by both source nodes and individual transmit power. We will show that this problem can be cast as a convex geometric program (GP). We note that both [12], [13] do not investigate this problem; it is important to analyze the effect of the per-stream transmit rate demanded by each source node on the overall sum-rate. Further, we consider more granular per-stream rate constraints unlike [13] that expresses QoS constraints in terms of upper-bounded MSE for each link (not transmit stream). *To the best of our knowledge, sum-rate maximization with per-stream rate constraints for TWR has not been considered earlier in the literature.* We show through extensive simulations that the proposed design yields significantly better performance than the state-of-the-art transceiver designs.

Notation: The symbols $\mathbf{A}$ and $\mathbf{a}$ denote a matrix and a column vector, respectively. $\text{Tr}(\mathbf{A})$, $\mathbf{A}^T$, $\mathbf{A}^H$, $\mathbf{A}^\dagger$, $[\mathbf{A}]_{i,j}$ and $\mathbf{A}^*$ are the trace, transpose, Hermitian, pseudoinverse, (i,j)th element and complex-conjugate of the elements of $\mathbf{A}$, respectively. We denote the Hadamard product of two matrices $\mathbf{A}$ and $\mathbf{B}$ as $\mathbf{A} \odot \mathbf{B}$ and an $N \times N$ identity matrix as $\mathbf{I}_N$. The notation $\text{diag}(x_1, \cdots, x_n)$ represents a diagonal matrix with x_1 to x_n as its diagonal elements. The notation $\boldsymbol{x} \succeq 0$ implies that all the elements of vector $\mathbf{x}$ are ≥ 0. The symbol $\mathbb{E}(\cdot)$ is the expectation operator.

II. System model

We consider MIMO TWR where two multi-antenna source nodes S_1 and S_2, as shown in Fig. 1, exchange data via a half-duplex non-regenerative multi-antenna relay. We assume that both source nodes have N_s antennas each, while the relay has N_r antennas. We also assume, as conventionally assumed in TWR literature [5], [6], that the direct links between two source nodes are absent due to high path loss and fading.

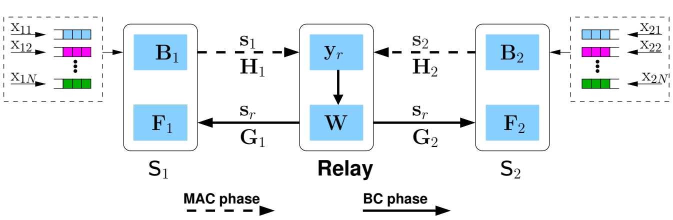


Fig. 1: QoS-constrained two-way relaying: Both source nodes have N streams to transmit, and each stream requires a particular rate.

In the first phase of TWR, commonly known as multiple access (MAC) phase, both source nodes simultaneously transmit their respective signals to the relay; the relay receives an $N_r \times 1$ signal given as

$$\mathbf{y}_r = \mathbf{H}_1\mathbf{s}_1 + \mathbf{H}_2\mathbf{s}_2 + \mathbf{n}_r. \tag{1}$$

Here the matrix $\mathbf{H}_i \in \mathbb{C}^{N_r \times N_s}$ is the MIMO channel for the $\mathsf{S}_i \rightarrow$Relay links for $i = 1, 2$,[1] and the vector $\mathbf{n}_r \in \mathbb{C}^{N_r \times 1}$ is the circularly-symmetric complex additive white Gaussian noise at the relay with $\mathbb{E}(\mathbf{n}_r\mathbf{n}_r^H) = \sigma_r^2\mathbf{I}_{N_r}$. The vector $\mathbf{s}_i \in \mathbb{C}^{N_s \times 1}$, transmit signal of the node S_i, is obtained by linearly precoding the normalized complex information data vector $\mathbf{x}_i \in \mathbb{C}^{N \times 1}$ using a precoder matrix $\mathbf{B}_i \in \mathbb{C}^{N_s \times N}$ such that $\mathbf{s}_i = \mathbf{B}_i\mathbf{x}_i$. We assume that with N_s antennas, both source nodes transmit $N = \min(N_s, N_r)$ independent source streams; therefore $\mathbb{E}(\mathbf{x}_i\mathbf{x}_i^H) = \mathbf{I}_N$. The precoded data vector $\mathbf{s}_i$ satisfies the transmit power constraint:

$$\text{Tr}(\mathbb{E}(\mathbf{s}_i\mathbf{s}_i^H)) = \text{Tr}(\mathbf{B}_i\mathbf{B}_i^H) = \text{Tr}(\boldsymbol{\Xi}_i) \leq P_i, \tag{2}$$

where P_i is the maximum transmit power of the node S_i.

In the second phase of TWR, commonly known as broadcast (BC) phase, the relay multiplies its received signal with a precoder matrix $\mathbf{W} \in \mathbb{C}^{N_r \times N_r}$. The $N_r \times 1$ signal vector transmitted by the relay is $\mathbf{s}_r = \mathbf{W}\mathbf{y}_r$. The maximum transmit power of the relay is P_r, which results in the following constraint on its transmit signal

$$\begin{aligned} P_r &\geq \text{Tr}\left(\mathbb{E}(\mathbf{s}_r\mathbf{s}_r^H)\right) \\ &= \text{Tr}\left(\mathbf{W}(\mathbf{H}_1\boldsymbol{\Xi}_1\mathbf{H}_1^H + \mathbf{H}_2\boldsymbol{\Xi}_2\mathbf{H}_2^H + \sigma_r^2\mathbf{I}_N)\mathbf{W}^H\right). \end{aligned} \tag{3}$$

The signal received by the node S_i in the BC phase is

$$\mathbf{y}_i = \mathbf{G}_i\mathbf{s}_r + \mathbf{n}_i = \underbrace{\mathbf{G}_i\mathbf{W}\mathbf{H}_i\mathbf{s}_i}_{\text{BI}} + \mathbf{G}_i\mathbf{W}\mathbf{H}_{\bar{i}}\mathbf{s}_{\bar{i}} + \mathbf{G}_i\mathbf{W}\mathbf{n}_r + \mathbf{n}_i.$$

The term labelled BI is the back-propagating interference. The matrix $\mathbf{G}_i \in \mathbb{C}^{N_s \times N}$ is the MIMO channel for the Relay$\rightarrow\mathsf{S}_i$ link, and the vector $\mathbf{n}_i$ is the circularly-symmetric complex additive noise at the node S_i with $\mathbb{E}(\mathbf{n}_i\mathbf{n}_i^H) = \sigma_i^2\mathbf{I}_{N_s}$. We assume that all the channel matrices are frequency-flat and constant over one cycle of MAC and BC phases, but vary independently across multiple cycles as in [5]–[7]. Since S_i knows $\mathbf{s}_i$, it can cancel the back-propagating interference from its received signal with the knowledge of necessary channel state information (CSI) as in [5]–[7]. The equivalent receive signal at S_i after cancelling the back-propagating interference from $\mathbf{y}_i$ is $\widehat{\mathbf{y}}_i = \mathbf{G}_i\mathbf{W}\mathbf{H}_{\bar{i}}\mathbf{B}_{\bar{i}}\mathbf{x}_{\bar{i}} + \mathbf{G}_i\mathbf{W}\mathbf{n}_r + \mathbf{n}_i$. The node S_i recovers its desired data by processing its received signal using a decoder matrix $\mathbf{F}_i \in \mathbb{C}^{N \times N_s}$:

$$\widetilde{\mathbf{y}}_i = \mathbf{F}_i\widehat{\mathbf{y}}_i = \mathbf{F}_i\mathbf{G}_i\mathbf{W}\mathbf{H}_{\bar{i}}\mathbf{B}_{\bar{i}}\mathbf{x}_{\bar{i}} + \mathbf{F}_i\mathbf{G}_i\mathbf{W}\mathbf{n}_r + \mathbf{F}_i\mathbf{n}_i. \tag{4}$$

III. Transceiver design

We begin this section by outlining the principles of GSVD-based transceiver design. We then briefly describe two existing GSVD TWR designs in [10], [11]. We conclude this section by proposing a novel modification that will significantly improve the sum-rate of the GSVD designs in [10], [11]. For the proposed design, we assume that the relay perfectly knows $\mathbf{H}_i$ and $\mathbf{G}_i$ and that it designs the precoders/decoders and distributes them to other nodes. The perfect channel knowledge at the relay and design/distribution of precoders/decoders by it is commonly assumed in the literature on joint design [6], [7].

[1] To avoid repetition, we assume that $i = 1, 2$ throughput this paper. Further, $\bar{i} = 2$ for $i = 1$ and $\bar{i} = 1$ for $i = 2$.

A. Principles of GSVD-based transceiver design

To design a transceiver based on the GSVD, the MAC-phase MIMO channels $\mathbf{H}_1$ and $\mathbf{H}_2$ are jointly decomposed using GSVD [14]:

$$\mathbf{H}_1 = \mathbf{T}_h^H \mathbf{\Sigma}_{h_1} \mathbf{U}_{h_1} \text{ and } \mathbf{H}_2 = \mathbf{T}_h^H \mathbf{\Sigma}_{h_2} \mathbf{U}_{h_2}. \tag{5}$$

Here $\mathbf{U}_{h_1} \in \mathbb{C}^{N \times N_s}$ and $\mathbf{U}_{h_2} \in \mathbb{C}^{N \times N_s}$ are matrices with N orthonormal rows, $\mathbf{\Sigma}_{h_1} \in \mathbb{R}^{P \times N} = [\mathbf{0}_{h_1}^T \; \bar{\mathbf{\Sigma}}_{h_1}^T]^T$ and $\mathbf{\Sigma}_{h_2} \in \mathbb{R}^{P \times N} = [\bar{\mathbf{\Sigma}}_{h_2}^T \; \mathbf{0}_{h_2}^T]^T$. The matrix $\bar{\mathbf{\Sigma}}_{h_i} \in \mathbb{R}_+^{N \times N} = \text{diag}(\sigma_{h_i,1}, \cdots, \sigma_{h_i,N})$, $\mathbf{0}_{h_i} = \mathbf{0}_{N \times (P-N)}$, and $\mathbf{T}_h \in \mathbb{C}^{P \times N_r}$ is a full-rank matrix, where $P = \text{rank}([\mathbf{H}_1 \; \mathbf{H}_2]) = \min(N_r, 2N_s)$. Next, the BC-phase MIMO channels $\mathbf{G}_1$ and $\mathbf{G}_2$ are jointly decomposed using GSVD [14]

$$\mathbf{G}_1 = \mathbf{U}_{g_1} \mathbf{\Sigma}_{g_1} \mathbf{T}_g^H \text{ and } \mathbf{G}_2 = \mathbf{U}_{g_2} \mathbf{\Sigma}_{g_2} \mathbf{T}_g^H. \tag{6}$$

Here $\mathbf{U}_{g_1} \in \mathbb{C}^{N_s \times N}$ and $\mathbf{U}_{g_2} \in \mathbb{C}^{N_s \times N}$ are matrices with N orthonormal rows, $\mathbf{\Sigma}_{g_1} \in \mathbb{R}^{N \times P} = [\mathbf{0}_{g_1} \; \bar{\mathbf{\Sigma}}_{g_1}]$ and $\mathbf{\Sigma}_{g_2} \in \mathbb{R}^{N \times P} = [\bar{\mathbf{\Sigma}}_{g_2} \; \mathbf{0}_{g_2}]$. The matrix $\bar{\mathbf{\Sigma}}_{g_i} \in \mathbb{R}_+^{N \times N}$=diag$(\sigma_{g_i,N}, \cdots, \sigma_{g_i,1})$, $\mathbf{0}_{g_i} = \mathbf{0}_{N \times (P-N)}$ and $\mathbf{T}_g \in \mathbb{C}^{N_r \times P}$ is a full-rank matrix, where $P = \text{rank}([\mathbf{G}_1^T \; \mathbf{G}_2^T]^T) = \min(N_r, 2N_s)$.

Following the decomposition, the MAC-phase transmit precoder of the node S_i is chosen as [10]

$$\mathbf{B}_i = \mathbf{U}_{h_i}^H \mathbf{\Lambda}_i. \tag{7}$$

Here $\mathbf{\Lambda}_i \in \mathbb{R}_+^{N \times N}$ is a diagonal power allocation matrix at S_i such that $\mathbf{\Lambda}_i \mathbf{\Lambda}_i^H = \text{diag}(\lambda_{i,1}, \cdots \lambda_{i,N})$. Similarly, the BC-phase decoder matrix for the node S_i is chosen as [10]:

$$\mathbf{F}_i = \mathbf{U}_{g_i}^H. \tag{8}$$

With the precoder/decoder in (7) and (8), the received signals at S_1 and S_2 in (4) can be expressed as

$$\begin{aligned}\widetilde{\mathbf{y}}_1 &= \mathbf{\Sigma}_{g_1} \mathbf{T}_g^H \mathbf{W} \mathbf{T}_h^H \mathbf{\Sigma}_{h_2} \mathbf{\Lambda}_2 \mathbf{x}_2 + \mathbf{\Sigma}_{g_1} \mathbf{T}_g^H \mathbf{W} \mathbf{n}_r + \mathbf{U}_{g_1}^H \mathbf{n}_1, \\ \widetilde{\mathbf{y}}_2 &= \mathbf{\Sigma}_{g_2} \mathbf{T}_g^H \mathbf{W} \mathbf{T}_h^H \mathbf{\Sigma}_{h_1} \mathbf{\Lambda}_1 \mathbf{x}_1 + \mathbf{\Sigma}_{g_2} \mathbf{T}_g^H \mathbf{W} \mathbf{n}_r + \mathbf{U}_{g_2}^H \mathbf{n}_2.\end{aligned} \tag{9}$$

To design the relay precoder $\mathbf{W}$, the earlier work in [10], [11] have considered two different approaches. In [10], the relay precoder orthogonalizes the end-to-end MIMO channels. To this end, the relay precoder is decomposed into a MAC precoder $\mathbf{W}_c \in \mathbb{C}^{P \times N}$, BC precoder $\mathbf{W}_b \in \mathbb{C}^{N \times P}$ and a power-allocation and permutation matrix $\mathbf{\Delta} \in \mathbb{R}_+^{P \times P}$ such that $\mathbf{W} = \mathbf{W}_b \mathbf{\Delta} \mathbf{W}_c$, where $\mathbf{\Delta}$ is an anti-diagonal matrix such that $\mathbf{\Delta}\mathbf{\Delta}^H = \text{diag}(\delta_1, \cdots, \delta_P)$. To orthogonalize the MIMO channels, precoders $\mathbf{W}_b$ and $\mathbf{W}_b$ are chosen as $(\mathbf{T}_g^H)^\dagger$ and $(\mathbf{T}_h^H)^\dagger$, respectively. The channel inversion, however, severely degrades the performance by enhancing the noise when the channels are ill-conditioned.

The design in [11] avoids channel-inversion by constructing $\mathbf{W}_b$ and $\mathbf{W}_c$ such that they have orthonormal columns and rows, respectively. This design, however, does not orthogonalizes the MIMO channels for *small* number of antennas, which leads to residual inter-stream interference between multiple transmit streams. This design ignores the residual inter-stream interference, and is shown to have better sum-rate than [10]. We will show later in Section V that the residual inter-stream interference, *if not ignored*, can significantly degrade the performance.

The objective of the relay precoder design herein, as discussed earlier, is to maximize sum-rate with per-stream transmit rate constraints. To achieve this, we do not diagonalize the end-to-end MIMO channels; we instead design the relay precoder to triangularize the MIMO channels. *This novel modification, as shown later in Section IV, is crucial in casting the optimization as a convex GP.*

The inter-stream interference due to triangular MIMO channels can be easily cancelled at the destination nodes by employing successive interference cancellation and need not be ignored as in [11]. Further, the MIMO channel triangularization is achieved by designing $\mathbf{W}_b$ and $\mathbf{W}_c$ with orthonormal columns and rows respectively. The proposed precoder, consequently, does not enhance the noise unlike [10]. Due to these two reasons, the proposed approach leads to better performance than in [10], [11]. We next discuss the proposed relay precoder design.

B. Proposed channel triangularization relay precoder

We first design the MAC-phase relay precoder $\mathbf{W}_c$ by factorizing the full-rank matrix $\mathbf{T}_h^H$ into a unitary matrix $\mathbf{Q} \in \mathbb{C}^{N_r \times N_r}$ and an upper-triangular matrix $\mathbf{R} \in \mathbb{C}^{N_r \times P}$ using QR factorization [14]

$$\mathbf{T}_h^H = \mathbf{QR} = \begin{bmatrix} \mathbf{Q}_h & \mathbf{Q}_0 \end{bmatrix} \begin{bmatrix} \mathbf{R}_h \\ \mathbf{0} \end{bmatrix}. \tag{10}$$

The bottom $(N_r - P)$ rows of matrix $\mathbf{R}$ are zeros. The matrix $\mathbf{Q}$ is therefore partitioned into $\mathbf{Q}_h \in \mathbb{C}^{N_r \times P}$ and $\mathbf{Q}_0 \in \mathbb{C}^{N_r \times (N_r - P)}$. Also, $\mathbf{R}_h \in \mathbb{C}^{P \times P}$ is an upper-triangular matrix. To reduce $\mathbf{T}_h^H$ to an upper-triangular matrix, we choose $\mathbf{W}_c = \mathbf{Q}_h^H$. Note that $\mathbf{W}_c$ is not unitary; only its rows are orthonormal. We now design the BC-phase relay precoder $\mathbf{W}_b$ by factorizing the full-rank matrix $\mathbf{T}_g^H$ into a lower-triangular matrix $\mathbf{L} \in \mathbb{C}^{P \times N_r}$ and a unitary matrix $\mathbf{V} \in \mathbb{C}^{N_r \times N_r}$ using LQ factorization [14]

$$\mathbf{T}_g^H = \mathbf{LV} = \begin{bmatrix} \mathbf{L}_g & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{V}_g \\ \mathbf{V}_0 \end{bmatrix}. \tag{11}$$

The last $(N_r - P)$ columns of matrix $\mathbf{L}$ are zeros. The matrix $\mathbf{V}$ is therefore partitioned into $\mathbf{V}_g \in \mathbb{C}^{P \times N_r}$ and $\mathbf{V}_0 \in \mathbb{C}^{(N_r - P) \times N_r}$. The matrix $\mathbf{L}_g \in \mathbb{C}^{P \times P}$ is a lower-triangular matrix. To reduce $\mathbf{T}_g$ to a lower-triangular matrix, we choose $\mathbf{W}_b = \mathbf{V}_g^H$. Note that $\mathbf{W}_b$ is not a unitary matrix; it only has orthonormal columns. The precoder $\mathbf{W}$ is given as

$$\mathbf{W} = \mathbf{V}_g^H \mathbf{\Delta} \mathbf{Q}_h^H. \tag{12}$$

With the above relay precoder, the received signal at S_1 and S_2 in (9) can be expressed as

$$\begin{aligned}\widetilde{\mathbf{y}}_1 &= \mathbf{\Sigma}_{g_1} \mathbf{L}_g \mathbf{\Delta} \mathbf{R}_h \mathbf{\Sigma}_{h_2} \mathbf{\Lambda}_2 \mathbf{x}_2 + \underbrace{\mathbf{\Sigma}_{g_1} \mathbf{L}_g \mathbf{\Delta} \mathbf{Q}_h^H \mathbf{n}_r + \mathbf{U}_{g_1}^H \mathbf{n}_1}_{\widetilde{\mathbf{n}}_1}, \\ \widetilde{\mathbf{y}}_2 &= \mathbf{\Sigma}_{g_2} \mathbf{L}_g \mathbf{\Delta} \mathbf{R}_h \mathbf{\Sigma}_{h_1} \mathbf{\Lambda}_1 \mathbf{x}_1 + \underbrace{\mathbf{\Sigma}_{g_2} \mathbf{L}_g \mathbf{\Delta} \mathbf{Q}_h^H \mathbf{n}_r + \mathbf{U}_{g_2}^H \mathbf{n}_2}_{\widetilde{\mathbf{n}}_2}.\end{aligned} \tag{13}$$

The lower-triangular $\mathbf{L}_g$, anti-diagonal $\boldsymbol{\Delta}$ and upper-triangular $\mathbf{R}_h$ result in the reflected lower-triangular structure of the end-to-end channel between $\widetilde{\mathbf{y}}_i$ and $\widetilde{\mathbf{x}}_{\bar{i}}$ as

$$\begin{bmatrix} \widetilde{y}_{i,1} \\ \widetilde{y}_{i,2} \\ \vdots \\ \widetilde{y}_{i,N-1} \\ \widetilde{y}_{i,N} \end{bmatrix} = \begin{bmatrix} 0 & 0 & \cdots & 0 & \times \\ 0 & & & \times & \times \\ \vdots & & \iddots & \vdots & \vdots \\ 0 & \times & \cdots & \times & \times \\ \times & \times & \cdots & \times & \times \end{bmatrix} \begin{bmatrix} x_{\bar{i},1} \\ x_{\bar{i},2} \\ \vdots \\ x_{\bar{i},N-1} \\ x_{\bar{i},N} \end{bmatrix} + \widetilde{\mathbf{n}}_i. \tag{14}$$

With this channel structure, the Nth transmit stream $(x_{\bar{i},N})$ does not experience inter-stream interference and is detected first. The $(N-n)$th transmit stream is then detected by successively subtracting the interference from *already-detected* streams numbered from $(N-n+1)$ to N, $n=1$ to $N-1$. After detection, the signal-to-noise ratio (SNR) of the nth receive stream of the source nodes S_1 and S_2 respectively are

$$\begin{aligned} \mathrm{SNR}_{1,n}(\boldsymbol{\delta},\boldsymbol{\lambda}_2) &= \frac{\delta_{\widetilde{n}}\lambda_{2,\widetilde{j}}(\sigma^2_{g_1,n}\sigma^2_{h_2,\widetilde{j}}|[\mathbf{L}_g]_{\widetilde{n},\widetilde{n}}[\mathbf{R}_h]_{\widetilde{j},\widetilde{j}}|^2)}{\sigma_r^2\sum_{l=1}^{P}\delta_l[\mathbf{L}_{g_1}\odot\mathbf{L}^*_{g_1}]_{n,l}+\sigma_1^2}, \\ \mathrm{SNR}_{2,n}(\boldsymbol{\delta},\boldsymbol{\lambda}_1) &= \frac{\delta_n\lambda_{1,\widetilde{j}}(\sigma^2_{g_2,n}\sigma^2_{h_1,\widetilde{j}}|[\mathbf{L}_g]_{n,n}[\mathbf{R}_h]_{j,j}|^2)}{\sigma_r^2\sum_{l=1}^{P}\delta_l[\mathbf{L}_{g_2}\odot\mathbf{L}^*_{\bar{g}_2}]_{n,l}+\sigma_2^2}. \end{aligned} \tag{15}$$

Here $n=1$ to N, $\widetilde{n}=P-N+n$, $j=P-n+1$ and $\widetilde{j}=N-n+1$. Further $\boldsymbol{\delta}=[\delta_1,\cdots,\delta_P]$ and $\boldsymbol{\lambda}_i=[\lambda_{i,1},\cdots,\lambda_{i,N}]$. In the denominator of the above SNR expressions, we denote $\mathbf{L}_{g_i}=\boldsymbol{\Sigma}_{g_i}\mathbf{L}_g$. The above SNR expressions can be derived from (13) after simple algebraic manipulations and by noting that $\mathbf{Q}_h^H\mathbf{n}_r$, $\mathbf{U}_{g_1}^H\mathbf{n}_1$ and $\mathbf{U}_{g_2}^H\mathbf{n}_2$ have the same statistical properties as $\mathbf{n}_r$, $\mathbf{n}_1$ and $\mathbf{n}_2$, respectively. *Note that the coefficients of the power-distribution variables, δ_l for $l=1$ to P, λ_{1n} and λ_{2n} for $n=1$ to N, are non-negative. We will use this fact to prove the convexity of the optimization problems*. The complete procedure for the proposed GSVD triangularization (GSVD-TRI) design is summarized in Algorithm 1.

Algorithm 1 GSVD-TRI design.

1: *GSVD design*: Design source precoders $\mathbf{B}_i$ and decoders $\mathbf{F}_i$, as in (7) and (8), to partially diagonalize $\mathsf{S}_i\rightarrow$relay$\rightarrow\mathsf{S}_{\bar{i}}$ MIMO channels.
2: *Channel triangularization*: Design relay precoder $\mathbf{W}$, as in (12), to triangularize the end-to-end MIMO channels using LQ and QR decompositions.
3: *Power allocation*: Jointly allocate power at the source nodes and relay to maximize sum-rate (see next section).

IV. Joint power allocation

We now jointly allocate power to optimize the sum-rate subject to the per-stream transmit rate required by both source node. We next prove two lemmas that will be useful in showing that the optimization is a GP; a GP has a posynomial objective and upper-bounded posynomial constraints [15, pg. 160].

Lemma 4.1: The transmit power of the node S_i, defined in (2), is a posynomial in optimization variable $\boldsymbol{\lambda}_i$, where $\boldsymbol{\lambda}_i=[\lambda_{i,1},\cdots,\lambda_{i,N}]$.

Proof: With the transmit precoder $\mathbf{B}_i$ designed in (7), we rewrite the transmit power of S_i as $\mathrm{Tr}(\mathbf{B}_i\mathbf{B}_i^H) = \mathrm{Tr}(\mathbf{U}_{h_i}^H\boldsymbol{\Lambda}_i\boldsymbol{\Lambda}_i^H\mathbf{U}_{h_i}) \overset{(a)}{=} \mathrm{Tr}(\mathbf{U}_{h_i}\mathbf{U}_{h_i}^H\boldsymbol{\Lambda}_i\boldsymbol{\Lambda}_i^H) = \sum_{j=1}^N\lambda_{i,j}$. In (a), we have used Tr($\mathbf{AB}$)=Tr($\mathbf{BA}$). Wee see that the constraint is a posynomial in $\boldsymbol{\lambda}_i$, as the coefficients of optimization variable $\lambda_{i,j},\forall j$ are positive [15, pg. 160]. ■

For notational convenience, the transmit power of the node S_i is henceforth denoted as $p_i(\boldsymbol{\lambda}_i)$.

Lemma 4.2: The transmit power of the relay, defined in (3), is a posynomial in $\boldsymbol{\lambda}_1$, $\boldsymbol{\lambda}_2$ and $\boldsymbol{\delta}$, where $\boldsymbol{\delta}=[\delta_1,\cdots,\delta_P]$.

Proof: This can also be proved on lines similar to Lemma 4.1. Please see [16] for details. The transmit power of the relay is now denoted as $p_r(\boldsymbol{\delta},\boldsymbol{\lambda}_1,\boldsymbol{\lambda}_2)$. ■

We next define the sum-rate as

$$R(\boldsymbol{\delta},\boldsymbol{\lambda}_1,\boldsymbol{\lambda}_2) = \frac{1}{2}\sum_{i=1}^{2}\sum_{n=1}^{N}\log_2(1+\mathrm{SNR}_{i,n}(\boldsymbol{\delta},\boldsymbol{\lambda}_{\bar{i}})) \tag{16}$$

$$\overset{(a)}{\approx} \frac{1}{2}\sum_{i=1}^{2}\sum_{n=1}^{N}\log_2(\mathrm{SNR}_{i,n}(\boldsymbol{\delta},\boldsymbol{\lambda}_{\bar{i}}) \tag{17}$$

$$= -\frac{1}{2}\log_2\prod_{n=1}^{N}\prod_{i=1}^{2}\mathrm{ISNR}_{i,n}(\boldsymbol{\delta},\boldsymbol{\lambda}_{\bar{i}}) \tag{18}$$

The factor of half is due to the half-duplex constraint. The term $\mathrm{ISNR}_{i,n}$ denotes the inverse of $\mathrm{SNR}_{i,n}$, that is $\mathrm{ISNR}_{i,n}$ $=1/\mathrm{SNR}_{i,n}$. Each $\mathrm{ISNR}_{i,n}$ term, due to positive coefficients of power distribution variables in $\mathrm{SNR}_{i,n}$ terms in (15), is a posynomial. The sum-rate in (16) can easily be shown as a ratio of two posynomials. As posynomials are not closed under division [15, pg. 161], the sum-rate is not a posynomial and its optimization therefore cannot be cast as a GP. We approximate the sum-rate as in (a) to cast the problem as a GP; the loss in the sum-rate due to this approximation is minor and becomes negligible in the high-SNR regime [17].

The QoS-constrained sum-rate optimization is now cast as

$$\begin{aligned} \underset{\{\boldsymbol{\lambda}_1,\boldsymbol{\lambda}_2,\boldsymbol{\delta}\}\succeq 0}{\text{Minimize}} \quad & \prod_{n=1}^{N}\prod_{i=1}^{2}\mathrm{ISNR}_{i,n}(\boldsymbol{\delta},\boldsymbol{\lambda}_{\bar{i}}) \\ \text{subject to} \quad & \mathrm{ISNR}_{1,n}(\boldsymbol{\delta},\boldsymbol{\lambda}_2)\le 1/(2^{r_{1n}}-1) \\ & \mathrm{ISNR}_{2,n}(\boldsymbol{\delta},\boldsymbol{\lambda}_1)\le 1/(2^{r_{2n}}-1) \\ & p_1(\boldsymbol{\lambda}_1)\le P_1, p_2(\boldsymbol{\lambda}_2)\le P_2 \\ & p_r(\boldsymbol{\delta},\boldsymbol{\lambda}_1,\boldsymbol{\lambda}_2)\le P_r \end{aligned} \tag{19}$$

Here first two constraints, which can be equivalently expressed as $\log_2(1+\mathrm{SNR}_{i,n}(\boldsymbol{\delta},\boldsymbol{\lambda}_{\bar{i}}))\ \ge\ r_{in}$, are on the per-stream transmit rate; here r_{in} is desired rate of nth stream of node i. The last three are transmit power constraints of different nodes. We have dropped $\frac{1}{2}\log_2$ factor from the objective as $\log(\cdot)$ is a monotonically increasing function. We see that the objective is a product of posynomials and is therefore a posynomial [15, pg. 161]. The problem is a GP as it has a posynomial objective and upper-bounded posynomial constraints.

V. Numerical results

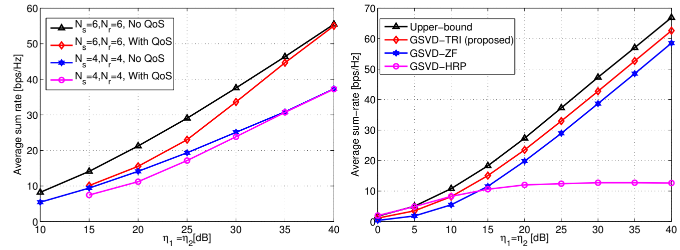
We now investigate the effect of the per-stream QoS constraints on the sum-rate of the proposed GSVD-TRI transceiver using Monte Carlo simulations. We assume Rayleigh flat-fading channels where the elements of $\mathbf{H}_i$ and $\mathbf{G}_i$ are independent and identically distributed complex Gaussian random variables with zero mean and variance h_i^2 and g_i^2 respectively. We fix the maximum transmit power and noise variance at all the nodes to unity and define the average SNR of the $\mathsf{S}_i \rightarrow$Relay and Relay$\rightarrow \mathsf{S}_i$ links as $\eta_i = h_i^2 = g_i^2$. We assume that both source nodes employ Gaussian signalling. The simulation results are averaged over 5000 statistically independent channel realizations.

In Fig. 2a, we plot the sum-rate of the proposed GSVD-TRI transceiver, with and without the per-stream QoS constraints for two different antenna configurations. We first focus on the scenario when each node has $N_s = N_r = 6$ antennas. The per-stream rate required by each source nodes is assumed to be 0.75 bps/Hz that is, $r_{1n} = r_{2n} = 0.75$ bps/Hz, $\forall n$. We make two key observations: 1) the per-stream rate requirement cannot be met below $\eta_1 = \eta_2 = 15$ dB; the per-stream rate requirement for $\eta_1 = \eta_2 \leq 15$ dB should therefore be reduced from the current one; and 2) the per-stream rate requirement leads to degradation in the sum-rate performance when compared with the scenario when there is no such rate requirement. This is because the system now sacrifices the overall sum-rate to satisfy the per-stream transmit rate of the individual streams. Moreover the degradation reduces at high SNR values where the probability of automatically meeting the per-stream rate requirement while maximizing the sum-rate is higher than at low SNR. For $N_s = N_r = 4$ antennas, where again $r_{1n} = r_{2n} = 0.75$ bps/Hz, $\forall n$, we see that the per-stream rate-requirement cannot be met below $\eta_1 = \eta_2 = 15$ dB. Further, similar to $N_s = N_r = 6$ antennas, the degradation due to the rate constraints reduces at high SNR values.

We now demonstrate the improved sum-rate of the proposed design over the other two GSVD-based designs: i) GSVD-ZF [Eq. (12), Eq. (13)] [10]; and ii) GSVD-HRP [Eq. (13), Eq. (15)] [11]. The improved sum-rate is because, as discussed earlier, the current transceiver overcomes the limitations of these two designs. Since both GSVD-ZF and GSVD-HRP designs do not incorporate QoS constraints, for the sake of fairness, we also maximize the sum-rate of the proposed design after dropping the QoS constraints. We also do not ignore the residual inter-stream interference of GSVD-HRP design. We see that the proposed GSVD-TRI design comprehensively outperforms both other designs.

VI. Conclusion

We investigated a GSVD-based joint transceiver design for MIMO TWR that triangularizes the end-to-end MIMO channels of the two source nodes. With MIMO channel triangularization, optimization of per-stream rate-constrained sum-rate reduce to joint optimal power allocation at the source and relay nodes, which is then cast as a convex geometric program. We analysed the impact of the per-stream rate constraints on the overall sum-rate. We also showed that the proposed design outperforms other state-of-the-art designs without rate constraints also.



(a) (b)

Fig. 2: a) Average sum-rate of the proposed GSVD-TRI transceiver design for two different antenna configurations; and b) Average sum-rate comparison of the GSVD-TRI design with the existing GSVD-ZF and GSVD-HRP designs; here $N_s = 6$ and $N_r = 10$.

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