Contents

Chapter 2

Economic dispatch of thermal units

2.1 Introduction

The complexity of interconnections and the size of the areas of electric power systems that are controlled in a coordinated way is rapidly increasing. This entails optimal allocation of the outputs of a large number of participating generators. Whether a generator should participate in sharing the load at a given interval of time is a problem of unit commitment. Once the unit commitment problem has been solved, it becomes a problem of optimal allocation of the available generations to meet the forecasted load demand for the current interval. At a modern-day energy management center, highly developed optimization techniques are used to determine not only the optimal outputs of the participating generators, but also the optimal settings of various control devices such as the tap settings of load tap changers (LTCs), outputs of VAR compensating devices, desired settings of phase shifters etc.

The desired objective for such optimization problems can be many, such as the minimization of the cost of generation, minimization of the total power loss in the system, minimization of the voltage deviations, and maximization of the reliability of the power supplied to the customers. One or more of these objectives can be considered while formulating the optimization strategy. Determination of the real power outputs of the generators so that the total cost of generation in the system is minimized is traditionally known as the problem of economic load dispatch (ELD). Majority of generating systems are of three types: nuclear, hydro, and thermal (using fossil fuels such as coal, oil and gas). Nuclear plants tend to be operated at constant output power levels. Operating cost of hydro units do not change much with the output. The operating cost of thermal plants, however, change significantly with the output power level. In this chapter, we will discuss the problem of ELD for power systems consisting of thermal units only as generators.

2.2 Economic dispatch problem (neglecting transmission losses)

First we formulate the ELD problem neglecting transmission losses. This is justified when a group of generators are connected to a particular bus-bar, as in the case of individual generating units in a power plant, or when they are physically located very close to each other. This ensures that the transmission losses can be neglected due to the short distance involved. One such system configuration is shown in Figure 2.1, where N thermal units are connected to a single bus-bar that is supplying a load P_{load}. Input to each unit is expressed in terms of cost rate (say \$/h). The total cost rate is the sum of cost rates of individual units. The essential operating constraint is that the sum of the power outputs must be equal to the load (note that we are neglecting power losses here).

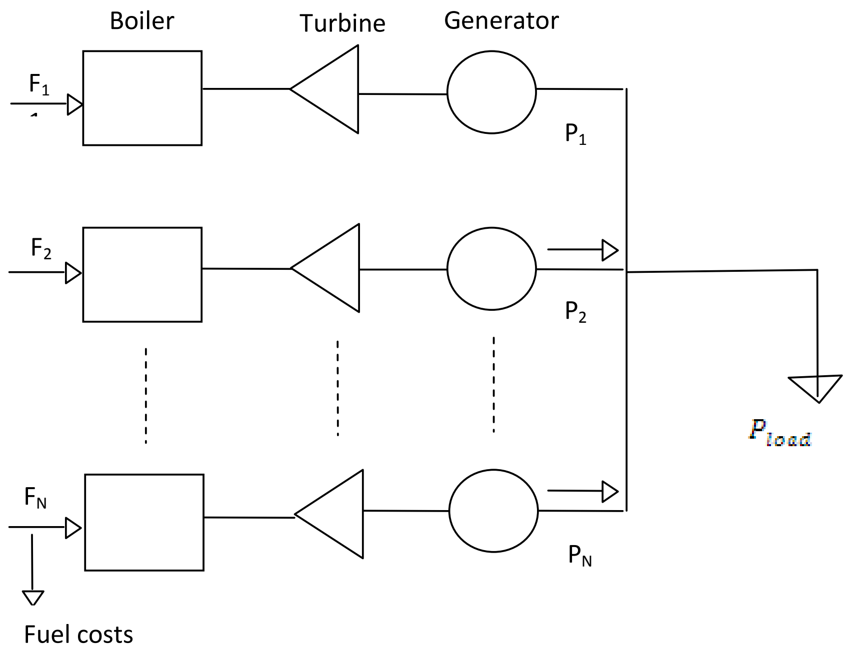

Figure 2.1: N thermal units connected to a bus to serve a load P_{load}

2.2.1 Fuel cost characteristics

The economic dispatch problem is the determination of generation levels such that the total cost of generation becomes minimum for a defined level of load. Now, for thermal generating units, the cost of fuel per unit power output varies significantly with the power output of the unit. Therefore one needs to consider the fuel cost characteristics of the generators while finding their optimal real power outputs. A typical fuel cost characteristics is shown below Figure 2.2.

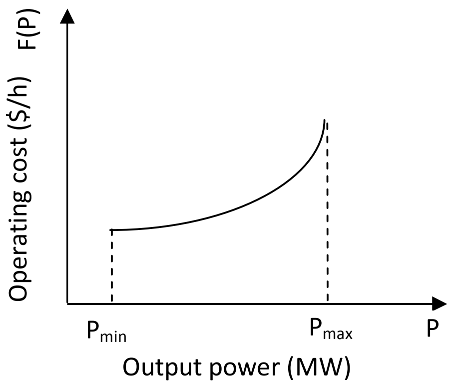

Figure 2.2: Typical fuel cost characteristics

Generally, the cost of labor, supply and maintenance are fixed. P_{min} is the output level below which it is uneconomical or technically infeasible to operate the units. P_{max} is the maximum output power limit. For formulating the dispatch problem, fuel costs are usually represented as a quadratic function of output power, as shown below.

$$F(P) = aP^2 + bP + c \tag{2.1}$$

2.2.2 Problem Formulation

Total fuel cost of operating N generators is given by,

$$\begin{aligned} F_T &= F_1(P_1) + F_2(P_2) + ... + F_N(P_N) \\ &= \sum_{i=1}^{N} F_i(P_i) \end{aligned} \tag{2.2}$$

Neglecting transmission losses, total generation should meet the total load. Hence, the equality constraint is,

$$\sum_{i=1}^{N} P_i = P_{\text{load}} \tag{2.3}$$

Based on the maximum and minimum power limits of the generators, following inequality constraints can be imposed:

$$P_{i,\text{min}} \leq P_i \leq P_{i,\text{max}} \qquad \forall i = 1, 2, ..., N. \tag{2.4}$$

This is a constrained optimization problem that can be solved by Lagrange multiplier method. The Lagrange method is formulated as,

$$\mathcal{L} = F_T + \lambda\phi \tag{2.5}$$

where $\phi = P_{\text{load}} - \sum_{i=1}^{N} P_i$ accounts for the equality constraint (2.3); λ is the Lagrange Multiplier. The necessary condition for F_T to be minimum is that the derivative of Lagrange function with respect to each independent variable is zero. Hence the necessary conditions for the optimization problem are:

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial P_i} &= \frac{\partial}{\partial P_i}\left\{\sum_{i=1}^{N} F_i(P_i) + \lambda(P_{\text{load}} - \sum_{i=1}^{N} P_i)\right\} \\ &= \frac{\partial F_i}{\partial P_i} - \lambda = 0; \forall i = 1, 2, ..., N,\end{aligned} \tag{2.6}$$

and

$$\frac{\partial \mathcal{L}}{\partial \lambda} = \phi = 0. \tag{2.7}$$

Rewriting (2.6)

$$\frac{\partial F_i}{\partial P_i} = \lambda; \forall i = 1, 2, ..., N \tag{2.8}$$

Equation (2.8) states that, to minimize the fuel cost, the necessary condition is to have all the incremental fuel costs same. Equation (2.8), along with (2.3) and (2.4) are called the coordination equations for economic load dispatch without considering network losses.

Note:
Using (2.1), fuel cost characteristics of all the generators are expressed as,

$$F_i = a_i P_i^2 + b_i P_i + c_i; \forall i = 1, 2, ..., N. \tag{2.9}$$

Using (2.8), the necessary conditions for the optimal solutions are given by,

$$\frac{\partial F_i}{\partial P_i} = 2a_i P_i + b_i = \lambda; \forall i = 1, 2, ..., N.. \tag{2.10}$$

Or,

$$P_i = \frac{\lambda - b_i}{2a_i}; \forall i = 1, 2, ..., N. \tag{2.11}$$

Substituting P_i from above in (2.3),

$$\sum_{i=1}^{N} \frac{\lambda - b_i}{2a_i} = P_{\text{load}}$$

Or,

$$\lambda = \left[\frac{P_{load} + \sum_{i=1}^{N} (b_i/2a_i)}{\sum_{i=1}^{N} (1/2a_i)} \right]. \tag{2.12}$$

Hence λ can be calculated by (2.10), and then P_i, i=1,2,...,N can be calculated by (2.9).

Example 2.1:
Two generating units of a power system are having the following cost curves:

$$F_1(P_1) = 0.05\ P_1^2 + 22\ P_1 + 120 \qquad \$/h$$
$$F_2(P_2) = 0.06\ P_2^2 + 16\ P_2 + 120 \qquad \$/h$$

(Note that the costs stated in the above equation are stated for demonstration purpose only. They are NOT indicative of actual costs in a real power system)

P_1,P_2 are in MW. Both the units operate all the time. Maximum and Minimum load on each unit are 100MW and 20 MW respectively. Determine the economic dispatch for the units for a total load of 80MW, neglecting the transmission lines losses.

Solution:
Using (2.10),

$$\lambda = \frac{80 + (\frac{22}{2*0.05}) + (\frac{16}{2*0.06})}{(\frac{1}{2*0.05}) + (\frac{1}{2*0.06})}$$
$$= (80 + 353.33)/18.33 = 23.64\ \$/MWh$$

Then, using (2.9),

$$P_1 = \frac{23.64 - 22}{2*0.05} = 16.36\ \ \text{MW}$$
$$P_2 = \frac{23.64 - 16}{2*0.06} = 63.64\ \ \text{MW}$$

Now, $P_{1,\min} = 20$ MW. Hence P_1 is fixed at 20MW and the rest of the demand is supplied by P_2. Therefore the economic dispatch is as followed:

$$P_1 = 20\ \text{MW}$$
$$P_2 = 80 - 20 = 60\ \text{MW}$$

2.2.3 Economic dispatch using gradient method

As discussed in Chapter 1, the basic principle of a gradient search method in that, the minimum of a function, $f(x)$ can be found by a series of steps going in the direction of maximum descent of $f(x)$. Hence the search should be directed towards $-\nabla f$.

For ELD problem, the objective is to minimize the total cost of generation, given by,

$$F_T = \sum_{i=1}^{N} F_i(P_i)$$

The equality constraint is,

$$\sum_{i=1}^{N} P_i = P_{\text{load}}$$

As seen in (2.5), the Lagrange function can be constructed as,

$$\mathcal{L} = \sum_{i=1}^{N} F_i(P_i) + \lambda(P_{\text{load}} - \sum_{i=1}^{N} P_i)$$

Now, the gradient of the Lagrange function can be expressed as,

$$\nabla\mathcal{L} = \begin{bmatrix} \frac{\partial \mathcal{L}}{\partial P_1} \\ \frac{\partial \mathcal{L}}{\partial P_2} \\ \vdots \\ \frac{\partial \mathcal{L}}{\partial P_N} \\ \frac{\partial \mathcal{L}}{\partial \lambda} \end{bmatrix} = \begin{bmatrix} \frac{\partial F_1}{\partial P_1} - \lambda \\ \frac{\partial F_2}{\partial P_2} - \lambda \\ \vdots \\ \frac{\partial F_N}{\partial P_N} - \lambda \\ P_{\text{load}} - \sum_{i=1}^{N} P_i \end{bmatrix} \tag{2.13}$$

The iterative equation then becomes the following:

$$\mathbf{x}^n = \mathbf{x}^{n-1} - \varepsilon \nabla \mathcal{L}, \tag{2.14}$$

where,

$$\mathbf{x}^n = \begin{bmatrix} P_1^{(n)} \\ P_2^{(n)} \\ \vdots \\ P_N^{(n)} \\ \lambda^{(n)} \end{bmatrix} \tag{2.15}$$

and

$$\mathbf{x}^{n-1} = \begin{bmatrix} P_1^{(n-1)} \\ P_2^{(n-1)} \\ \vdots \\ P_N^{(n-1)} \\ \lambda^{(n-1)} \end{bmatrix} \tag{2.16}$$

2.2.4 Algorithm for ELD using gradient method

Step 1: Select the starting values, $P_1^{(0)}$,$P_2^{(0)}$,...,$P_N^{(0)}$, where $\sum_{i=1}^{N} P_i^{(0)} = P_{\text{load}}$.

Step 2: Compute the initial $\lambda_i^{(0)}$ for each generator

$$\lambda_i^{(0)} = \frac{\partial F_i(P_i)}{\partial P}\bigg|_{P_i^{(0)}} ;\forall i = 1, 2, ..., N$$

Step 3: Compute initial average incremental cost

$$\lambda^{(0)} = (\tfrac{1}{N}) \sum_{i=1}^{N} \lambda_i^0$$

Step 4: Compute $\nabla\mathcal{L}$.

Step 5: If $|\nabla L| \leq \delta$, then go to step (8), otherwise go to Step 6. Here δ is a predefined small value.

Step 6: Update $\mathbf{x}^{(i)} = [P_1^{(i)}, P_2^{(i)}, \ldots, P_N^{(i)}, \lambda^{(i)}]^{\mathrm{T}} = \mathbf{x}^{(i-1)} + \varepsilon \nabla \mathcal{L}$

Step 7: Go to step 4.

Step 8: Stop.

2.3 Economic dispatch of thermal units considering network losses

This is the case of economically distributing the load among different plants of a power system. Figure 2.3 shows the schematic of such system. It is important to note here that the transmission system losses need to be considered here.

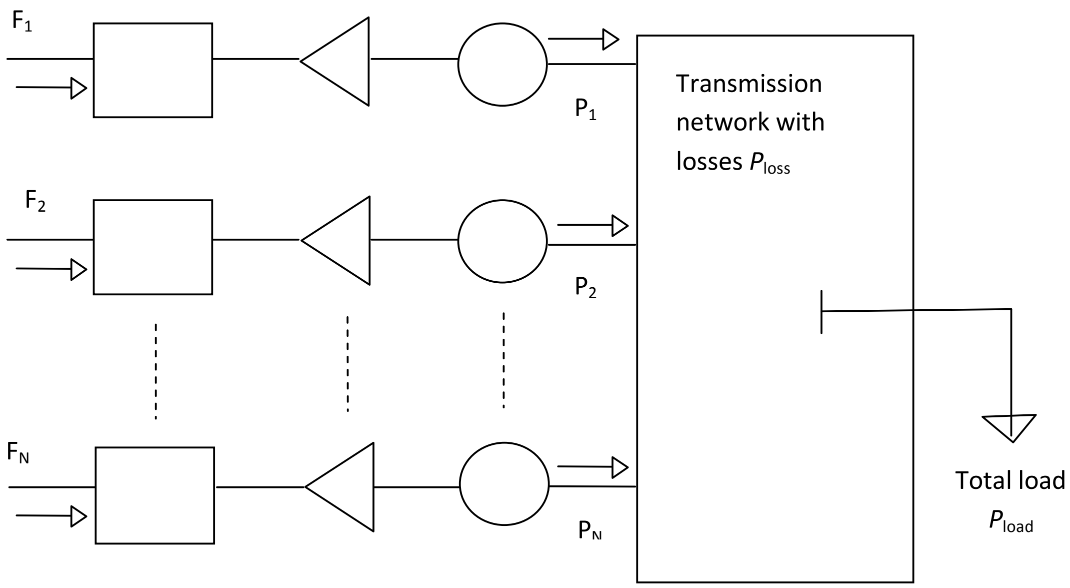

Figure 2.3: N thermal units serving P_{load} through the transmission network

In Example 2.1 we saw that for a given output, unit 1 had higher incremental cost compared to unit 2. For economic dispatch, unit 2 therefore was scheduled to produce more power than unit 1. This is the case in general when units are part of the same plant, or geographically closely located. For a unit with low incremental cost, operating cost may be higher, if the transmission line losses are very high (e.g. due to the large distance between the unit and the load). Therefore, one has to consider the transmission line losses for determining economic dispatch of units in a power system.

Total fuel cost rate is given by (2.2):

$$F_T = F_1(P_1) + F_2(P_2) + ... + F_N(P_N) = \sum_{i=1}^{N} F_i(P_i).$$

The power balance equation including transmission losses is now given by,

$$\phi = P_{\text{loss}} + P_{\text{load}} - \sum_{i=1}^{N} P_i = 0, \tag{2.17}$$

where P_{loss} is the total transmission loss in the system.

The problem here is to find P_i's that minimize F_T, subject to the constraint (2.17). Using the method of Lagrange multipliers,

$$\mathcal{L} = F_T + \lambda\phi$$

where ϕ is given by (2.17)

The necessary conditions to minimize F_T are as follows:

$$\frac{\partial \mathcal{L}}{\partial P_i} = 0; \forall i = 1, 2, \ldots, N$$

Or,

$$\frac{\partial}{\partial P_i}[\sum_{i=1}^{N} F_i(P_i) + \lambda(P_{\text{loss}} + P_{\text{load}} - \sum_{i=1}^{N} P_i)] = 0$$

Or,

$$\frac{\partial \mathcal{L}}{\partial P_i} = \frac{\partial F_i}{\partial P_i} + \lambda(\frac{\partial P_{\text{loss}}}{\partial P_i} - 1) = 0; \forall i = 1, 2, ..., N. \tag{2.18}$$

Rearranging above,

$$\lambda = \frac{\frac{\partial F_i}{\partial P_i}}{1 - \frac{\partial P_{\text{loss}}}{\partial P_i}}. \tag{2.19}$$

Above equation is often expressed as,

$$\lambda = P_{\text{fi}} \frac{\partial F_i}{\partial P_i}, \tag{2.20}$$

where P_{fi} is called the penalty factor of the plant, and is given by,

$$P_{\text{fi}} = \frac{1}{1 - \frac{\partial P_{\text{loss}}}{\partial P_i}} \tag{2.21}$$

Here $\frac{\partial P_{\text{loss}}}{\partial P_i}$ is the *incremental loss* for bus i. Equation (2.20) shows that the minimum cost operation is achieved when the incremental cost (IC) of each unit multiplied by its penalty factor is same for all generating units in the system.

Note:
Relating to the case of units in same plant, or generators connected to the same bus, (2.20) implies:

$$\lambda = P_{\text{f}1}\frac{\partial F_1}{\partial P_1} = P_{\text{f}2}\frac{\partial F_2}{\partial P_2} = \ldots = P_{\text{f}N}\frac{\partial F_N}{\partial P_N}.$$

When units are connected to the same bus, incremental change with transmission loss with change in generation is the same for all the units. Hence,

$$P_{\text{f}1} = P_{\text{f}2} = \ldots = P_{\text{f}N}.$$

Therefore,

$$\frac{\partial F_1}{\partial P_1} = \frac{\partial F_2}{\partial P_2} = \ldots = \frac{\partial F_N}{\partial P_N}, \tag{2.22}$$

which is same as in the case of units connected to a bus.

Equations (2.19) and (2.17) are collectively known as *coordination equations* for ELD considering transmission losses. Solution of ELD problem in the above case is a bit complex compared to the case without considering network losses. There are two basic approaches to solve this problem. One is the case of network loss formula and the other is the case of optimization tools incorporating power flow equations on constraints.

2.4 Transmission line loss equation

Transmission line loss equation, known as Kron's loss formula is given by,

$$P_{\text{loss}} = \mathbf{P}^{\text{T}}\mathbf{B}\mathbf{P} + \mathbf{B}_0^{\text{T}}\mathbf{P} + \mathbf{B}_{00}, \tag{2.23}$$

where $\mathbf{P}$ is the vector of all generator bus net outputs; $\mathbf{B}$ is a square matrix; $\mathbf{B}_0$ is a vector of same length as $\mathbf{P}$; $\mathbf{B}_{00}$ is a constant.

The $\mathbf{B}$-terms are called Loss-coefficients or $\mathbf{B}$-coefficients, and the $N \prod N$ symmetrical matrix $\mathbf{B}$ is simply known as the $\mathbf{B}$-matrix.

Equation (2.23) can be written as,

$$P_{\text{loss}} = \sum_{i=1}^{N}\sum_{j=1}^{N} P_i B_{ij} P_j + \sum_{i=1}^{N} B_{i0} P_i + B_{00}. \tag{2.24}$$

Referring to the coordination equation the equality constraint now becomes:

$$\phi = P_{\text{load}} + [\sum_{i=1}^{N}\sum_{j=1}^{N} P_i B_{ij} P_j + \sum_{i=1}^{N} B_{i0} P_i + B_{00}] - \sum_{i=1}^{N} P_i. \tag{2.25}$$

The derivative of Lagrange function now becomes,

$$\frac{\partial \mathcal{L}}{\partial P_i} = \frac{\partial F_i}{\partial P_i} - \lambda[1 - 2\sum_{j=1}^{N} B_{ij}P_j - B_{i0}]. \tag{2.26}$$

Note that the coordination equations are now coupled. Methods of solution are discussed in the following.

2.4.1 ELD using B-matrix formula

From (2.1),

$$F_i = a_i P_i^2 + b_i P_i + c_i$$

Or,

$$\frac{\partial F_i}{\partial P_i} = 2a_i P_i + b_i$$

Now, from (2.20),

$$\lambda = P_{\text{fi}} \frac{\partial F_i}{\partial P_i} = P_{\text{fi}}(2a_i P_i + b_i) \tag{2.27}$$

where,

$$P_{\text{fi}} = \frac{1}{1 - \frac{\partial P_{\text{loss}}}{\partial P_i}} = \frac{1}{1 - 2\sum_{j=1}^{N} B_{ij}P_j - B_{i0}}. \tag{2.28}$$

The flowchart for solving the ELD problem is presented in the next section.

2.4.2 Flowchart for ELD using B-matrix loss formula

2.5 Economic dispatch using optimization techniques

ELD is essentially a cost minimization problem. Optimization techniques such as linear programming, quadratic programming, heuristic methods (such as genetic algorithm), dynamic programming etc. can be used to solve the ELD problem. Solving the ELD problem using linear programming is discussed briefly in the next section.

2.5.1 ELD using linear programming

The quadratic cost function of generators can be linearized as,

$$\begin{aligned} F_i(P_i) &\approx F_i(P_i^{(0)}) + \left.\frac{\partial F_i(P_i)}{\partial P_i}\right|_{P_i^{(0)}} \Delta P_i \\ &= b\Delta P_i + c, \end{aligned} \tag{2.29}$$

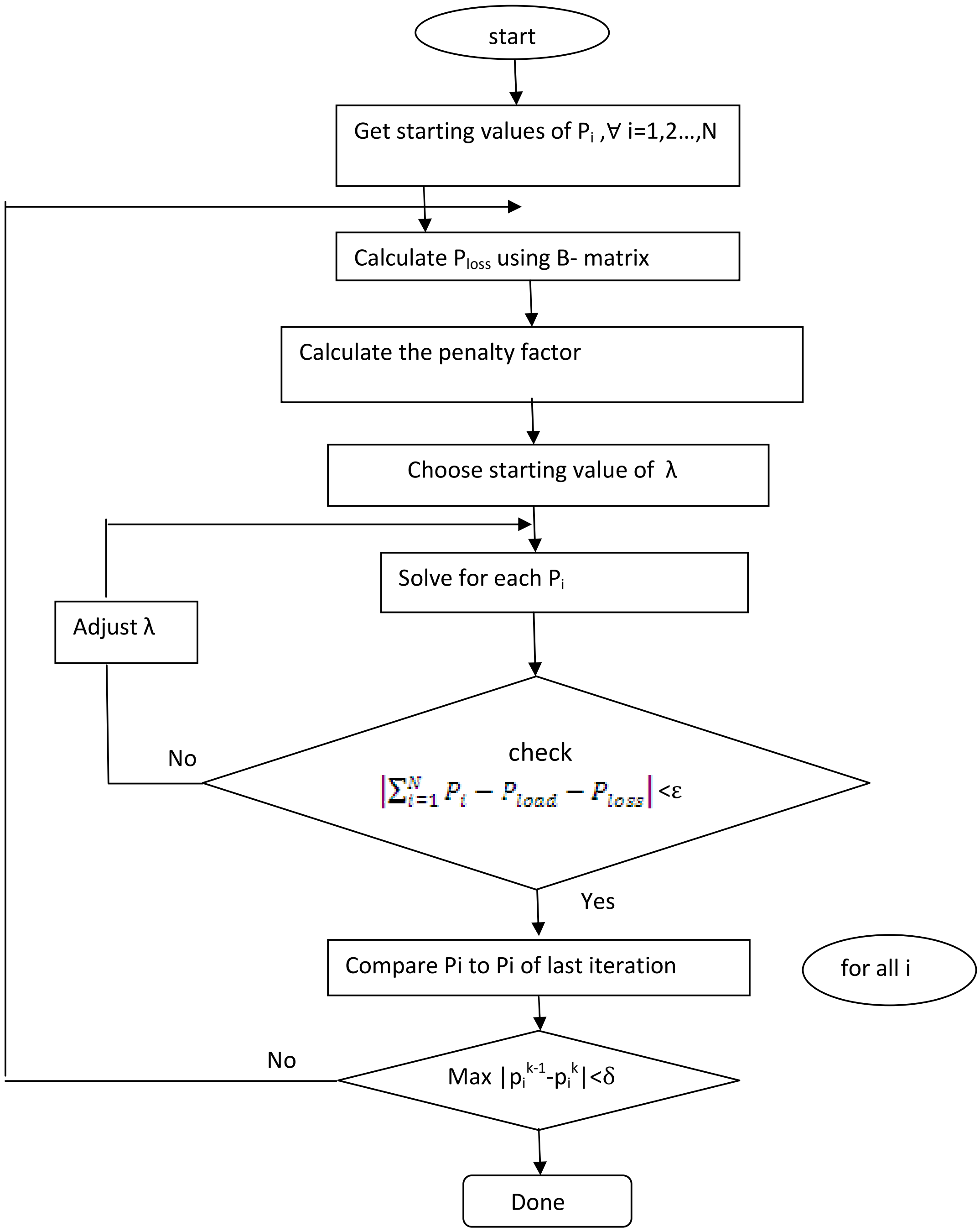

Figure 2.4: Economic load dispatch with updated penalty factors

where $b = \frac{\partial F_i(P_i)}{\partial P_i}\Big|_{P_i^{(0)}}$ (assumed constant for small change in power); $c = F_i(P_i^{(0)})$.

The equality constraint is,

$$\phi = P_{\text{load}} + P_{\text{loss}} - \sum_{i=1}^{N} P_i = 0.$$

Hence,

$$\Delta\phi = \sum_{i=1}^{N} \frac{\partial \phi}{\partial P_i} \Delta P_i = \sum_{i=1}^{N} (\frac{\partial P_{\text{loss}}}{\partial P_i} - 1)\Big|_{P_i^{(0)}} \Delta P_i = 0$$

Or,

$$\sum_{i=1}^{N} (\frac{\partial P_{\text{loss}}}{\partial P_i} - 1)\Big|_{P_i^{(0)}} \Delta P_i = 0. \tag{2.30}$$

Using (2.29) and (2.30), ELD can be solved by linear programming. The basic algorithm is presented in the following.

2.5.2 Algorithm for ELD using linear programming

Step 1: Select set of initial control variables.

Step 2: Solve power flow problem to obtain a feasible solution for the power balance equality constraint.

Step 3: Linearize the objective function and formulate linear programming problem

Step 4: Solve LP problem and obtain optimal incremental control variables: P_i

Step 5: Update control variables: $P_i^{K+1} = P_i^K + \Delta P_i$

Step 6: Obtain power flow solution and update control variables

Step 7: If $\Delta P \leq \delta, \forall i = 1, 2, ..., N$, then stop otherwise go to step 3.

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[2] A. J. Wood, B. F. Wollenberg, *Power Generation Operation and Control.* New York: John Wiley & Sons, Inc., second ed., 2006.