Design Response Spectrum

- Earlier Attenuation Models Provided relationships in terms of PGA/PGV etc.
- For design purpose we need response spectrum
- We cannot design a structure for response spectrum of one ground motion as the ground motion in a given site is highly uncertain
- Peak and valley of response spectrum of one ground motion differs from others
- Hence, there was a need to develop smoothed response spectrum for design purpose considering uncertainties of ground motion at a given site
- The smoothed response spectrum used for design is called design response spectrum
First Documented Design Spectrum

- Developed GW Housner
- Considered 8 accelerograms from West Coast of USA
- Evaluated response spectrum for each of these ground motions
- Normalized each spectrum to have a PGA/g of 0.2
- Averaged and smoothed
- Can be used for any PGA/g by scaling w.r.t 0.2

![First Documented Design Spectrum](image.png)
Construction Design Spectrum

✓ Three methods are used for construction of Design Response Spectrum

1. Characteristic Response spectrum shape & specified PGA from attenuation and local site condition - Temporal model – 2% in 50 year

2. N.M. New mark & W.J. Hall approach

3. Method based on statistical correlation to directly derive average RS ordinates

Peak Ground Acceleration and Response Spectrum Shape Method

✓ Elastic Spectrum

1. Obtain response spectrum for an ensemble of ground motions representative of that site

2. Normalize each response spectrum with its PGA

3. At each period, evaluate mean and mean plus standard deviation values of the ensemble and then plot mean and mean plus standard deviation spectra

Generally shapes are different for different soil conditions: Rock site, medium stiff, alluviums site etc. For engineering purpose, smoothed shapes are used.
FIGURE 9.2 (a) Original and (b) normalized response spectra. (Reproduced with permission of Earthquake Engineering Research Institute from Seed, H.B. and Idriss, I.M., *Ground Motions and Soil Liquefaction during Earthquakes*, Earthquake Engineering Research Institute, Oakland, CA, 1983.)

FIGURE 9.3 Average response spectra for different site conditions. (Reproduced with permission of Earthquake Engineering Research Institute from Seed, H.B. and Idriss, I.M., *Ground Motions and Soil Liquefaction during Earthquakes*, Earthquake Engineering Research Institute, Oakland, CA, 1983.)
Inelastic Spectrum – For seismic design of structures, reduced force and supplied ductility (Ductility capacity) are considered

Assumption:

- Force deformation behaviors is elasto-plastic
- Max inelastic deformation = max elastic deformation

(not for individual record in building design codes but for average sense)
Nonlinear Response Spectrum

\[ F_y = ku_y = k \frac{u_{\text{max}}}{\mu} = k \frac{SD(\omega_n, \xi)}{\mu} = \omega_n^2 m \frac{SD(\omega_n, \xi)}{\mu} \]

\[ = m \frac{PSA(\omega_n, \xi)}{\mu} = \frac{SA/g}{\mu} \ast \omega \]

Yield acceleration \( \ddot{u}_y = \frac{F_y}{m} \approx \frac{SA(\omega_n, \xi)}{\mu} \)

However, \( \ddot{u}_y \rightarrow PGA \rightarrow \text{Independent of force-deformation behavior} \)

Hence, there is a transition zone between elastic and inelastic spectrum starting from PGA.

Design Force

\[ F_y = \frac{SA/g}{\mu} \omega = c \omega \rightarrow e = \frac{SA(\omega_n, \xi)/g}{\mu} \quad e = \text{seismic coefficient} \]
Linear and Elastoplastic Force-Deformation

FIGURE 9.6  Inelastic acceleration design spectrum derived from elastic counterpart.

Linear and Elastoplastic Response Spectra

Box 9.1  Procedure to Construct Design Spectrum Using Peak Ground Acceleration and Response Spectrum Shape

1. Estimate the peak ground acceleration expected at the site under consideration for an acceptable probability of exceedance.
2. Select a spectral shape according to the site's soil conditions.
3. Multiply the ordinates of the selected spectral shape by the estimated peak ground acceleration to obtain an elastic design spectrum.
4. Divide the ordinates of the elastic design spectrum by a preselected ductility factor to determine the corresponding inelastic design spectrum.
5. Draw a transition line between the elastic spectral acceleration at zero period and the inelastic spectral acceleration at the period that defines the left corner of the elastic spectrum.
Accuracy of Elastoplastic Response Spectra in Average Sense

Newmark – Hall Approach

- Elastic Spectrum –
  - Normalization of PGA for generation of spectral shape is doubtful
  - 1973 Newmark – Hall reported that some spectral ordinates are dominated by PGA & some by PGD
Normalized Response Spectra – 1940 El Centro

Newmark – Hall Approach

✓ Elastic Spectrum –

✓ Normalization of PGA for generation of spectral shape is doubtful

✓ 1973 Newmark – Hall reported that some spectral ordinates are dominated by PGA & some by PGD

✓ They suggested Evaluation of Spectral shape as follows

1. \( \omega_n < 0.05 \text{Hz} \); \( \text{SD} = \text{PGD} \)
2. \( 0.1 < \omega_n < 0.3 \text{Hz} \) \( \text{SD} = \text{PGD} \times \text{constant amplification factor (D)} \)
3. For \( 0.3 < \omega_n < 2 \text{Hz} \) intermediate frequency \( \text{SD} = \text{PGV} \times V \)
4. For high frequencies \( 2 < \omega_n < 8 \text{Hz} \)
   \( \text{SA} = A \times \text{PGA} \)
5. For \( \omega_n > 20 \text{Hz}, 33 \text{Hz} \)
Newmark – Hall Approach

- Elastic Spectrum –
  - They considered (for a given site) a large number of motions (28 records) & came out with A, V, D mean & values for different damping ratios
  - They also suggested obtain directly from attenuation relationship & temporal model if PSHA
  - They also said, if is only known

\[
\frac{PGA \times PGD}{PGV^2} = 6.0, \text{ where } PGD \text{ is in } \text{in} / \text{s}^2
\]

Later these ratios are calculated for large number of GMS & different local site conditions. They depend on M, epicentral dist, \( T_D \), etc.

Amplification Factors

<table>
<thead>
<tr>
<th>Table 9.1</th>
<th>Newmark–Hall Amplification Factors for the Construction of Elastic Design Spectra</th>
</tr>
</thead>
<tbody>
<tr>
<td>Damping Ratio (%)</td>
<td>Mean</td>
</tr>
<tr>
<td></td>
<td>A</td>
</tr>
<tr>
<td>0.5</td>
<td>3.68</td>
</tr>
<tr>
<td>1</td>
<td>3.21</td>
</tr>
<tr>
<td>2</td>
<td>2.74</td>
</tr>
<tr>
<td>3</td>
<td>2.46</td>
</tr>
<tr>
<td>5</td>
<td>2.12</td>
</tr>
<tr>
<td>7</td>
<td>1.89</td>
</tr>
<tr>
<td>10</td>
<td>1.64</td>
</tr>
<tr>
<td>20</td>
<td>1.17</td>
</tr>
</tbody>
</table>

Newmark-Hall Spectrum Construction

Box 9.2  Procedure to Construct Elastic Design Spectrum Using Newmark–Hall Approach

1. Estimate for the site under consideration the expected peak values of ground acceleration, ground velocity, and ground displacement.
2. Draw in a tripartite graph lines perpendicular to the spectral displacement, spectral pseudo-velocity, and spectral acceleration axes, with ordinates, respectively, equal to the
   a. Peak ground displacement times D
   b. Peak ground velocity times V
   c. Peak ground acceleration times A
   where D, V, and A are the amplification factors given in Table 9.1.
3. Draw a line perpendicular to the spectral acceleration axis with an ordinate equal to the peak ground acceleration.
4. Draw a transition line between the frequencies of 8 and 33 Hz, linking the lines drawn in Step 2 with the line drawn in Step 3.
5. Connect all five lines.

Newmark-Hall Spectrum Construction-Later Study

TABLE 9.2
Average v/a, ad/v², and d/a Ratios Obtained in 1976 Mohraz Study

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>v/a (in./s/g)</th>
<th>ad/v²</th>
<th>d/a (in./g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rock</td>
<td>24</td>
<td>5.3</td>
<td>8</td>
</tr>
<tr>
<td>&lt;30 ft of alluvium underlain by rock</td>
<td>30</td>
<td>4.5</td>
<td>11</td>
</tr>
<tr>
<td>30–200 ft of alluvium underlain by rock</td>
<td>30</td>
<td>5.1</td>
<td>12</td>
</tr>
<tr>
<td>Alluvium</td>
<td>48</td>
<td>3.9</td>
<td>23</td>
</tr>
</tbody>
</table>

Newmark–Hall Approach

- Inelastic Spectrum –
  - They constructed nonlinear response spectra using the approach mentioned early…i.e., to use the concept of equal displacement
  - They found something very interesting
Total Deformation Spectrum

**FIGURE 9.9** Yield-deformation spectra for elastoplastic systems with 2% damping corresponding to the N–S accelerogram from the 1940 El Centro earthquake. (Reproduced with permission of Earthquake Engineering Research Institute from Newmark, N.M. and Hall, W.J., *Earthquake Spectra and Design,* Earthquake Engineering Research Institute, Oakland, CA, 1982.)

Total Deformation Spectrum

**FIGURE 9.10** Total-deformation spectra for elastoplastic systems with 2% damping corresponding to the N–S accelerogram from the 1940 El Centro earthquake. (Reproduced with permission of Earthquake Engineering Research Institute from Newmark, N.M. and Hall, W.J., *Earthquake Spectra and Design,* Earthquake Engineering Research Institute, Oakland, CA, 1982.)
Findings

1. For frequencies <0.3 Hz (i.e., region of constant elastic spectral displacements), the total inelastic displacement response is approximately equal to the elastic displacement response (i.e., μ = 1) for all μ.
2. For frequencies between 0.3 and 2 Hz (i.e., region of constant elastic spectral velocities), the total inelastic displacement response is approximately equal to the elastic displacement response for all μ.
3. For frequencies between 2 and 8 Hz (i.e., region of constant elastic spectral accelerations), the total inelastic displacement response is not equal to the elastic displacement response, but depends on μ instead.
4. For frequencies >33 Hz, the yield acceleration response is nearly the same for all ductility factors and equal to the peak ground acceleration.

Point 3: Equal displacement is not good for constant acceleration sensitive region- (See high frequency region of total-displacement plot in earlier slide)

Newmark – Hall: Inelastic Spectrum

They found that deformation energy dissipated by elastic system is same as that of inelastic system instead of equal deformation for this region

**FIGURE 9.11** Force-deformation curves for linear and elastoplastic systems.
Newmark – Hall: Inelastic Spectrum

\[ \frac{1}{2} k u_0^2 = \frac{1}{2} k u_y^2 + (u_{\text{max}} - u_0) ku_y \]

\[ \frac{u_0^2}{u_y^2} = 1 + 2 \left( \frac{u_{\text{max}}}{u_y} - 1 \right) = 2\mu - 1 \]

\[ R_\mu = \frac{1}{\mu} \]

\[ u_y = \frac{u_0}{\sqrt{2\mu - 1}} = \frac{SD}{\sqrt{2\mu - 1}} = \frac{SA/\omega^2}{\sqrt{2\mu - 1}} \]

So the ductility reduction factors for construction of Nonlinear Newmark – Hall spectrum are

For constant displacement and velocity region \( R_\mu = \frac{1}{\mu} \)

For constant acceleration and velocity region \( \sqrt{2\mu - 1} \)

They did not considered, the following points

1. Effect of damping
2. Soil properties
3. Different force deformation behavior etc.

Several studies have been conducted to far to evaluate ductility reduction factor

Newmark-Hall Yield Spectrum

FIGURE 9.12 Construction of inelastic design spectrum using Newmark–Hall approach.
For a given site, design spectrum can be constructed by using empirical equations of seismic hazard analysis – e.g., Attenuation model, earthquake magnitude, fault type.. etc

Drawbacks: for various damping ratios, load-deformation behavior etc., this approach is very cumbersome