# Local entropy generation for saturated two-phase flow

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## ARTICLE INFO

## ABSTRACT

This paper addresses the estimation of local entropy generation rate for diabatic saturated two-phase flow of a pure fluid. Two different approaches have been adopted for this thermodynamic characterization: the separated flow model using the classical vapor flow quality, and the mixture model, using the thermodynamic vapor quality. Based on these two models, two distinct expressions for the local entropy generation have been proposed. The analysis explicitly shows the contribution of heat transfer and pressure drop respectively to the local entropy generation. The contribution due to phase-change process is also determined using the mixture model. The developed formulation is applied to analyze the thermodynamic performance of enhanced heat transfer tubes under different conditions. It is shown that enhanced tubes may be a relevant solution for reducing entropy generation at low mass velocities whereas smooth tubes remain the best solution at higher ones.

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## 1. Introduction

In this paper, we focus our attention on the local entropy generation during diabatic saturated two-phase flow. The primary motivation of the study is to quantify local entropy generation under such flow conditions. In the literature, efforts to evaluate numerous heat transfer related processes in terms of entropy generation have been promoted. Bejan [1] studied entropy generation through heat and fluid flow of a single-phase fluid. He proposed an entropy generation number given by $N_S = \dot{S}'_{gen,enhanced}/\dot{S}'_{gen,smooth}$. More recently, Bejan [2] made a review on the thermodynamic optimization (or entropy generation minimization) of flow geometry in engineering flow systems. It was observed that design improvements are registered by spreading the imperfections (e.g. flow resistances) through the system. An expression was also proposed for the entropy generation during a diabatic single-phase flow indicating that irreversibilities come from two main causes: heat transfer and pressure drops. Based on the entropy generation theorem, Zimparov [3] has proposed equations for enhanced heat transfer surfaces to include the effect of fluid temperature variation along the length of a tubular heat exchanger during single-phase flow. This general evaluation criteria add new information to Bejan's entropy generation minimization method, assessing the two objectives simultaneously. They may help to identify inappropriate enhanced surfaces, in terms of irreversibilities, thereby assisting in designing better heat transfer equipment. The article ended with the estimation of heat transfer and fluid friction characteristics of ten spirally corrugated tubes. Thermodynamic optimization method of tree-shaped flow geometries during single-phase flow was also proposed by Zimparov et al. [4]. They optimized the performance of several classes of simple systems consisting of T- and Y-shaped assemblies of ducts, channels and streams. They assumed a laminar and fully developed flow and developed analytical equations for determining the entropy generation in these geometries. Abbassi [5] has performed an entropy generation analysis in a uniformly heated microchannel heat sink (MCHS) by using the porous medium approach based on extended Darcy equation for fluid flow, and two-equation model for heat transfer. They investigated the effect of influential parameters such as, channel aspect ratio, thermal conductivity ratio of the fluid and the solid and effective porosity of the MCHS on the total entropy generation. Lior et al. [6] has reviewed the methods developed for analyzing the space and time dependent exergy and irreversibility fields in transport processes. They also emphasized the fact that only few articles suggest keys to process improvements. Sahiti et al. [7] developed an optimization model on the basis of the entropy generation minimization for different double-pipe heat exchanger (air/water) flow lengths and different length of circumferential pin fins. Their conclusions are derived on the basis of the behavior of entropy generation number as a function of Reynolds number. Owing to the fact that the spatial distribution of the air-side heat transfer coefficient is difficult to establish accurately, their formulation is based on the overall heat transfer coefficient, and cannot therefore be termed as a 'local' expression. Haddad et al. [8] studied the entropy generation due to laminar forced convection in the

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## Nomenclature

| | |
|---|---|
| $A$ | cross section, $m^2$ |
| $Ar$ | area ratio between enhanced and smooth tubes based on the internal diameter |
| $Be$ | Bejan number $Be = d\dot{S}'_{heattransfer}/d\dot{S}'$ |
| $c_p$ | specific heat capacity at constant pressure, J/kg K |
| $c_x$ | specific heat capacity at constant vapor quality, J/kg K |
| $dz$ | element of discretization, m |
| $D$ | tube diameter, m |
| $Ec$ | Eckert number $Ec = U^2/(c_p \Delta T)$ |
| $G$ | mass velocity, $kg/m^2\,s$ |
| $h$ | specific enthalpy, J/kg |
| $L$ | length, m |
| $m$ | mass, kg |
| $\dot{m}$ | mass flow rate, kg/s |
| $N_S$ | entropy generation number |
| $p$ | pressure, Pa |
| $P$ | perimeter, m |
| $q$ | heat flux, $W/m^2$ |
| $\dot{Q}$ | heat rate, W |
| $s$ | specific entropy, J/kg K |
| $d\dot{S}'$ | entropy generation rate per unit length, W/K m |
| $\delta t$ | characteristic time, s |
| $T$ | temperature, K |
| $\Delta T$ | wall superheat, K |
| $U$ | velocity, m/s |
| $v$ | specific volume, $kg/m^3$ |
| $x$ | vapor quality |
| $z$ | longitudinal abscissa, m |

### Greek symbols

| | |
|---|---|
| $\beta$ | parameter |
| $\Gamma$ | slip ratio ($U_v/U_l$) |
| $\rho$ | density, $kg/m^3$ |

### Subscripts

| | |
|---|---|
| enhanced | refers to enhanced tubes |
| gen | generated |
| in | inlet |
| l | liquid |
| lv | liquid to vapor |
| heattransfer | heat transfer |
| m | mixture |
| min | minimum |
| t | total |
| tp | two-phase |
| sat | saturation |
| smooth | refers to smooth tubes |
| v | vapor |
| w | wall |

entrance region of a concentric cylindrical annulus. They found that the entropy generation was inversely proportional to both Reynolds number and the dimensionless entrance temperature. The results also showed that increasing Eckert number and/or the radius ratio will increase the entropy generation.

In addition to these single-phase studies, some authors have also developed theories on entropy generation for two-phase flow. Collado [9] has established an expression for the entropy-based boiling curve for subcooled flow boiling. The work is based on the classical 1-D entropy formulation for a steady boiling flow. However, in this article, the pressure drop term has been neglected. This might be inappropriate for the case of flow boiling in mini or microchannels, for which, high friction losses are expected. Besides, the specific contributions, due to heat transfer, pressure drop and phase-change, to the entropy generation, have not been explicitly expressed. More recently, a mathematical model was presented by Saechan and Wongwises [10] for determining optimal configuration of plate finned tube condensers (air-refrigerant type). The analysis was based on the minimum entropy generation of air and refrigerant. The equations were based on the overall heat transfer coefficient. The entropy generation number, $N_S$, was used for optimizing their architecture (e.g. number of tube rows, fin spacing, tube diameter, etc.). They concluded their article by saying that most of the entropy generation occurred on the air side. Another theoretical model for two-phase flow characterization was developed by Bilicki et al. [11] by the method of irreversible thermodynamics. They considered only particulate fluid flows (i.e. not annular, stratified nor churn flow regimes) and subdivided their fluid volume into a large but finite number of cells. They wrote the conservation laws for two-phase flow. Their model is however complicated and the authors needed some appropriate approximations or even transformations to use their model. Finally, they restricted their model to a 1-D homogeneous model but they were able to predict some experimental data. They also obtained expressions for rate of local entropy generation per unit volume. Shiba and Bejan [12] optimized the global performance of

a counterflow heat exchanger serving as a condenser in a vapor-compression-cycle refrigeration system by minimizing the total entropy generation rate. Their numerical results showed how the optimal configuration responded to changes in specified external parameters such as refrigeration load, fan efficiency, volume and weight. This interesting work was however devoted to the system and not the two-phase flow itself, which is the objective of the present paper. Vargas and Bejan [13] considered the fundamental problem of thermodynamically matching the two streams, one hot and the other cold, so that the irreversibility generated by their thermal interaction is minimum. Each stream experienced a change of phase. The authors showed that the optimum was marked by an optimal ratio between the stream mass flow rates, and an optimal ratio between the two heat exchanger sizes when the total heat transfer area was fixed. This study was also devoted to the system approach and not the local aspect of two-phase flow.

From the literature review, we highlight the fact that, while quite some attempts have been made in the past to estimate entropy generation in single-phase flows, almost no study has been performed on the estimation of local entropy generation for a diabatic saturated two-phase flow. Moreover, most of the earlier studies are dedicated to single-phase flow or, at best, to global space-time averaged analysis of two-phase flow on a system level. To begin with, we will redefine some thermodynamic relations for 1-D diabatic saturated two-phase flow of a pure fluid based on two different approaches: the separated flow model and the mixture model. Thereafter, we will propose two new local entropy generation equations based on these two models respectively. Finally, we will demonstrate the use of this formulation with a case study of enhanced heat transfer tubes.

## 2. Energy and mass conservation for a diabatic saturated two-phase flow

Two different approaches may be found in the literature for modeling two-phase flow: the relatively simpler homogeneous

model which assumes a slip ratio ($\Gamma = U_v/U_l$) equal to unity (both phases travel at the same velocity) and the more detailed separated flow models. However, if we want to treat two-phase flow from a more holistic viewpoint, both these conventional models have inherent limitations and the use of the mixture model is necessary. We will present the basic equations for the separated flow model and for the mixture model in the next sections.

### 2.1. Separated flow model

Usually, the classical vapor quality (also called flow quality) equation used for a diabatic two-phase flow is,

$$x = \frac{\dot{m}_v}{\dot{m}_v + \dot{m}_l} \tag{1}$$

Therefore, the two-phase enthalpy and two-phase entropy are expressed as:

$$h_{tp} = xh_v + (1-x)h_l \tag{2}$$

and

$$s_{tp} = xs_v + (1-x)s_l \tag{3}$$

The two-phase specific volume is defined as:

$$v_{tp} = xv_v + (1-x)v_l \tag{4}$$

Assuming that the kinetic and gravity terms are much less than the thermal energy component, first law of thermodynamics gives:

$$\frac{\delta \dot{Q}}{\dot{m}_t} = \mathrm{d}(h_{tp}) = h_{lv}\mathrm{d}x + x\mathrm{d}h_v + (1-x)\mathrm{d}h_l \tag{5}$$

Furthermore, we know that

$$\begin{aligned} \mathrm{d}h_v &= T_v\mathrm{d}s_v + v_v\mathrm{d}p_v \\ \mathrm{d}h_l &= T_l\mathrm{d}s_l + v_l\mathrm{d}p_l \end{aligned} \tag{6}$$

Assuming that $T_v = T_l = T_{sat}$ and $\mathrm{d}p_v = \mathrm{d}p_l = \mathrm{d}p$, neglecting capillary contribution, we get

$$\begin{aligned} \frac{\delta \dot{Q}}{\dot{m}_t} &= h_{lv}\mathrm{d}x + (x\mathrm{d}s_v + (1-x)\mathrm{d}s_l)T_{sat} + (xv_v + (1-x)v_l)\mathrm{d}p \\ &= h_{lv}\mathrm{d}x + c_{x,tp}\mathrm{d}T_{sat} + v_{tp}\mathrm{d}p \end{aligned} \tag{7}$$

The heat capacity of the two-phase flow at constant vapor quality is defined as:

$$c_{x,tp} = xc_{x,v} + (1-x)c_{x,l} \tag{8}$$

with

$$\begin{aligned} c_{x,l} &= T_{sat}\left(\frac{\mathrm{d}s_l}{\mathrm{d}T_{sat}}\right)_{x=0} \\ c_{x,v} &= T_{sat}\left(\frac{\mathrm{d}s_v}{\mathrm{d}T_{sat}}\right)_{x=1} \end{aligned} \tag{9}$$

Using the Clapeyron equation ($\mathrm{d}p/\mathrm{d}T_{sat} = h_{lv}/(T_{sat}v_{lv})$), Eq. (7) becomes:

$$\mathrm{d}x = \frac{\delta \dot{Q}}{\dot{m}_t h_{lv}} - \underbrace{\frac{(v_{lv}T_{sat}c_{x,tp} + h_{lv}v_{tp})}{h_{lv}^2}\mathrm{d}p}_{\text{flashing effect}} \tag{10}$$

where $\delta \dot{Q}$ is the heat supplied to the fluid and $\dot{m}_t$ is the total mass flow rate of the two-phase flow (constant).

In Eq. (10), the second term refers to the enthalpy rearrangement between the phases due to the spontaneous flashing effect of the two-phase flow, which becomes significant when the pressure drop is high.

With this particular model, both phases travel at different velocities. The distance covered by the vapor and the liquid is thus different over the same time $\delta t$.

### 2.2. Mixture model

The mixture theory presented in this paper was proposed by Collado [14] and is based on the Reynolds transport theory [15]. In this paper, we will go one step further from Collado [4]: we will not necessarily assume a constant slip ratio (as was originally done in Ref. [15]) and we will also explicitly take into account the pressure drop contribution to flow irreversibilities. The reasoning of the theory is quite simple: if the phases travel at different velocities, they cannot cover the same distance during the same time. As a result, for the same length of control volume, the time scale is different for each phase: $\delta t_v$ refers to the characteristic time of the vapor phase and $\delta t_l$ refers to the characteristic time of the liquid phase. To take into account the fact that the temporal scales are distinct for both the phases, a time scaling factor should be included, which, as shown hereafter, is the slip ratio ($\Gamma$).

Consider a fixed mass of fluid $m_m$ composed of saturated vapor and liquid phase, flowing in a diabatic channel:

$$m_m = m_v + m_l \tag{11}$$

If we follow (i.e. we track) the same fixed mass of fluid through time and space inside the pipe, we can divide all the terms by $\delta t_l$:

$$\frac{m_m}{\delta t_l} = \frac{m_v}{\delta t_l} + \frac{m_l}{\delta t_l} \tag{12}$$

or

$$\frac{m_m}{\delta t_l} = \frac{\delta t_v}{\delta t_l}\frac{m_v}{\delta t_v} + \frac{m_l}{\delta t_l} \tag{13}$$

with

$$\frac{\delta t_l}{\delta t_v} = \frac{U_v}{U_l} = \Gamma \tag{14}$$

Finally, for the limit case $\delta t_l \to 0$, we obtain

$$\dot{m}_m = \frac{\dot{m}_v}{\Gamma} + \dot{m}_l = \rho_m U_l A \tag{15}$$

where $A$ is the cross section of the channel. Note that $\dot{m}_t$ is the total mass flow rate of the inlet fluid, which usually is liquid. Therefore, the time reference associated with $\dot{m}_t$ must be that of the liquid. The mixture density is thermodynamically defined as:

$$\rho_m = \left(\frac{x_m}{\rho_v} + \frac{1-x_m}{\rho_l}\right)^{-1} = \alpha\rho_v + (1-\alpha)\rho_l \tag{16}$$

where $\alpha$ is the volumetric fraction of the vapor in the flow boiling mixture, which can be measured with reasonable accuracy from X- or $\alpha$-ray methods [16].

The mixture vapor quality corresponds to the thermodynamic vapor quality and is expressed as:

$$x_m = \frac{m_v}{m_v + m_l} = \frac{\dot{m}_v}{\dot{m}_v + \Gamma \dot{m}_l} = \frac{\dot{m}_v}{\dot{m}_m} \tag{17}$$

We may express the thermodynamic vapor quality as a function of the classical vapor quality (flow quality) given by Eq. (1):

$$x_m = \frac{x}{x + \Gamma(1-x)} \tag{18}$$

Eq. (18) has been plotted in Fig. 1 for different slip ratios. As expected, higher the slip ratio, greater is the deviation between thermodynamic and conventional vapor quality; the former being always lower than the latter for $\Gamma > 1$. For $\Gamma = 1$, we obtain $x_m = x$.

The elementary derivative of the thermodynamic vapor quality is given by the following relation:

$$dx_m = \frac{\Gamma}{(x + \Gamma(1-x))^2}dx - \frac{x(1-x)}{(x + \Gamma(1-x))^2}d\Gamma \quad (19)$$

The mixture enthalpy and entropy (still tracking the same constant mass) are thermodynamically expressed as:

$$h_m = x_m h_v + (1 - x_m)h_1 \quad (20)$$

and

$$s_m = x_m s_v + (1 - x_m)s_1 \quad (21)$$

The equations obtained in this section (mixture model) are applicable irrespective of the equilibrium state of the flow, i.e. the liquid and vapor temperatures may exhibit different values, i.e. $h_v$, $s_v$ to be calculated at $T_v$ and $h_1$, $s_1$ to be calculated at $T_1$ ($T_v$ may or may not be equal to $T_1$). However, in this study, local thermodynamic equilibrium is assumed in each discretized cell. As a consequence, we will take all the properties at $T_v = T_1 = T_{sat}$. Note also, that for $\Gamma = 1$, the mixture model and the separated flow models are equivalent.

## 3. Local entropy generation for saturated two-phase flow

### 3.1. Separated flow model

Assume a saturated two-phase flow in a channel flowing at a certain mass flow rate $\dot{m}_t$ as shown in Fig. 2. According to the second law of thermodynamics applied to a control volume of length $dz$, we may write the following relation:

$$d\dot{S}'dz = d(\dot{m}_v s_v + \dot{m}_1 s_1) - \frac{\delta \dot{Q}}{T_w} \quad (22)$$

where $d\dot{S}'$ is the entropy generation rate per unit length, $\delta \dot{Q}$ is the heat rate applied to the control volume and $T_w$ is the wall
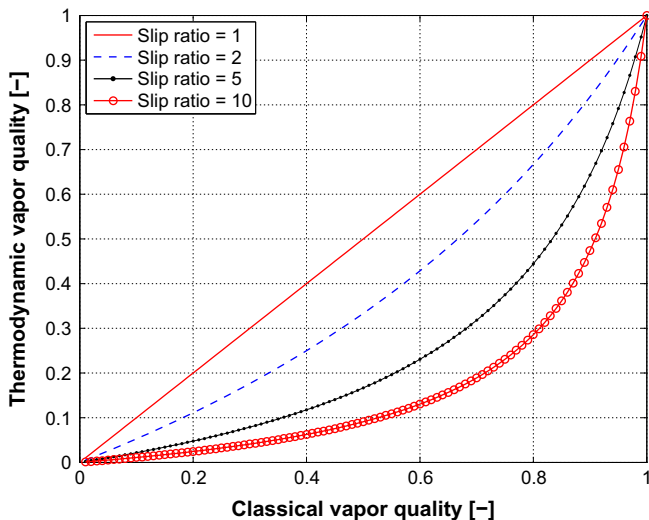


**Fig. 1.** Thermodynamic vapor quality as a function of the classical vapor quality.
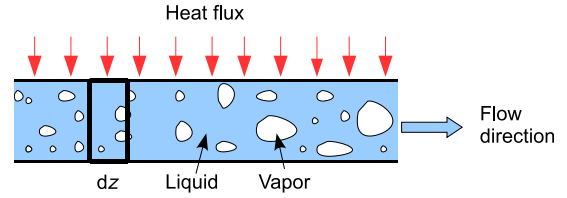


**Fig. 2.** Schematic of a diabatic saturated two-phase flow in a channel.

temperature which is assumed to be the temperature of the system boundary, as a first approximation. Furthermore, we know that $dh_{tp} = T_{sat}ds_{tp} + v_{tp}dp$ and $\dot{m}_t ds_{tp} = d(\dot{m}_v s_v + \dot{m}_1 s_1)$, thus we get:

$$d\dot{S}'dz = \frac{\dot{m}_t}{T_{sat}}dh_{tp} - \frac{\dot{m}_t v_{tp}}{T_{sat}}dp - \frac{\delta \dot{Q}}{T_w} \quad (23)$$

Combining the second law constraint with the energy equations, i.e. Eqs. (23) and (5), it is simplified as:

$$d\dot{S}'dz = \frac{\delta \dot{Q}}{T_{sat}} - \frac{\delta \dot{Q}}{T_w} - \frac{\dot{m}_t v_{tp}}{T_{sat}}dp \quad (24)$$

For an adiabatic flow, Eq. (24) reduces to,

$$d\dot{S}' = \frac{\dot{m}_t v_{tp}}{T_{sat}}\left(-\frac{dp}{dz}\right) \quad (25)$$

For a diabatic flow, we can write that $T_w = T_{sat} + \Delta T$ ($\Delta T$ being the superheat of the wall), as a consequence, we get,

$$d\dot{S}' = \frac{\delta \dot{Q}}{dz}\left[\frac{\Delta T}{T_{sat}^2(1 + \Delta T/T_{sat})}\right] + \frac{\dot{m}_t v_{tp}}{T_{sat}}\left(-\frac{dp}{dz}\right) \quad (26)$$

In general, $\Delta T/T_{sat} << 1$. Besides, we know that $q = h\Delta T = \delta \dot{Q}/(dzP)$ (assuming a constant perimeter $P$ of the channel with respect to $z$). Thus, this yields,

$$\underbrace{d\dot{S}'}_{\text{entropy generation}} = \underbrace{\frac{q^2 P}{h T_{sat}^2}}_{\text{heat transfer contribution}} + \underbrace{\frac{\dot{m}_t v_{tp}}{T_{sat}}\left(-\frac{dp}{dz}\right)}_{\text{pressure drop contribution}} \quad (27)$$

where $q$ is the heat flux and $h$ is the heat transfer coefficient at the tube wall.

Eq. (27) can be applied to any diabatic two-phase flow. The term $(-dp/dz)$ indirectly takes into account part of the slip ratio effect. However, the equation is not complete because it does not account for the entire phase-change contribution.

### 3.2. Mixture model

The separated flow model and the mixture model are different, as has been discussed earlier. We now introduce a link between both models by defining a factor $F$:

$$dh_{tp} = \frac{\delta \dot{Q}}{\dot{m}} = Fdh_m \quad (28)$$

According to Fig. 1 and assuming that $dh_{tp} \approx h_{lv}dx$ and $dh_m \approx h_{lv}dx_m$, it is easy to see that:

a) at low $x$ and as $\Gamma > 1$, $dx > dx_m \Rightarrow dh_{tp} > dh_m, \Rightarrow F > 1$
b) at high $x$ and as $\Gamma > 1$, $dx < dx_m \Rightarrow dh_{tp} < dh_m, \Rightarrow F < 1$

As a result, from (b) we see that using $F = \Gamma$ (as has been done in Collado's theory) in this study is inappropriate. However, there exists a relation between the vapor flow quality and the thermodynamic vapor quality, which is given by Eqs. (18) and (19). Furthermore, the vapor flow quality can be expressed using a simple energy balance (Eq. (10)). As a result, it is possible to formulate an alternative and more comprehensive energy balance for the mixture model, which would link the heat flux, the mass flow rate and the mixture enthalpy. This energy balance formulation is described below.

$$
\begin{aligned}
F &= \frac{\delta \dot{Q}}{\dot{m}_t dh_m} \\
&= \left[ \beta + \frac{(c_{x,m} - \beta c_{x,tp})v_{lv}T_{sat} + h_{lv}(v_m - \beta v_{tp})}{h_{lv}\delta \dot{Q}}\dot{m}_t dp \right. \\
&\quad \left. - \frac{x(1-x)\dot{m}_t h_{lv}\beta}{\Gamma \delta \dot{Q}}d\Gamma \right]^{-1}
\end{aligned}
\tag{37}
$$

or

$$
F = \frac{\Gamma h_{lv}\frac{\delta \dot{Q}}{dz}}{\beta \Gamma h_{lv}\frac{\delta \dot{Q}}{dz} - \left[ (c_{x,m} - \beta c_{x,tp})v_{lv}T_{sat} + h_{lv}(v_m - \beta v_{tp}) \right]\Gamma \dot{m}_t \left( -\frac{dp}{dz} \right) - x(1-x)\dot{m}_t h_{lv}^2 \beta \frac{d\Gamma}{dz}}
\tag{38}
$$

Assume a saturated two-phase flow mixture flowing in a channel as shown in Fig. 2. Starting with Eq. (10), we can write:

$$
\frac{\delta \dot{Q}}{\dot{m}_t} = h_{lv}dx + \frac{(v_{lv}T_{sat}c_{x,tp} + h_{lv}v_{tp})}{h_{lv}}dp
\tag{29}
$$

Combining with Eq. (19), we get

$$
\frac{\delta \dot{Q}}{\dot{m}_t} = h_{lv}\left[ \frac{dx_m\left(x + \Gamma(1-x)\right)^2 + x(1-x)d\Gamma}{\Gamma} \right] + \frac{(v_{lv}T_{sat}c_{x,tp} + h_{lv}v_{tp})}{h_{lv}}dp
\tag{30}
$$

or

$$
\frac{\delta \dot{Q}}{\dot{m}_t}\beta = h_{lv}dx_m + h_{lv}\beta\frac{x(1-x)d\Gamma}{\Gamma} + \frac{\beta h_{lv}v_{tp} + \beta c_{x,tp}v_{lv}T_{sat}}{h_{lv}}dp
\tag{31}
$$

with

$$
\beta = \frac{\Gamma}{(x + \Gamma(1-x))^2}
\tag{32}
$$

It is interesting to note that $\beta = 1$ when $\Gamma = 1$. Since

$$
dh_m = h_{lv}dx_m + c_{x,m}dT_{sat} + v_m dp
\tag{33}
$$

we finally get:

$$
\begin{aligned}
dh_m &= \beta\frac{\delta \dot{Q}}{\dot{m}_t} + \frac{(c_{x,m} - \beta c_{x,tp})v_{lv}T_{sat} + h_{lv}(v_m - \beta v_{tp})}{h_{lv}}dp \\
&\quad - \frac{x(1-x)h_{lv}\beta}{\Gamma}d\Gamma
\end{aligned}
\tag{34}
$$

where

$$
c_{x,m} = x_m c_{x,v} + (1 - x_m)c_{x,l}
\tag{35}
$$

and

$$
\begin{aligned}
c_{x,l} &= T_{sat}\left( \frac{ds_l}{dT_{sat}} \right)_{x_m=x=0} \\
c_{x,v} &= T_{sat}\left( \frac{ds_v}{dT_{sat}} \right)_{x_m=x=1}
\end{aligned}
\tag{36}
$$

Based on Eq. (34), we can introduce a factor $F$ which gives the relation between $\delta \dot{Q}$ and $dh_m$.

The arising of the factor $F$ dividing the heat rate is justified by the former mass balance. As a matter of fact, the time scales of the liquid and the vapor must be different because the phase velocities are not equal. Therefore, $F$ would act as a scaling factor between the two different time scales associated with the two-phases respectively. For an adiabatic two-phase flow, $dh_m = 0$, and we get

$$
\left[ (c_{x,m} - \beta c_{x,tp})v_{lv}T_{sat} + h_{lv}(v_m - \beta v_{tp}) \right]\Gamma dp = x(1-x)h_{lv}^2 \beta d\Gamma
\tag{39}
$$

As a result, for an adiabatic two-phase flow

$$
F = \frac{1}{\beta}
\tag{40}
$$

Note that, at low vapor flow qualities, the pressure drop and phase-change terms are negligible compared to the heat transfer term in the expression of $F$. Thus, we can say that $F \approx \Gamma$ which is the factor determined experimentally by Collado et al. [17] for such conditions at low $x$. Furthermore, when $\Gamma = 1$, we get $F = 1$. Now it is possible to write the expression of the entropy generation for the two-phase flow mixture:

$$
d\dot{S}'dz = \dot{m}_t ds_m - \frac{\delta \dot{Q}}{T_w F}
\tag{41}
$$

or

$$
d\dot{S}'dz = \frac{\dot{m}_t}{T_{sat}}dh_m - \frac{\dot{m}_t v_m}{T_{sat}}dp - \frac{\delta \dot{Q}}{T_w F}
\tag{42}
$$

By definition, $\delta \dot{Q}/F\dot{m}_t = dh_m$, and we get

$$
d\dot{S}'dz = \frac{\delta \dot{Q}}{T_{sat}F} - \frac{\delta \dot{Q}}{T_w F} - \frac{\dot{m}_t v_m}{T_{sat}}dp
\tag{43}
$$

For the same reasons explained in the previous section, we can write

$$
\underbrace{d\dot{S}'}_{\text{entropy generation}} = \underbrace{\frac{q^2 P}{h T_{sat}^2 F}}_{\text{heat transfer contribution}} + \underbrace{\frac{\dot{m}_t v_m}{T_{sat}}\left( -\frac{dp}{dz} \right)}_{\text{pressure drop contribution}}
\tag{44}
$$

which is the entropy generation rate (with the time scale of the liquid) per unit length. Again, we can determine the heat transfer and pressure drop contributions to the entropy generation. When

$\Gamma = 1$, Eq. (44) is equivalent to Eq. (27), as expected. This new expression is explicit and highlights the effect of the slip ratio on the entropy generation, not only in the d$p$ term but also in $F$. The term with d$\Gamma$ in the expression of $F$ (Eq. (38)) corresponds to the phase-change contribution which was not taken into account in Eq. (27). In that sense, this new expression based on the mixture model is more comprehensive than expression (27).

## 4. Applications

We now highlight the application of Eqs. (27) and (44) in practical saturated two-phase flow boiling configurations. Usually, it is accepted that the use of enhanced tubes for flow boiling is a good way to increase heat transfer compared to smooth tubes. Meanwhile, the pressure drops are much higher with enhanced tubes than with smooth tubes. So, one of the important questions is: Are enhanced tubes more suitable under all flow conditions?

Relations (27) and (44) have been applied to a smooth tube of 10 mm inside diameter and an enhanced tube of the same equivalent diameter, both having a length of 3 m. The heat transfer coefficient for the smooth tube has been calculated with the flow pattern based model by Wojtan et al. [18]. The pressure drops have been determined using the flow pattern based model by Moreno Quibén

and Thome [19] for smooth tubes. Regarding the enhanced tube, the model by Koyama et al. [20] has been used for calculating the heat transfer coefficient of the microfin tube. The pressure drops have been determined using the correlation by Kuo and Wang [21]. The area ratio ($Ar$) between the enhanced and the smooth tubes is arbitrarily set at 1.5. The void fraction has been determined using the Steiner [22] version of the Rouhani and Axelsson [23] drift flux void fraction model. Tube orientation is horizontal. As a result, the gravitational pressure drop is non-existent. The acceleration and frictional pressure drop have been taken into account.

### 4.1. Results

The results of the simulations using the separated flow model are shown in Fig. 3(a). The entropy generation has been plotted versus the mass velocity for R-134a at a saturation temperature of 10 °C for a heat flux of 10 kW/m$^2$. It clearly appears that for a mass velocity less than 245 kg/m$^2$ s, the use of the enhanced tube is more relevant because the entropy generation is lower. Actually, at low mass velocities, the benefit of the enhanced tubes comes from the heat transfer contribution whereas the pressure drop contribution remains small. At mass velocities greater than 245 kg/m$^2$s, the use of enhanced tubes is no more advantageous because the pressure drop penalty is too high.
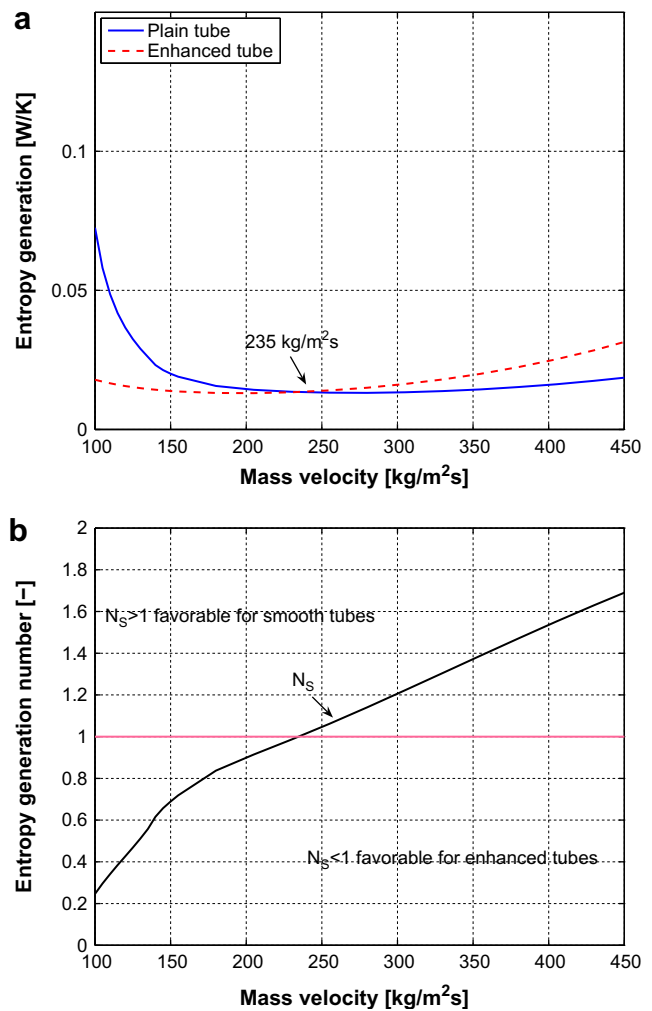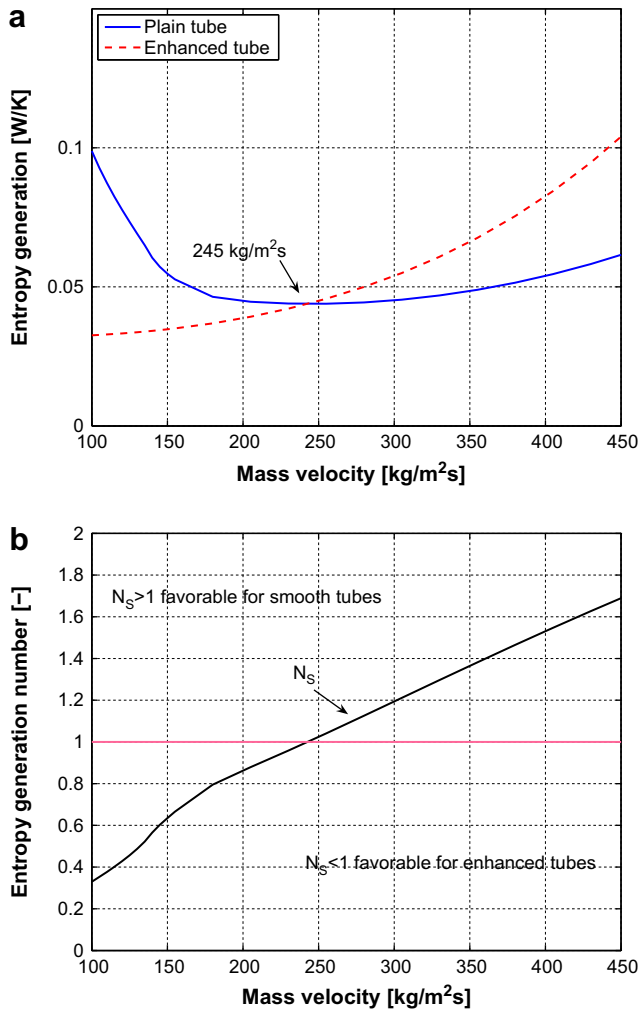


**Fig. 3.** Simulations using the separated flow model for R-134a, $T_{sat} = 10\,°C$, $x_{in} = 0.3$, $q = 10$ kW/m$^2$, $D = 10$ mm and $L = 3$ m. (a) Entropy generation for smooth and enhanced tubes as a function of the mass velocity. (b) Entropy generation number as a function of the mass velocity.
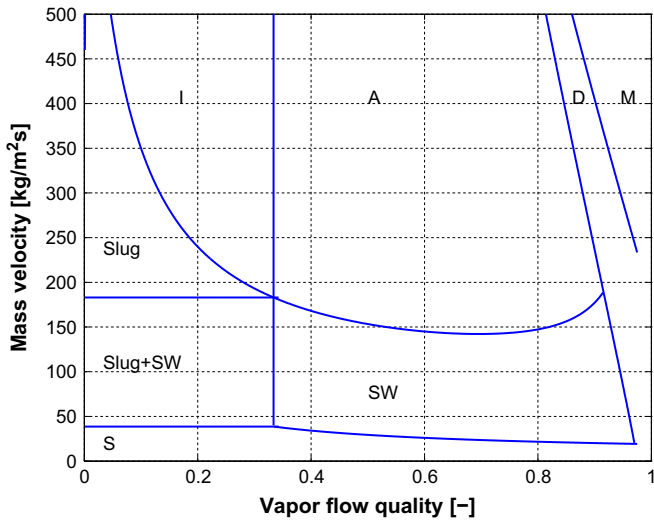
**Fig. 4.** Simulations using the mixture model for R-134a, $T_{sat} = 10\,°C$, $x_{in} = 0.3$, $q = 10$ kW/m$^2$, $D = 10$ mm and $L = 3$ m. (a) Entropy generation for smooth and enhanced tubes as a function of the mass velocity. (b) Entropy generation number as a function of the mass velocity.

**Fig. 5.** Flow pattern map for R-134a, $T_{sat} = 10\,^\circ C$, $q = 10\ kW/m^2$, $D = 10\ mm$ and $G = 250\ kg/m^2 s$ (Wojtan et al. [24]): S: Stratified; SW: Stratified wavy; I: Intermittent; A: Annular; D: Dryout; and M: Mist flow.

Another way to present the results could be the calculation of the entropy generation number. In our case, $N_S$ is defined as:

$$N_S = \frac{d\dot{S}'_{enhanced}}{d\dot{S}'_{smooth}} \tag{45}$$

Fig. 3b shows the results for this entropy generation number using the separated flow model. The same conclusions as before may be drawn.

Regarding the mixture model, simulations are presented in Fig. 4 for the same flow and geometric conditions as those of Fig. 3. It is clear that for a mass velocity less than 235 kg/m²s, the use of the enhanced tubes is more relevant because the entropy generation is lower.

Entropy generation is directly related to flow patterns. Indeed, heat transfer coefficients or pressure drops are based on the different flow patterns encountered during evaporation and are shown in Fig. 5:

- for $G < 170\ kg/m^2 s$, slug and stratified wavy, stratified wavy and dryout,
- for $G > 170\ kg/m^2 s$, slug, intermittent, annular, dryout and mist flow.

Describing each flow pattern based model for heat transfer and pressure drop is not the purpose of this work. However, it is important to understand that entropy generation is directly related to the flow configurations and therefore it is important to study the flow regimes and their transitions.

Heat transfer and pressure drop contributions are depicted in Fig. 6 using the Bejan number which is defined as:

$$Be = \frac{d\dot{S}'_{heattransfer}}{d\dot{S}'} \tag{46}$$

The Bejan number shows the importance of heat transfer contribution to the total entropy generation. We can see that at low mass velocity, both models give the same results. However, at high mass velocities, the pressure drop contribution for the mixture model is higher due to the phase-change contribution. As a consequence, the entropy generation per unit length will be larger. We also clearly see that at low mass velocity, entropy generation is mainly due to the heat transfer whereas at high mass velocity, entropy generation is mostly created by pressure drop, which is
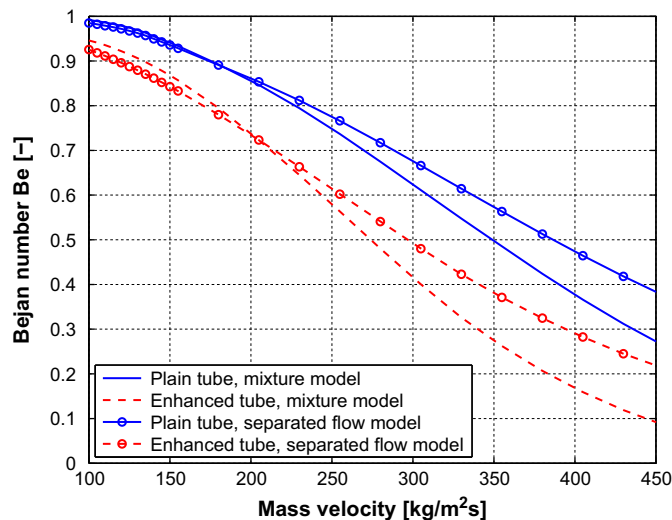




**Fig. 6.** Bejan number as a function of the mass velocity for both tubes and the two models.
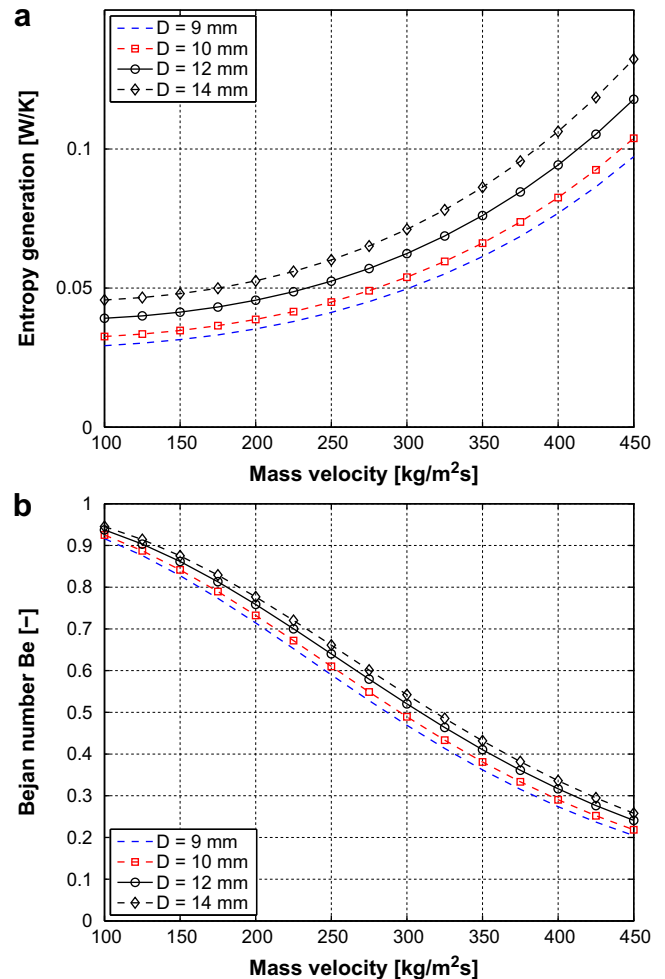
**Fig. 7.** Influence of the diameter using the separated flow model for R-134a, $T_{sat} = 10\,^\circ C$, $x_{in} = 0.3$, $q = 10\ kW/m^2$ and $L = 3$ m (enhanced tube). (a) Entropy generation versus mass velocity. (b) Bejan Number versus mass velocity.

higher for the enhanced tube. Finally, it follows that, enhanced tubes are a relevant solution for reducing entropy generation at low mass velocities whereas smooth tubes are better at higher ones.

The influence of the diameter on the entropy generation has been plotted in Figs. 7(a) and 8(a) for the enhanced tube. As it can be observed, a relevant solution for reducing the entropy generation is to decrease the diameter of the channel. This would promote the heat transfer coefficient much more than the pressure drop, yielding a decrease of the entropy generation. This increase of the heat transfer contribution is shown in Figs. 7(b) and 8(b). The Bejan number decreases when the diameter decreases. Investigations of diabatic saturated two-phase flow in mini-channels is thus relevant.

Beyond the meaning of these results, one should be aware that minimization of irreversibilities of an inherently irreversible process should also be judged at the system level. Therefore, this work should be extended to the system level in a second step. Nevertheless, these sample applications show the simplicity as well as the capability of the equations developed in this paper.

## 4.2. Uncertainty calculations

Uncertainty calculations related to the entropy generation are discussed hereafter. The respective equations are iteratively solved at the local saturation temperature corresponding to each
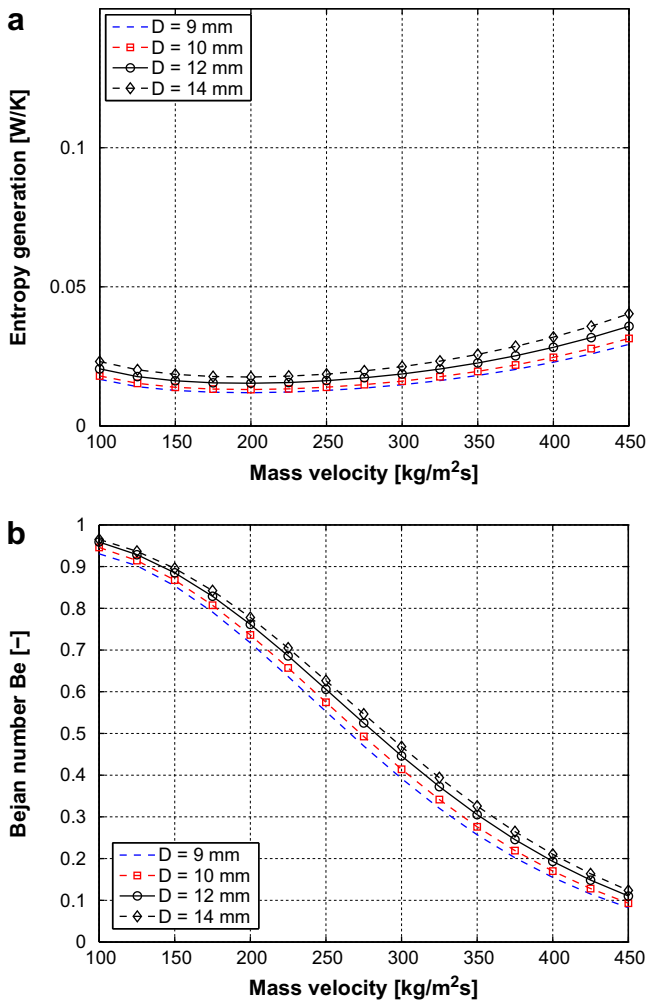


**Fig. 8.** Influence of the diameter using the mixture model for R-134a, $T_{sat} = 10\,°C$, $x_{in} = 0.3$, $q = 10\,kW/m^2$ and $L = 3\,m$ (enhanced tube). (a) Entropy generation versus mass velocity. (b) Bejan Number versus mass velocity.



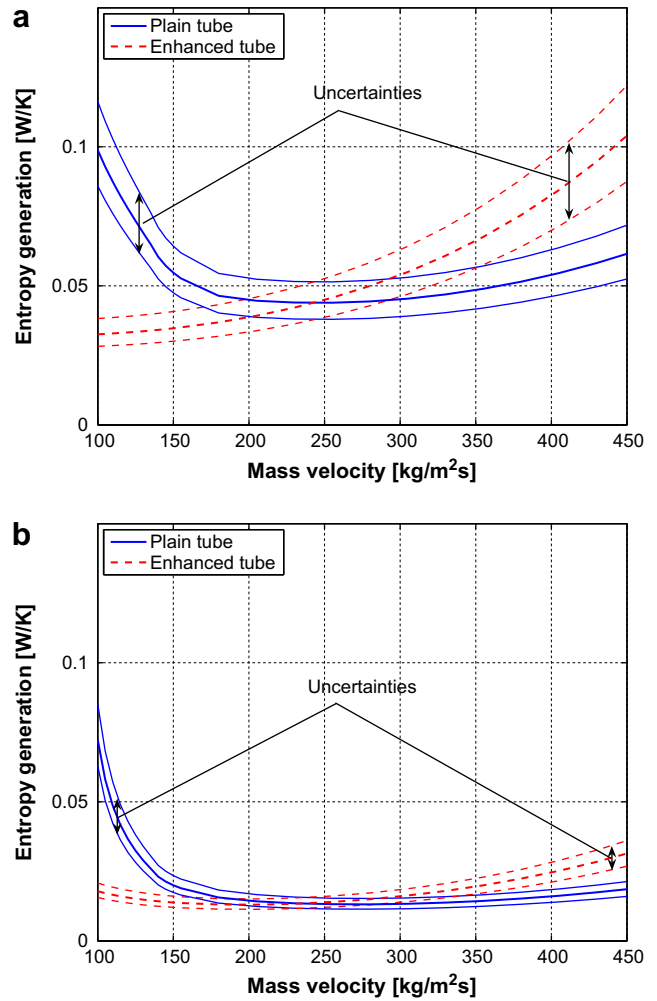**Fig. 9.** Uncertainty calculations for R-134a, $T_{sat} = 10\,°C$, $x_{in} = 0.3$, $q = 10\,kW/m^2$, $D = 10\,mm$ and $L = 3\,m$, using an uncertainty of $\pm 20\%$ on the heat transfer coefficient and the pressure drop. (a) Separated flow model. (b) Mixture model.

discretized cell, with a convergence criteria of 0.001 K. The local thermo-physical properties at the corresponding saturation temperature are successively used. The tube is discretized into 5000 elementary control volumes (which is sufficient for grid independent results) and typically, the solution converges in 3–4 cycles. The relevant uncertainty in the estimation of entropy generation comes only from the models or correlations used for the heat transfer coefficient and the pressure drop. These models are empirical methods, which usually exhibit an error of $\pm 20\%$. This typical uncertainty on the prediction methods has been simulated in Fig. 9. We see that, the uncertainty is lower for the mixture model than for the separated model. The uncertainty is really dependent on the models used for the pressure drop or the heat transfer coefficient calculation. The prediction methods used here are believed to be the current best methods available in the open literature.

## 5. Conclusions

A diabatic saturated two-phase flow of a pure fluid has been thermodynamically characterized based on two different approaches: the separated flow model using the classical flow quality and the mixture model using the thermodynamic vapor quality. Two expressions for the local entropy generation have been

proposed in this paper based on these two models. These equations show the heat transfer, pressure drop and phase-change (for the mixture model only) contributions to entropy generation. The developed expressions of the entropy generation also present many advantages, namely:

- They highlight the importance of accurately studying the heat transfer coefficients, the pressure drops and the void fraction, which allows the calculation of the slip ratio.
- They are local expressions which can be used in many geometries, e.g. smooth tubes, enhanced tubes, microchannels, etc.
- They can be used for evaporation or condensation.
- They propose an objective quantity for characterizing the geometries (using the same fluid) or choosing the fluids (using the same geometry).

Using these relations, it is shown that enhanced tubes may be a relevant solution for reducing entropy generation at low mass velocities whereas smooth tubes remain the best solution at higher ones. Besides, under certain operating conditions, decreasing the channel diameter would promote the heat transfer and reduce the entropy generation.

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