THERMO-HYDRODYNAMICS OF DEVELOPING FLOW IN A RECTANGULAR MINI-CHANNEL ARRAY

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ABSTRACT

Thermo-hydrodynamic performance of hydrodynamically and thermally developing single-phase flow in an array of rectangular mini-channels has been experimentally investigated. The array consists of fifteen rectangular parallel mini-channels of width 1.1±0.02 mm, depth 0.772±0.005 mm (hydraulic diameter 0.907 mm), inter-channel pitch of 2.0 mm, machined on a copper plate of 8.0 mm thickness and having an overall length of 50 mm. Deionized water used as the working fluid, flows horizontally and the test section is heated directly using a thin mica insulated, surface-mountable, stripe heater (constant heat flux boundary condition). Reynolds number between 200 and 3200 at an inlet pressure of about 1.1 bar, are examined. The laminar-to-turbulent transition is found to occur at Re ≈ 1100 for average channel roughness of 3.3 μm. The experimental pressure-drop under laminar (Re < 1100) and turbulent flow conditions (Re > 1100) closely match with the correlations in literature on developing flow. The experimental Nusselt numbers for both laminar and turbulent flow are found within satisfactory range of values estimated from theoretical correlations existing in the literature on developing flows. Thus, the study reveals that conventional theory, which predicts thermo-hydrodynamics of developing internal flows, is largely applicable for the mini-channels used in this study. No new physical phenomenon or effect is observed.

NOMENCLATURE

A_{cs}	Area of cross section (m^2)
C_p	Specific heat at constant pressure (J/kg·K)
D_h	Hydraulic diameter (mm)
f	Friction factor (Darcy)
G	Mass flux (kg/m^2s)
g	Acceleration due to gravity (m/s^2)
h	Heat transfer coefficient (W/m^2K)
k	Thermal conductivity (W/mK)
L	Length of the channel (m)
$\dot{m}$	Mass flow rate (kg/s)
P	Pressure (N/m^2)
$\dot{Q}$	Heat energy (W)
q''	Heat flux (W/m^2)
T	Temperature (K)
u	Velocity (m/s)
w	Width of channel (m)
X_h	Hydrodynamic entry length (m)
X_{th}	Thermal entry length (m)

Greek symbols

α	Height/width (aspect) ratio of rectangular channels
ρ	Mass density (kg/m^3)
μ	Dynamic viscosity (Ns/m^2)
ε	Surface roughness parameter (m)
λ	Wall thermal conductivity (W/mK)

Non-dimensional Numbers

Gz	Graetz number
Nu	Nusselt Number
Po	Poiseuille Number
Pr	Prandtl Number
Re	Reynolds Number

Subscripts

app	Apparent
avg	Average
cf	Constricted flow
f	Fluid
g	Gas
h	Hydraulic
l	Liquid
sat	Saturation
t	Tube, transition to turbulent
w	Wall
x	Distance from inlet

INTRODUCTION

In recent years, there is a rapid growth of applications which require high heat transfer rates and fluid flows in relatively small passages. Some examples which demand such flow conditions are electronics cooling, space thermal management, MEMS devices for biological and chemical analyses etc. The development of new applications requiring cooling of components in a confined space has motivated researchers to focus on the prediction of the thermo-hydrodynamic performance of mini and microchanels.

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Nominally, microchannels can be defined as channels whose dimensions are less than 1 mm and greater than 1 µm [1]. Currently most researched microchannels fall into the range of 30 to 300 µm. Channels having dimensions of the order of mm can be defined as minichanels. Despite the fundamental simplicity of laminar flow in straight ducts, experimental studies of mini/microscale flow have often failed to reveal the expected relationship between the transport parameters. Furthermore, data of simultaneously developing flows, which inherently provide high species transport coefficients, are not very abundant.

LITERATURE REVIEW

Prior contributions that are relevant to the present investigation are reviewed briefly. As regards the pressure drop characteristics of internal flows, from a historical perspective, Darcy (1857), Fanning (1877), Mises (1914), and Nikuradse (1933) did the pioneering work to relate the pressure drop in pipes to various parameters like relative roughness, Reynolds number, and transition from laminar to turbulent etc. Later Colebrook (1939) proposed a well known correlation. Moody (1944) presented the Colebrook's results in a graphical format correlating f_{darcy} as a function of flow Re (laminar and turbulent regions) over a relative roughness (ε/D) range of 0 to 5%. In laminar region, relative roughness shows very little effect on the overall pressure drop. In contrast, in turbulent region, f increases with Re and asymptotically reaches a constant value at higher Re; this asymptotic value increases with increasing relative roughness [2-5].

As discussed earlier, now the focus has shifted from macro sized channels/pipes to mini/micro counterparts. As regards heat transfer, the potential of microchannels in high heat flux removal application was first highlighted in 1981 when Tuckerman and Pease [6] studied the fluid flow and heat transfer characteristics in microchannels, and demonstrated that the electronic chips could be effectively cooled by forced convective flows of water through microchannels fabricated either directly in the silicon chip or in the circuit board. In their study a very high heat flux removal rate was obtained as due to small characteristic length of the micro-scale channels.

As regards friction factor in mini/micro systems, earliest study was reported in 1983. Shortly after the initial work of Tuckerman and Pease [6], Wu and Little [7] conducted experiments to measure flow friction characteristics alongside heat transfer characteristics of gases flowing in silicon/glass microchannels having trapezoidal cross-section with a D_h=55.81, 55.92 and 72.38 µm. Gases were used as test fluids (N_2, H_2, Ar); the measured values of f_{darcy} were larger (10–30%) than those predicted by the conventional theory. The authors concluded that the deviations are due to the large ε/D and its asymmetric distribution on the channel walls.

After these initial works, research was carried out in the next 10 years to study the flow and heat transfer in the microchannel or microtube. The potential of the microchannel cooling technology was confirmed and evaluated in those works. However, dissimilar and contradicting phenomena of flow and heat transfer in microchannels or micro-tubes were also observed in some of these works.

Peng et al. [8, 9] measured both the heat transfer and flow friction for single phase convection of water through rectangular microchannels having hydraulic diameters of 0.133-0.367 mm and distinct geometric configurations with aspect ratios of 0.333-1.0. Their measurements of both heat transfer and flow friction indicated that the laminar heat transfer ceased at a Reynolds number of 200-700. The results indicated that the geometric configuration had a significant effect on the single-phase convective heat transfer and flow characteristics. The laminar heat transfer was found to be dependent upon the aspect ratio and the ratio of D_h to the center-to-center distance of the micro-channels. The turbulent flow resistance was usually smaller than that predicted by classical relationships. The Re corresponding to flow transition to fully developed turbulent flow became much smaller than the ordinary channel flow (≈ 400-900). Empirical correlations were suggested for calculating both heat transfer and pressure drop.

Wang and Peng [10] also experimentally studied the forced flow convection of water and methanol in rectangular microchannels. They found that the fully developed turbulent convection was initiated at Reynolds numbers in the range of 1000-1500, and that the conversion from the laminar to transition region occurred in the range of 300-800. They also observed that the heat transfer behavior in the laminar and transition regions was quiet unusual and complicated, and strongly influenced by many factors such as liquid temperature, velocity, channel dimension. The results provide significant data and considerable insight into the behavior of the forced-flow convection in micro-channels.

Adams et al. [11] have experimentally investigated the single phase turbulent forced convection of water through circular channels of diameters 0.76 and 1.09 mm. They found that Nu for the microchannels are higher than those predicted by the traditional correlations for turbulent flows. Their data suggested that the extent of enhancement in the convection increased as the channel diameter decreased and the Reynolds number increased.

Wu and Cheng [12] conducted experiment on laminar convective heat transfer and pressure drop of deionized water in trapezoidal silicon microchannels having different geometric parameters, surface roughness, and surface hydrophilic properties. They found that laminar Nu and apparent friction constant depend greatly on different geometric parameters. The Nu and apparent friction constant both increase with the increase in surface roughness. The experimental results also showed that the Nu increases almost linearly with the Reynolds number at low Re ($Re < 100$), but increases slowly at $Re > 100$. They further developed dimensionless correlations for Nu and friction constant.

Agostini et al. [13] have done friction factor and heat transfer coefficient experiments with liquid flow of R134a in rectangular mini-channels. Two test sections made of aluminum multi-port extruded (MPE) tubes (channels dimensions were - Tube-1: 1.11-1.22 mm and Tube-2: 0.72-0.73 mm) were tested. Correlations available for large tubes in the literature were found to predict there results reasonable well. Laminar to turbulent transition occurred around Re ≈ 2000. Also, they pointed out the importance of estimating uncertainties in reporting data on mini-channels.

Morini [14] has summarized the experimental work in mini/micro channels ($D_h \leq 1.0$ mm) done till 2004. It is pointed out that in many cases the experimental data of Po and Nu disagree with the conventional theory but they also appear to be inconsistent with each other. Several reasons have been proposed to account for these differences. Rarefaction/ compressibility effects, viscous dissipation, electro osmotic effects, property variation effects, channel surface conditions (relative roughness and morphology) and experimental uncertainties have been invoked to explain the anomalous behavior of transport mechanisms in mini/micro channels.

Steinke and Kandlikar [15, 16] point out the importance of specifying exact boundary conditions for comparison of data from various sources. For example, in many studies, the heat flux is only applied to three sides of the channel, while, for comparative analysis, a uniformly applied heat flux is invoked. Also, since the heat transfer in microchannels is very large, the associated differential temperatures are small. They highlight the importance of accurate temperature measurements for estimating transfer coefficients. They also point out, as many others have done in the past that simultaneously developing flows are the most complex and much more accurate data is needed in this regime.

Hetsroni et al. [17, 18] have analyzed and reviewed a large body of data in circular, triangular, rectangular and trapezoidal mini/ micro channels with D_h from 60 µm - 2000 µm. They discuss the effects of geometry, axial heat flux due to thermal conduction through the working fluid and channels walls and energy dissipation in the fluid. They also discuss the entrance effects (inlet and outlet manifold design), effect of wall roughness, interfacial effects and measurement accuracy. As regards surface roughness, they also conclude that, as done by Schlichting [19] in the classical text, the presence of roughness on the wetted pipe surface favors an early laminar to turbulent flow transition. A need for a systematic approach to quantify the effect of surface roughness is also highlighted.

Reynaud et al. [20] have also undertaken pressure drop and heat transfer measurements of 2D mini-channels ranging from 300 µm to 1.12 mm. All their experimental results are largely in good agreement with classical theory of conventional channels. Some observed deviations are explained either by macroscopic effects (mainly entry and viscous dissipation) or by imperfections of the experimental apparatus.

Kandlikar et al. [21] and later Taylor et al. [22] point out that since the modern mini/micro fluidic systems routinely violate the 5% relative roughness threshold, as set forth by the classical works of Nikuradse, Moody etc. mentioned earlier, due to the inherent limitations of microfabrication techniques, there is a need to modify the Moody's Diagram. They propose a concept of D_{cf}, as follows:

$$D_{cf} = D_t - 2\varepsilon \tag{1}$$

Re and f were redefined based on the constricted flow diameter; Moody diagram was replotted by using the above new definitions. Later, they tested various mini/micro-channels and found that the transition from laminar to turbulent flow is seen to occur at Re well below 2100 because of roughness effects.

Caney et al. [23] tested a 1.0 mm^2 aluminum rectangular channel 420 mm long with flow Re ≈ 310 - 7790. They found that experimental Po and Nu show a good agreement with classical correlations for conventional channels.

Recently, Hrnjak and Tu [24] studied fully developed flow frictional pressure (Re ≈ 112 - 9180) in rectangular microchannels with D_h range of 69.5-304.7µm, height-to width ratio range of 0.09-0.24, and relative roughness range of 0.14–0.35%. In the laminar region, Poiseuille number (Po) of both liquid and vapor R134a flow in microchannels with smoother surfaces ($R_a/D_h < 0.3\%$) agree with the analytical solution based on the Navier–Stokes equation. The critical Reynolds numbers were found to be marginally smaller than the conventional values (i.e. Re ≈ 2300). In the turbulent region, the friction factors are found to be considerably larger than that predicted by the Churchill's (1977) equations for smooth tubes.

The relatively few works available in literature in the field of microscale thermal-hydraulics (especially on mini-channel regime, 3.0 mm ≥ D_h ≥ 200 µm) reveal contradictory conclusions and there are still important discrepancies between the results obtained by different researchers. This can be largely attributed to experimental uncertainties, effect of roughness, manifold design and control of boundary conditions in the experiment. Secondly, very limited combined fluid flow and heat transfer studies are available in literature for developing flow in mini-micro channels. No two models can be compared with each other because exactly matching sets of experimental results are also difficult to get in the literature. So, there is a need of complete series of data for simultaneously developing flow, which is not available in literature [25]. The aim of the present work is to fill this gap.

EXPERIMENTAL DETAILS AND PROCEDURE

The experimental facility is designed and constructed as illustrated schematically in Figure 1-a. The test section consists of an array of fifteen rectangular parallel mini-channels (w = 1.1±0.02 mm, d = 0.772±0.005 mm; D_h= 0.907 mm), machined on a copper plate of 8x92x132 mm^3 with each channel length = 50 mm. Channels are connected by inlet and outlet headers of 50x20x4 mm^3. Figure 1-b shows the actual photograph of the mini-channel test section. The grooved channels are covered by transparent polycarbonate sheet enabling flow visualization (boiling experiments were also done but not reported here) and aiding insulation from top of the test section. Channel roughness parameters are measured at different locations using laser surface profilometer and then averaged to estimate the effective roughness. The effective value of R_a is found to be 3.3 µm.

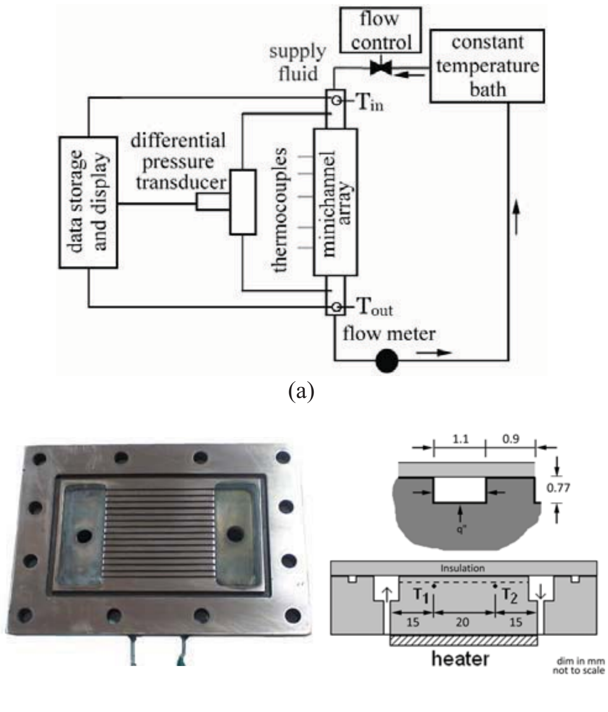


Figure 1. (a) SCHEMATIC LAYOUT OF THE EXPERIMENTAL SETUP (b) COPPER PLATE MINI-CHANNEL ARRAY (c) DETAILS OF THE MINI-CHANNEL ARRAY

A mica insulated strip-heater (50x50x1 mm^3) is attached below the copper plate using thermal paste to heat the incoming working fluid under constant heat flux condition. Compared to the channel size, the heater-block has a very large heat capacity. Thus, it is reasonable to assume that the heat flux on the test specimen is constant along the three sides of the channels. The top side is insulated, as shown in Figure 1 below.

The working fluid (distilled and deionized water) at a fixed temperature (maintained by a constant temperature bath) is allowed to pass through the mini-channel array via the inlet and outlet headers. A digital variac controls the power supply to the strip-heater. The fluid temperature at inlet and outlet of the test section are measured using two J-type thermocouples suitably located in the inlet and outlet headers. Two more J-type thermocouples are placed 15 mm from the both ends of channel, centrally along the channel length, in order to calculate the wall temperature, as shown in Figure 1-c.

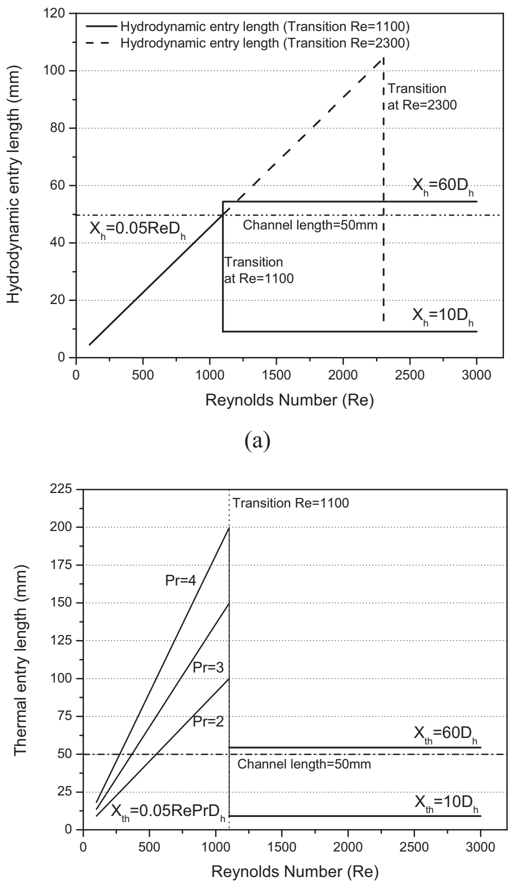
The pressure drop across the test section is measured using a differential pressure transducer (Honeywell: FP2000). It can measure in the range of 0-35000 Pa with an accuracy of 0.1% of full scale reading after calibration. Data acquisition is carried out using a PCI-DAQ (NI® TBX-68T) and is designed on LabView® platform. Proper insulations are provided wherever necessary so as to minimize the heat losses to the environment, which are found to be below 12 %.

RESULTS AND DISCUSSION

Pressure Drop

The test section dimensions and experimental parameters are chosen in such a manner that the flow in the channels is always developing (both hydrodynamically and thermally) either along the full length of the channel or at least for some length of the channel from the inlet. Figures 2-a, b, show the hydrodynamic and thermal entry length estimations with respect to the flow Re, based on standard equations [4], as noted on Figure 2. As per the classical theory, at Re ≈ 1100, the hydrodynamic entry length in laminar flow region is 49.89 mm, which is approximately equal to total channel length, i.e. 50 mm. If the laminar-to-turbulent flow transition is believed to take place at Re = 2300, then for all Re > 1100, the flow along the entire length of channel will be developing in nature. For Re < 1100, the flow will fully-develop somewhere inside the channel, as highlighted in Figure 2-a. Interestingly, experimental observations (please refer to the next section) indicate that the slope of the Po vs Re significantly changes at Re ≈ 1100, suggesting the inception of transitional flows i.e. a drift away from laminarity. Considering that the flow is indeed starting to get turbulent (transition to turbulent flow) at Re ≈ 1100, conventional theory suggests that the hydrodynamic entry length will then vary between $10 \cdot D_h$ to $60 \cdot D_h$. If the upper limit of $X_h = 60 \cdot D_h$ is considered valid, then flow along the entire length of channel will remain developing in nature for all Re > 1100. It should be noted, however, that Figure 2 can, at best, be treated as a theoretical guideline; in practical reality the boundaries between "developing" and "fully developed" may be fuzzy due to intrinsic perturbations and inherent system/hardware design limitations, which induces "non-ideal" flow characteristics.

As regards the thermal boundary layer development, since the applicable Prandtl number is in the range of Pr ≈ 3 - 4, under laminar flow conditions, flow is thermally developing along the entire channel length (refer Figure 2-b; unless for Re < 350, in which case the flow will be thermally developing



(a)

(b)

Figure 2. VARIATION OF (a) HYDRODYNAMIC ENTRY LENGTH (b) THERMAL ENTRY LENGTH WITH FLOW Re

for some length of the channel only). For the case of turbulent flow, there is a possibility that flow thermally fully develops within the channel length, as suggested by Figure 2-b. Typically, a length of $\geq 10 \cdot D_h$ is considered sufficient for full hydrodynamic and thermally turbulent flow development.

As regards total pressure drop, it is primarily due to friction only as the acceleration component is considered to be negligible and gravitational component is equal to zero.

For hydrodynamically developing flow, Poiseuille Number (Po) is not constant and can be calculated as follows:

$$Po_x = 2\Delta P / \rho u^2 x_+ \qquad (2)$$

where $x_+ = x / Re \cdot D_h$, the dimensionless axial distance in the flow direction for the hydrodynamic entrance region.

Since in the present range of experimental Reynolds numbers the flow will be a combination of "developing" and "fully developed", this local Poiseuille number is integrated along the applicable developing length and developed length, to find the net average Poiseuille number across the length of the channel considered, i.e.,

$$Po_{avg} = \frac{1}{L}\left(\int_0^{x_h} Po_x \cdot dx + \int_{x_h}^{L} Po_{developed} \cdot dx \right) \qquad (3)$$

Figure 3 shows the observed pressure drop in the array as a function of flow Re. The change of slope in the trend at Re ≈ 1100 is observed for all experiments. As noted earlier in Figure 2-a, the Reynolds number limit for the flow to remain

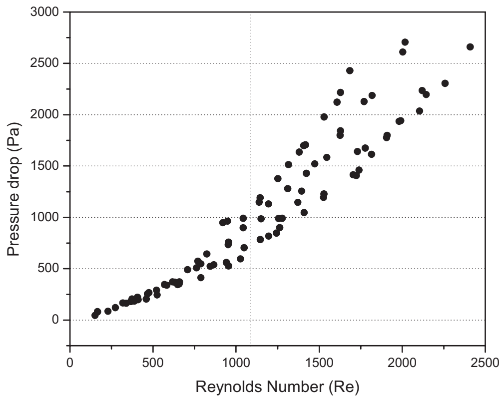


Figure 3. EXPERIMENTAL PRESSURE DROP vs FLOW Re

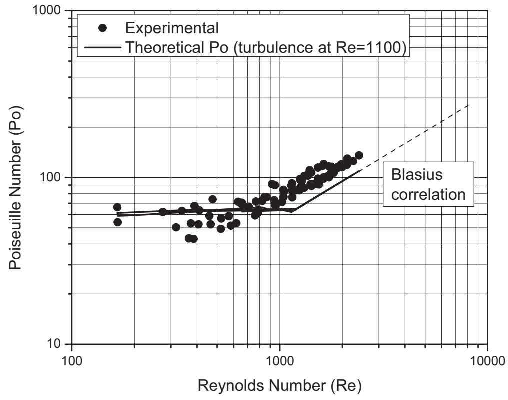


Figure 4. Po vs. Re FOR THE ARRAY

completely developing inside the entire channel length is also Re = 1100. Thus, this is a unique situation where the flow is simultaneously affected by the inception of laminar-to-turbulent flow transition and the development of the velocity boundary layer. At this stage, it is difficult to clearly differentiate between the two effects and assign explicitly discernable explanation to the change of slope at Re ≈ 1100. Also it is noted that there is considerable scatter in the data obtained after Re ≈ 1100.

The transition from laminar to turbulent flow has been reported to occur in mini-channels at Re considerably below 2300 (e.g., Wu and Little [7], Wang and Peng [10], Steinke and Kandlikar [16] etc.). Kandlikar et al. [26] conducted experiments with SS tubes and observed an early transition occurrence for ε/D_h of 0.355%. They suggested the following equations to describe the effect of roughness for fully developed flow conditions:

$$Re_{t,cf} = 2300 - 18750\left(\varepsilon / D_{h.cf}\right); 0 < \left(\varepsilon / D_{h.cf}\right) \le 0.08 \quad (4)$$

$$Re_{t,cf} = 800 - 3270\left(\varepsilon / D_{h.cf} - 0.08\right); 0.08 < \left(\varepsilon / D_{h.cf}\right) \le 0.15$$

where, $D_{h,cf} = D_h - 2\varepsilon$ (5)

Equivalent roughness, ε is estimated by:

$$\varepsilon = R_{pm} + F_p \quad (6)$$

where, R_{pm} is the average maximum profile peak height, and F_p is the floor distance to the mean line in roughness charts.

For the present experiments, $\varepsilon/D_{h,cf} = 0.0293$ application of Eq. 4 and 5 suggests transition at Re = 1750 which is somewhat less that the observed value of Re = 1100.

For fully developed laminar flow in circular channels, Poiseuille number (Po) remains constant and equals 64. For rectangular cross section, Poiseuille number decreases with increase in aspect ratio (α) of the cross section. For the microchannel under study, Po is found to be 58.4 [2].

The theoretical vs. experimental Po as a function of flow Re is shown in Figure 4. The Blasius correlation for fully developed turbulent flow [4] is also shown for comparison by considering the critical Re = 1100. In laminar flow regime (at low Re), flow will be fully developed for most of the part of the channel length and therefore the experimental Po is in reasonable range of the predicted values. As the flow Re increases, the flow development length inside the channel increases (as per Figure 2-a) and the deviation from the predicted fully-developed value is clearly seen. As the flow drifts away from laminarity, Po becomes a strong function of flow Re. With the simultaneous superposition of flow development phenomenon in the entire length of the channel after Re ≈ 1100 and incipience of transitional behavior, the experimental Po is greater than the fully developed turbulent behavior, as predicted by the Blasius correlation. As can be seen from the figure, it is expected that at very high Re values, with fully turbulent flows, the Po will indeed match the predictions by the Blasius correlation.

Since flow along the channel is developing up to certain length and fully developed thereafter, the experimental results are compared with correlations available for developing flow.

Shah [27] proposed a correlation to predict pressure drop in developing laminar flow in circular ducts:

$$Po_x = \frac{2\Delta P}{\rho u^2 x_+} = \left[13.74\sqrt{x_+} + \frac{1.25 + 64x_+ - 13.74\sqrt{x_+}}{1 + 0.00021\left(x_+\right)^{-2}}\right]\frac{1}{x_+}$$

where $x_+ = x / Re\,D_h$ (7)

A duct is classified as (a) Long duct if $x_+ \ge 0.06$, (b) Short duct if $x_+ \le 0.001$. Eq. (7) is applicable for long ducts only. Earlier Shapiro et al. [28] proposed a short duct correlation:

$$Po_x = 2\Delta P / \left(\rho u^2 x_+\right) = 13.74 / \sqrt{x_+} \quad (8)$$

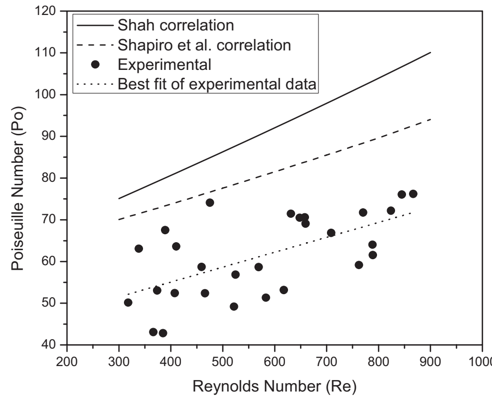


Figure 5. Po vs. Re FOR DEVELOPING LAMINAR FLOW

The developing laminar region in our experiments lie neither in long duct nor in short duct region. To the best of our knowledge no correlation exists in the literature for the range $0.001 \le x_+ \le 0.06$. As the hydraulic diameter increases, the working range for the same Reynolds number would have fallen under "short duct" domain. In such case the correlation proposed by Shapiro et al. [28], i.e. Eq. (8) need to be used. This correlation predicts less pressure drop than the correlation by Shah [27], i.e. Eq. (7). From experimental observation, as depicted in Figure 5, it is found that experimental Po is always less than theoretical Po given by Eq. (7) or (8). This deviation is well explained in the background of the fact that we are working with rectangular channels. As mentioned earlier, for fully developed flow in rectangular channels under consideration, Po =58.4 while for circular channels it is equal to 64, i.e., the former is about 8.5% less. This trend is maintained in developing flow also when compared to the predictions of Shapiro et al.

Heat Transfer

The local heat transfer coefficient for single-phase forced convection flow in mini-channels can be calculated using:

$$h = \dot{Q}_{conv} / A_{cs}\left(T_w - T_f\right) \tag{9}$$

where, T_f is the average of the inlet and outlet fluid temperature and $A_{cs} = 0.25\cdot\pi\cdot D_h^2$ is the area of cross-section of the channel. The corresponding Nusselt number is given by:

$$Nu = h \cdot D_h / k_f \tag{10}$$

Figure 6 depicts the complete experimental data set for the variation of local Nusselt number with Re, at two different Pr. Nusselt numbers are found to vary between 4 and 19; corresponding heat transfer coefficient varied from 2000-12000 W/m^{2}K. For high Re, in all the experiments, heat transfer coefficient at location 'T_2' is found to be lower than that at location 'T_1' (T_1, T_2 at x = 15 and 35 mm respectively; refer Figure 1-c). This is in accordance with the known fact that for developing flows under uniform heat flux condition, Nusselt number decreases along the length of the channel (For details refer to Figures 7 and 8 also).

An attempt is made to confirm experimental data with existing theory for thermally developing region. Experimental data in laminar region of thermally developing flow are compared with correlations proposed by Churchill and Ozoe [29], Sieder Tate [30], Stephan and Preuβer [31], and Shah and London [2]; all these available correlations are applicable for circular cross-section ducts with uniform heat flux.

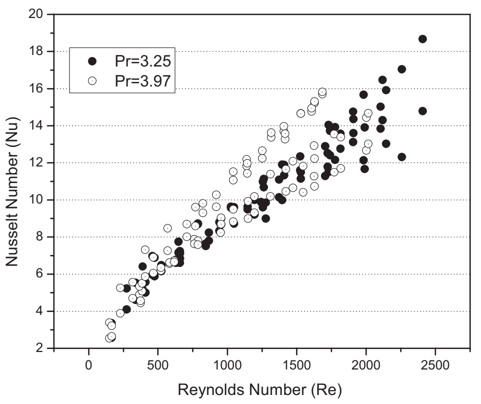


Figure 6. Nu_X vs. Re AT DIFFERENT LOCATIONS

Churchill and Ozoe [29] proposed a correlation for thermally developing flow in circular ducts with uniform heat flux:

$$\frac{Nu_x}{4.364\left[1+\left(Gz/29.6\right)^2\right]^{1/6}} = \left\{1+\left(\frac{Gz/19.04}{\left[1+\left(Pr/0.0207\right)^{2/3}\right]^{1/2}\left[1+\left(Gz/29.6\right)^2\right]^{1/3}}\right)^{3/2}\right\}^{1/3} \tag{11}$$

where, $Gz = (\pi/4)x_*$ and $x_* = x/(D_h \cdot Re \cdot Pr)$

Incropera and DeWitt [32] presented a correlation attributed to Sieder and Tate [30], which is of the form:

$$Nu = 1.86\left(Re\,Pr\,D_h/L\right)^{1/3}\left(\mu_f/\mu_w\right)^{0.14} \tag{12}$$

Stephan and Preuβer [31] proposed a correlation as follows:

$$Nu = 4.364 + \frac{0.086\left(Re\,Pr\,D_h/L\right)^{1.33}}{1+0.1Pr\left(Re\,D_h/L\right)^{0.83}} \tag{13}$$

Shah and London [2] proposed a correlation as follows:

$$Nu = 1.953\left(Re\,Pr\,D_h/L\right)^{1/3};\left(Re\,Pr\,D_h/L\right) \ge 33.3 \tag{14a}$$

$$Nu = 4.364 + 0.0722\left(Re\,Pr\,D_h/L\right); Re\,Pr\,D_h/L < 33.3 \tag{14b}$$

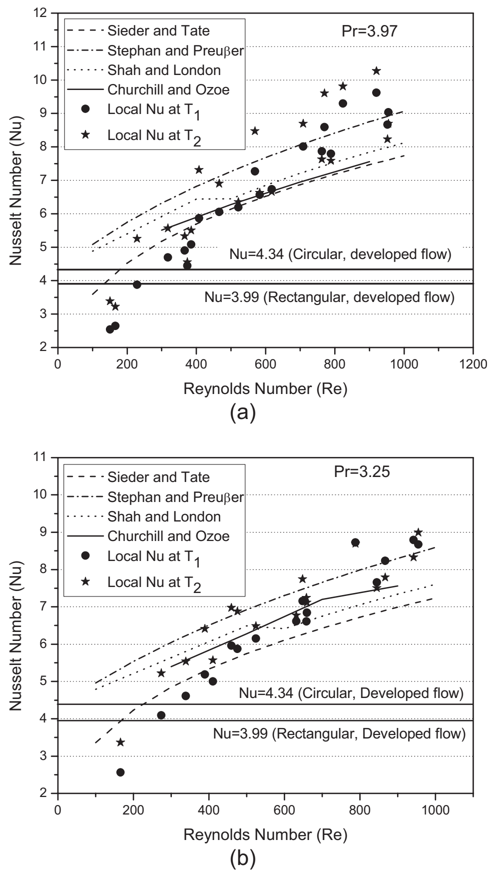


Figure 7. COMPARISION OF EXPERIMENTAL AND THEORETICAL VALUES OF Nu FOR LAMINAR FLOW (a) Pr = 3.97, (b) Pr = 3.25

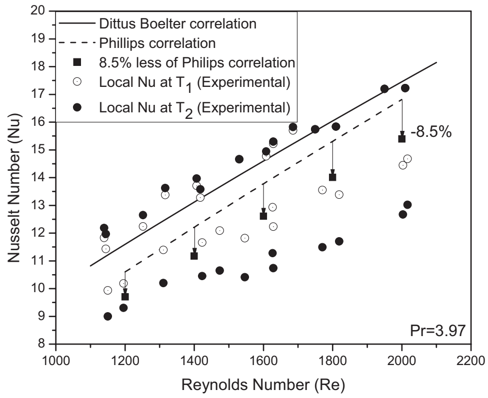
Comparison of these experimental values with theoretical Nusselt numbers calculated using Eq. 11-14 are presented in Figure 7-a, b respectively for Pr = 3.97 and 3.25. Figure 7-a, b suggest that present experimental values are in accordance with theoretical relations as in Eq. 11-14. Point to remember here is that the above used theoretical correlations are for circular tubes in contrast to rectangular channels used in present case.

Kays and Crawford [4] suggested a correlation to find Nusselt number for fully developed laminar flow in rectangular ducts with constant heat flux condition, i.e.,

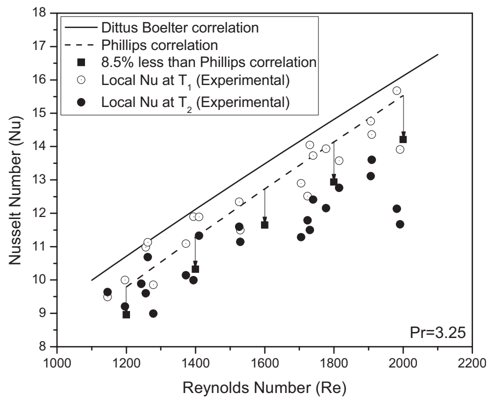
$$Nu = 8.235\begin{pmatrix} 1-1.883\alpha+3.767\alpha^2-5.814\alpha^3 \\ +5.361\alpha^4-2\alpha^5 \end{pmatrix} \tag{15}$$

where, α is the aspect ratio (=channel height/channel width). In the considered case α = 0.7, so using Eq. 15, one can find Nu = 3.99. For fully developed laminar flow in circular channels Nu = 4.34 [4].

Earlier it was shown that Poiseuille number for developed laminar flow for rectangular duct (Po = 58.6) is less than the value for circular duct (Po = 64) by 8.4%. Similarly, the Nusselt number for developed laminar flow in rectangular ducts is also less than that of circular ducts by 8.1%. Hence, our study shows a similar reduction in both Nusselt and Poiseuille number for simultaneously developing flow in rectangular ducts, in comparison to circular ducts of same hydraulic diameter.



(a)



(b)

Figure 8. COMPARISION OF EXPERIMENTAL AND THEORETICAL Nu FOR TURBULENT FLOW
(a) Pr 3.97 (b) Pr 3.25

For fully developed turbulent region in a circular duct, the value of Nu can be predicted using Dittus-Boelter correlation [32] as follows:

$$Nu = 0.023 \cdot Re^{0.8} \cdot Pr^{0.4} \tag{16}$$

Here, Nu is constant irrespective of the location as flow is fully developed.

Philips [33] suggested the following correlation to predict Nu in a circular duct in the turbulent developing region.

$$Nu_x = 0.012\left[1+\left(D_h/x\right)^{\frac{2}{3}}\right]\left[Re^{0.87}-280\right]Pr^{0.4} \tag{17}$$

The theoretical Nusselt number thus obtained is compared with the experimental value in Figure 8; a satisfactory match between the theoretical and experimental is obtained. The experimental results are more in compliance with Phillips correlation [33] than Dittus Boelter correlation [32] because Phillips correlation [33] incorporates the effect of thermally developing turbulent flow as well. It is clearly seen from the comparison of results presented in Figures 7 and 8 that heat transfer coefficient under turbulent conditions increased as compared to low Re, when laminar flow conditions existed.

SUMMARY AND CONCLUSIONS

Thermo-hydrodynamic study of simultaneously developing single-phase flow through a mini-channel array has been experimentally studied. The mini-channel array consists of fifteen rectangular parallel channels (w = 1.1±0.02 mm, d = 0.772±0.005 mm, and D_h= 0.907 mm), machined on a copper plate of 8 mm thick. Flow Re was varied from 200-3200 while flow Pr was maintained in the range of 3 - 4. The main conclusions of the study are:

(i) Developing flows provide very high heat transfer coefficients in the entrance regions and therefore of interest for mini/micro scale high heat flux removal applications.

(ii) In general, the conventional theory which predicts thermo-hydrodynamics of internal flows is well applicable for the channels used in this study. No additional physical effects were observed.

(iii) Experimental data suggests an early laminar to turbulent transition near Re ≈ 1100. This is primarily attributed to the channel roughness morphology and possibly by corner swirls generated due to the construction of inlet/outlet manifolds of the array.

(iv) The experimental and theoretical Po as well as Nu are well correlated with the available models for developing flow. Comparison was made with circular channels of equivalent diameter as well as with fully developed flow conditions.

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