INFLUENCE OF DROP SHAPE AND COALESCENCE ON DROPWISE CONDENSATION OVER TEXTURED SURFACES

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ABSTRACT

The authors of the present study have developed a mathematical model of dropwise condensation on the underside of horizontal and inclined surfaces [1-2]. The model is general enough to deal with a variety of fluids ranging from water to liquid sodium. The condensation process is seen to be quasi-periodic, starting from nucleation to growth and coalescence, followed by the movement of unstable drops that initiates fresh nucleation on exposed portions of the substrate. From the distribution of drops of various sizes, the instantaneous surface averaged heat transfer coefficient and wall shear stress are computed. Heat transfer rate is high through small drops and small under large drops. The largest drop diameter achieved depends on gravitational stability with respect to the interfacial forces at the three-phase contact line. Drop instability controls the periodicity of the condensation process and as a result, the heat transfer coefficient and wall shear stress. The developed model (i) simplifies drop shapes as spheroidal and (ii) assumes droplet coalescence process as instantaneous. Including additional details in the multi-scale condensation model makes it computationally intractable. To examine the consequence of these two simplifications on the overall dropwise condensation process, the sensitivity of the heat transfer coefficient and wall shear stress to drop deformation and coalescence has been studied in the present work. The shape of the drop on the underside of an inclined surface has been determined using a software tool based on the principle of minimization of potential and surface energies. Velocity and timescales of droplet coalescence are determined from high speed imaging. With the drop shape correctly determined, results show that improved wall shear stress and heat transfer coefficient are both smaller but the overall integrated predictions are not severely sensitive to these additional details. With coalescence, large heat fluxes attained for short time duration do not contribute much to the surface-averaged value. On the other hand, large shear stresses are momentarily created and these can impact the life of surface coatings.

KEY WORDS: Condensation; Measurement and Instrumentation; Heat Exchangers; Drop formation, Coalescence; Modeling

1. INTRODUCTION

Condensation occurs on a solid wall when the surface temperature falls below the local saturation temperature of the adjacent vapor. On specially treated surfaces, liquid droplets will appear at specific nucleation sites. As condensation proceeds, these droplets grow, coalesce with neighboring drops, and may fall-off or start to slide down the cold wall. The process is cyclic and drop instability and its lyophobicity will prevent the formation of a continuous liquid film. Such a phase-change process, termed dropwise condensation (Carey, [3]), is heterogeneous in which vapor condenses in the form of discrete liquid drops on or underneath a cold solid substrate. Dropwise condensation can be sustained only on specially textured surfaces (Rose, [4]; Vemuri et al., [5]; Rausch et al., [6]). The heat transfer coefficient during dropwise condensation can be quite high, for example, up to 30 times greater than the filmwise mode, when tested with Langmuir-Blodgett surfaces [4] and 5-20 times better when a promoter layer is used (Koch et al., [7]).

1 The phenomenon of coalescence is limited to the instant when two liquid interfaces merge; the events that follow are called post-coalescence, during which the newly formed mass from the original drops equilibrates in time. In this study, the term - coalescence is used in a wider sense, encompassing the post-coalescence till complete equilibrium is attained.

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Leaching of the coated or textured substrate can alter its wettability characteristics and result in aging, eventually altering the hydrodynamics of dropwise condensation. Apart from heat fluxes, it is thus important to know the nature of wall shear stresses created during drop movement.

Drops are expected to form at individual nucleation sites, while the area between the drops remains inactive with respect to condensation (Leach et al., [8]). The diameter of the smallest drop at nucleation can be estimated from thermodynamic considerations (Carey, [3]). Subsequently, for a given degree of subcooling, the drop grows by direct condensation at a rate determined by the interfacial heat transfer coefficient, conduction resistance to heat transfer through the drop, capillary resistance and substrate resistance, if any. Larger drops also grow by coalescence with their neighbors. When a certain critical size is reached, drops become gravitationally unstable, fall-off, or slide along the surface, wiping other drops along their path. Fresh nucleation sites are thus revealed and the condensation process repeats in a cyclic manner.

Vemuri and Kim [9] and Kim and Kim [10] developed mathematical models of dropwise condensation based on a single condensate drop combined with a population balance model for the entire surface. There are, however, some limitations while using this approach. The condensate drop was assumed to be hemispherical with a circular base. This approximation fails on inclined and variably textured surfaces and is further limited by the fact that drop instability cannot be correctly predicted (Annapragada et al., [11]). The effect of inclination was accounted for to an extent by the 2-circle approximation (El Sherbini and Jacobi [12]). Leipertz [13] discussed the importance of the drop shape on local heat transfer and instability.

The authors of the present study have developed a multiscale dropwise condensation model starting from the atomic scale nucleation, progressing towards the growth of individual droplets, coalescence, and drop instability, thus closing the dropwise condensation cycle [1-2]. Though the model is quite comprehensive, the drop-level treatment is simplified in the sense that its shape is determined by the 2-circle approximation. In addition, the process of coalescence is taken to be instantaneous. The objectives of the present study are to explicitly study drop deformation and the process of coalescence in the context of dropwise condensation. In addition, the sensitivity of wall heat transfer rates and shear stresses occurring during the dropwise condensation process to these two phenomena are examined.

2. MATHEMATICAL MODEL

The overall mathematical model described previously by the authors [1-2] comprises four primary steps (Fig. 1). These are, determination of three-dimensional drop shapes from a force equilibrium equation, expressions for the critical radius at drop instability (fall-off on horizontal substrates and slide-off/fall off on inclined substrates), correlations for heat flux and shear stress under an isolated moving drop, and simulation of the overall condensation cycle. Quantities of interest emanating from the model are the condensing droplet patterns, cycle times, area coverage of drops, instantaneous and time-averaged heat fluxes and wall shear stresses. The sensitivity of the data to an accurately estimated drop shape and the details of the coalescence process are interest in the present study. Sample condensation patterns underneath a horizontal surface and over a vertical surface are shown respectively in Figs. 2 and 3. Validation studies against experiments are discussed by the authors in [2].

2.1 Determination of the drop shape

The number of liquid drops involved in a dropwise condensation process is large. Moreover, several generations of drops must be tracked simultaneously to capture the process interactions. For computational simplicity, several authors have treated the drops being formed as part of a spheroidal frustum, with a circular base [3-6]. An extension of this approach is the use of the 2-circle approximation, which is particularly applicable for drops located on inclined surfaces. However, such a simplification in drop shape results in erroneous estimation of not only the critical diameter of the drop at instability, but also in the subsequent heat transfer rates and wall shear stresses induced under sliding conditions. Hence, it is of interest to examine the significance of the correctly determined three-dimensional shape of a drop on the quantities of interest.
Fig. 1: Flow chart of the mathematical model for multiscale simulation of dropwise condensation.

For a drop sitting on a horizontal surface (or underneath), the shape is determined by its equilibrium contact angle and volume, apart from material and interfacial properties. On an inclined surface, the unique contact angle is replaced by the distribution of the angle around the three-phase contact line. A variety of approximations have been reported for the contact angle variation (from advancing angle to receding angle), including linear and cosine functions. El Sherbini and Jacobi [12] conducted experiments to investigate the three-dimensional shape of a drop on and underneath inclined surfaces. Their results show that contact angle variation along the circumference of drop is best fitted by a third-degree polynomial in the azimuthal angle. The vertical cross-sectional profile was predicted by the 2-circle method sharing a common tangent, while the footprint was approximated as an ellipse. Many researchers have argued that the leading and trailing angles of a drop at criticality on or underneath an inclined substrate are equivalent to the advancing and receding contact angles respectively (Cheng et al. [14]). Their difference, namely the contact angle hysteresis, is a constant for given liquid-substrate combination at a given temperature. In the present study this hysteresis is prescribed and remains constant throughout the simulation. The variation of contact angle at the three-phase contact line is taken to be cubic function in the azimuthal angle.

The shape of a static drop supported on a solid surface is governed by the Young-Laplace equation that balances weight, surface tension, and internal pressure (Pozrikidis [15]). In three dimensions, the equation is difficult to solve and alternative approaches are preferred. A variational approach has been proposed to compute the three-dimensional drop shape, wherein the overall energy of the drop (sum of pressure, gravitational, and interfacial energy) is successively minimized. This step has been achieved using Surface-Evolver® (Brakke, [16]; Santos and White [17]), an open source software. Complete information of the three-dimensional shape and its footprint can be extracted under equilibrium conditions. The overall numerical methodology is as follows: To initiate the solution, an imaginary cube of liquid of a given volume is taken and its overall energy is minimized to derive the drop shape under equilibrium conditions. In order to provide the surface energy at the three-phase contact line, the variation of contact angle as a function of the azimuthal angle $\phi$ needs to be specified. The following contact angle variation proposed by El Sherbini and Jacobi [12] has been adopted:
\[
\theta(\phi) = 2\left(\frac{\theta_{\text{adv}} - \theta_{\text{rcd}}}{\pi}\right)\phi^3 - 3\left(\frac{\theta_{\text{adv}} - \theta_{\text{rcd}}}{\pi^2}\right)\phi^2 + \theta_{\text{adv}}
\]  

(1)

Here, \(\theta_{\text{adv}}\) is the advancing angle, \(\theta_{\text{rcd}}\) is the receding angle and \(\phi\) is the azimuthal angle. The steps for obtaining the shape of three-dimensional non-symmetric drops using Surface-Evolver® have been discussed by the authors elsewhere (Bhutani et al., [18]; also see [19]). The correct drop shape serves as an input to the determination of the critical size beyond which the drop would fall off from a horizontal surface or slide away on an inclined surface.

Typical footprints of drops on a vertical surface as a function of volume are shown in Fig. 4. The footprint is seen to become increasingly non-circular at higher volumes.

Fig. 2: Spatio-temporal drop distribution from initial drop formation to first fall-off of drop during dropwise condensation of water vapor at 303 K with sub-cooling of \(\Delta T = 5\) K underneath a horizontal substrate of contact angle 90º. Nucleation site density is \(10^6\) cm\(^{-2}\). Fall-off is seen to occur at about 50.2 min.
Fig. 3: Spatio-temporal distribution of drop during dropwise condensation of water vapor at 303 K with subcooling of $\Delta T = 5$ K along a vertical substrate. Contact angle hysteresis is $17^\circ$ and the size of the substrate is $20 \text{ mm} \times 20 \text{ mm}$. Nucleation site density is $10^6 \text{ cm}^{-2}$.

Fig. 4: Effect of volume on footprint of a water drop at equilibrium on a vertical surface; advancing angle $110^\circ$ and contact angle hysteresis $20^\circ$. 
2.2 Modeling of sliding drop underneath an inclined substrate

Once the drop becomes unstable, it is set in motion along (inclined) or away (horizontal) from the surface. Sliding instability is likely to occur first at smaller drop sizes because gravity overcomes contact angle hysteresis rather than the surface tension component related to the contact angle itself. For fall-off, gravity has to overcome adhesion generated by the average contact angle at the 3-phase contact line. During sliding, the fluid medium forming the drop is set in motion, generating a circulation pattern within. These flow details are needed to determine local and global wall heat transfer rates and shear stresses over a part of the condensation cycle.

Flow and heat transfer are computed within the moving deformed drop by solving the appropriate equations of motion in three dimensions. The frame of reference for flow and transport calculations is fixed within the liquid drop, the wall moving relative to it at a constant speed. These unsteady three dimensional calculations are required for each drop moving over the surface within the overall condensation cycle. Such calculations are computationally expensive. As an alternative, simulations have been carried out at the scale of individual drops by solving the Navier-Stokes and energy equations. Correlations of space-averaged skin friction coefficient and Nusselt number are derived from these simulations and embedded in the condensation program. Correlations are expressed as a function of Reynolds number and contact angle for various Prandtl number fluids and serve as input to the overall condensation model depicted in Fig. 1.

The skin friction coefficient averaged over the entire surface, by taking the true three-dimensional shape of the droplet, is obtained as:

\[ \bar{C}_f = 64.2 \, Re^{0.97} \theta_{avg}^{1.58} \]  

(2)

In the 2-circle approximation, the average skin friction coefficient is obtained as:

\[ \bar{C}_f = 58 \, Re^{-0.97} \theta_{avg}^{1.57} \]  

(3)

Both correlations show a near reciprocal dependence of skin friction coefficient on Reynolds number. This trend is qualitatively similar to fully developed flow in internal geometries. Equations 2 and 3 show that skin friction decreases with increasing contact angle, namely the hydrophobicity of the surface. Thus, highly hydrophobic surfaces would be less prone to wear arising from droplet motion over them. The older approach (Equation 3) estimates the average skin friction coefficient to be smaller in comparison to that obtained by taking the true shape of the droplet (Equation 2) obtained by Surface-Evolver®. This result can be traced to the fact that drop deformation on an inclined surface is truly three-dimensional. Hence, the distortion of fluid movement within the drop is greater, increasing the wall shear stress.

In the range of \(3.5 < Pr < 7\), the old and the new correlations for drop level heat transfer are obtained as:

Present: \(\overline{\text{Nu}}_{id} = 12.4 \, Re^{0.166} \, Pr^{0.1} \theta_{avg}^{0.77} \)  

(4)

Previous: \(\overline{\text{Nu}}_{id} = 9.48 \, Re^{0.166} \, Pr^{0.1} \theta_{avg}^{0.77} \)  

(5)

The dependence of the average heat transfer coefficient on Reynolds number is seen to be quite mild. The dependence of heat transfer coefficient on the drop shape is strong through the contact angle. Nusselt number prediction in the improved approach (Equation 4) with Surface-Evolver® is greater than the 2-circle approximation (Equation 5). The reason for this difference is again due to the truly three-dimensional drop shape that distorts the flow field and augments wall shear stress and thus, the heat transfer.

Table 1 summarizes the condensation parameters on a horizontal, 45° inclined, and a vertical surface with a subcooling of 2 K. For a horizontal surface, pendant drops are formed that experience fall-off instability at a critical radius of \(r_{max}\) as noted in column 3. For the other two configurations, the critical drop size is given by \(r_{crit}\). The previous and the present approaches in drop shape calculations are compared. The present approach is seen to generate larger drops with larger area of coverage. Fall-off and slide-off instabilities are delayed. The present approach yields smaller average Nusselt numbers owing to enhanced conduction resistance of the larger drops. The surface and cycle-averaged skin friction coefficient is smaller because of a longer time period for which drops grow and are therefore, stationary, for longer periods.
Table 1: Comparison of condensation parameters for water vapor with the old and the new approaches of determining the drop shape. The skin friction coefficient and Nusselt number are cycle and area-averaged.

<table>
<thead>
<tr>
<th>Approach</th>
<th>Substrate orientation</th>
<th>$r_{\text{max}}$ ($r_{\text{crit}}$) (mm)</th>
<th>Area of coverage (%)</th>
<th>Time for first fall/slide-off (min)</th>
<th>Time elapsed between successive sliding events (min)</th>
<th>$C_f$ (skin friction coefficient)</th>
<th>$\text{Nu}$ (Nusselt number)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-circle approximation</td>
<td>0°</td>
<td>4.25</td>
<td>78</td>
<td>57</td>
<td>5.9</td>
<td>0</td>
<td>220</td>
</tr>
<tr>
<td></td>
<td>45°</td>
<td>2.12</td>
<td>72</td>
<td>42</td>
<td>1.7</td>
<td>$0.122 \times 10^{-4}$</td>
<td>290</td>
</tr>
<tr>
<td></td>
<td>90°</td>
<td>1.49</td>
<td>64</td>
<td>15</td>
<td>0.77</td>
<td>$0.987 \times 10^{-4}$</td>
<td>430</td>
</tr>
<tr>
<td>Present (Surface-Evolver®)</td>
<td>0°</td>
<td>4.32</td>
<td>82</td>
<td>59.2</td>
<td>6.35</td>
<td>0</td>
<td>187</td>
</tr>
<tr>
<td></td>
<td>45°</td>
<td>2.60</td>
<td>76</td>
<td>43.5</td>
<td>1.98</td>
<td>$0.961 \times 10^{-5}$</td>
<td>247</td>
</tr>
<tr>
<td></td>
<td>90°</td>
<td>1.54</td>
<td>67</td>
<td>21</td>
<td>0.9</td>
<td>$0.902 \times 10^{-4}$</td>
<td>380</td>
</tr>
</tbody>
</table>

2.3 Condensation patterns

The spatio-temporal drop distributions, from initial nucleation to drop instability are shown in Figs. 5(a-b) for condensation of water vapor over a vertical surface. The 2-circle approximation and the analysis based on the 3D drop shape are compared. The drop shape is characterized by an advancing angle ($\theta_{\text{adv}}$) = 110° and contact angle hysteresis ($\Delta\theta$) = 20°. The saturation temperature is 315 K and the degree of subcooling between the vapor and the substrate is 2 K. Figs. 5(a) and 5(b) show that the average footprint of the drop over the substrate at a given time instant is noticeably greater in the present approach, when compared to the 2-circle approximation.

Condensation patterns are time-dependent and best discussed in terms of statistical properties. The effect of drop shape on instantaneous drop coverage area, available exposed nucleation sites, wall heat transfer and wall shear stress are summarized in Figs. 6 (left) and 6 (right). Fig. 6(left-a) shows the area of coverage obtained by the modified approach is higher than the 2-circle approximation. Fig. 6(left-a) also shows that the area of coverage builds up when drops grow by condensation and coalescence, but decreases sharply as those drops which have attained criticality slide away on the substrate, cleaning the substrate with more drops coming in their way. These trends are also visible in Fig. 5. Since the present detailed approach results in larger drops to be formed, the first fall-off/slide-off is delayed (see inset, Fig. 6(left-a)). Fig. 6(left-b) shows the variation of available nucleation sites with respect to time. It is determined from the open area of the substrate where nucleation sites are present but are not covered by the growing drops. Nucleation sites are also revealed during coalescence and falloff/slide-off events. The drop coverage area and available nucleation sites change rapidly at early times, eventually reaching a cyclic state later, with a well-defined average.

The temporal variation of heat transfer through the entire condensing substrate is presented in Fig. 6(right-a). At an early stage of condensation, drops are small and offer small conduction resistance. Hence, their growth rates are high, and a large heat flux passes through the substrate. At later times, a cycle of drop formation, merger, and subsequent departure (due to droplets attaining criticality) is obtained leading to a dynamic steady-state in heat transfer about a time-average. Fig. 6(right-a) shows that the instantaneous heat transfer rate obtained with the improved approach is less than that computed with the 2-circle approximation. The reduction in heat transfer rate arises primarily due to the fact that larger drops get formed at criticality, as noted earlier.
Temporal variation of wall shear stresses during drop motion over the condensing substrate is shown in Fig. 6(right-b). Unlike heat transfer, shear stresses are non-zero only during drop motion. Hence, in the new approach, shear stresses are affected by a higher coefficient of skin friction but a smaller time for motion since drops are stationary for a longer period. Hence, with respect to the prediction of the drop shape, the trend here is not conclusive. The shear stresses determined using the present approach for the drop shape show marginally lower values at certain time instants within the condensation cycle.

Time-averaged Nusselt number and average skin friction coefficient for the entire surface are shown in Fig. 7. The surface orientation is vertical. Fig. 7(a) shows the variation of average Nusselt number with respect to degree of sub-cooling. The condensation parameters are selected as in Fig. 5, except that the degree of subcooling is an independent variable. The variation of average skin friction coefficient with respect to degree of subcooling is shown in Fig. 7(b). The effect of increasing subcooling is to increase the rate of condensation, increase the growth rates of drops, reduce the cycle time and increase the number of drops sliding at any given instant. Hence, Nusselt number and skin friction coefficient monotonically increase with subcooling. The Nusselt number and average skin friction coefficient with the present approach are less than the earlier approach of estimating the drop shape with 2-circle approximation because the corresponding drop volume at criticality is greater.

Fig. 5 Drop distributions from initial nucleation to the instance of first drop slide-off on a vertical surface for condensing fluid water. Drop shape determined by (a) 2-circle approximation (previous) and (b) present. Advancing angle 110°, hysteresis = 20° and degree of sub-cooling 2 K.
Fig. 6 LEFT (a) Variation of area of coverage with respect to time by the present approach compared with the 2-circle approximation [12] while inset shows a larger view of commencement of the first slide-off; (b) cyclic variation of available nucleation sites with respect to time.

Fig. 6 RIGHT (a) Cyclic variation of instantaneous wall heat transfer rate during condensation of water, and (b) wall shear stress. Advancing angle 110°, hysteresis = 20°, subcooling = 2K, orientation = vertical.

Fig. 7 (a) Variation of average surface Nusselt number with respect to the degree of sub-cooling and (b) Variation of average surface skin friction coefficient with respect to sub-cooling for condensation of water vapor on a vertical substrate. Advancing angle 110°, hysteresis = 20°.
3. COALESCEANCE OF TWO DROPS

The discussion here is on the merger of two small liquid drops located on a textured lyophobic surface. We restrict our discussion to dropwise condensation on (i) a vertical surface and (ii) one where the outward normal has at least one component in the direction of the gravity vector. The drops are composed of the same liquid while the coalescence process takes place in a vapor environment. The drops are not pinned anywhere at the three-phase contact line and the footprint of the resulting coalesced drop evolves with time. In the discussion below, it is assumed that the droplet coalescence process studied under ambient conditions (in quiescent air) will carry over to quiescent vapor condensation over a cold substrate. In the context of dropwise condensation, the coalescence process is responsible for introducing an additional velocity in the liquid mass, length scale due to merger, and timescales due to release and dissipation of excess surface energy; these additional process attributes can be suitably incorporated in the mathematical model of dropwise condensation.

It is now understood that drops merge with the formation of a bridge between the respective liquid interfaces. The bridge evolves in time, eventually leading to a unified interface of the combined droplet. The liquid velocities generated at this early stage, in conjunction with the excess available free energy due the coalescence, may deform the interface further. The magnitude of initial velocity generated depends on the internal pressure difference across the drops. Thus, factors such as drop volume and contact angle, which dictate the interface curvature, become determining factors at early times, immediately following coalescence. On a long term basis, the contact angle at the three-phase contact line will approach the equilibrium value, while bulk fluid motion generated due to excess free energy gets dissipated by viscosity. The total time for post-coalescence equilibrium will, usually, correspond to the timescale of bridge growth added to the timescale of relaxation – either in the bulk by viscosity or dissipation at the three-phase contact line due to its motion in the process of reaching the new equilibrium.

Coalescing liquid drops will experience a variety of forces whose relative magnitudes change with time. These forces are related to surface tension, gravity, viscosity, and inertia. Derived quantities such as pressure are decided by these primary forces. The ratio of a pair of forces will generate dimensionless parameters that delineate the relevant limiting mechanisms. Apart from fluid and interfacial properties, the drop size is an important parameter. It is to be expected that surface tension would be strong particularly in comparison to gravity and inertia forces when the drop dimension is small. Though not distinguishable in terms of dimensionless parameters, gravity forces are expected to be more important for pendant drops than sessile. In the following discussion, the drops are taken to merge under ambient conditions. Hence, surface tension would be that appearing at the liquid-air interface. Further, density difference between the pair of fluids is practically equal to the liquid density.

Let $R$ be a length scale and $U$, a velocity scale; in the discussions below, $R$ is taken to be the characteristic dimension of the footprint of the drops after coalescence. Symbols $\rho$, $\sigma$, and $\mu$ refer to density, surface tension, and the dynamic viscosity. For the coalescence of two stationary drops, the velocity scale is implicit in the process and cannot be independently prescribed. Various choices of $U$ are discussed in the following section. Similarly, the process does not have a single natural timescale. The quantities of interest to the present study are various timescales appropriate for the respective forces leading to the total time of coalescence for a pair of drops. Forming dimensionless groups, we have the following numbers:

- **Inertia - Surface tension, Weber number:**
  \[ We = \left( \frac{\rho U^2 R}{\sigma} \right) \]

- **Inertia - Gravity, Froude number:**
  \[ Fr = \frac{U^2}{gR} \]

- **Inertia - Viscosity, Reynolds number:**
  \[ Re = \frac{\rho U R}{\mu} \]

- **Gravity - Surface tension, Bond number:**
  \[ Bo = \left( \frac{\rho g R^2}{\sigma} \right) \]

- **Viscosity - Surface tension, Ohnesorge number:**
  \[ Oh = \frac{\mu}{\sqrt{\rho R \sigma}} \]
Table 2: Magnitudes of dimensionless parameters arising in the coalescence of two water drops under ambient conditions; velocity scale $U$ is taken as $\sigma/\rho R^{0.5}$ where $R$ is the base radius.

<table>
<thead>
<tr>
<th>Volume, $\mu l$ ($U, m/s$)</th>
<th>Base radius, (mm)</th>
<th>$We$</th>
<th>$Fr$</th>
<th>$Re$</th>
<th>$Bo$</th>
<th>$Oh$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.06 (0.48)</td>
<td>0.306</td>
<td>1</td>
<td>76.50</td>
<td>183</td>
<td>0.013</td>
<td>$5.46 \times 10^{-3}$</td>
</tr>
<tr>
<td>0.60 (0.33)</td>
<td>0.659</td>
<td>1</td>
<td>16.50</td>
<td>268</td>
<td>0.060</td>
<td>$3.73 \times 10^{-3}$</td>
</tr>
<tr>
<td>1.0 (0.30)</td>
<td>0.782</td>
<td>1</td>
<td>11.73</td>
<td>292</td>
<td>0.085</td>
<td>$3.40 \times 10^{-3}$</td>
</tr>
<tr>
<td>2.0 (0.27)</td>
<td>0.985</td>
<td>1</td>
<td>7.38</td>
<td>328</td>
<td>0.135</td>
<td>$3.00 \times 10^{-3}$</td>
</tr>
<tr>
<td>4.0 (0.24)</td>
<td>1.240</td>
<td>1</td>
<td>4.66</td>
<td>368</td>
<td>0.213</td>
<td>$2.70 \times 10^{-3}$</td>
</tr>
<tr>
<td>6.0 (0.22)</td>
<td>1.420</td>
<td>1</td>
<td>3.55</td>
<td>394</td>
<td>0.289</td>
<td>$2.50 \times 10^{-3}$</td>
</tr>
<tr>
<td>16.0 (0.19)</td>
<td>1.969</td>
<td>1</td>
<td>1.85</td>
<td>464</td>
<td>0.539</td>
<td>$2.16 \times 10^{-3}$</td>
</tr>
</tbody>
</table>

Experiments described in Section 3.2 are with water under ambient conditions. For water at 30ºC, the fluid properties are: $\rho = 995.6$ kg/m$^3$; $\mu = 79.77 \times 10^{-3}$ Pa·s; $\sigma = 70 \times 10^{-3}$ N/m. For a liquid drop of volume $V$ and contact angle $\theta$, the base radius $R$ under equilibrium conditions is approximately determined by the formula,

$$V = \frac{\pi}{3} r^3 (2 - 3 \cos \theta + \cos^3 \theta); \quad R = r (1 - \cos^2 \theta)$$

(8)

For the experiments discussed in the following sections, $\theta = 90^\circ$. Hence,

$$R = \left(\frac{3\pi}{2} V\right)^{1/3}$$

(9)

To estimate the dimensionless numbers defined above, the velocity scale can be taken as $\sigma/\mu$, $(\sigma/\rho R)^{0.5}$ or $(gR)^{0.5}$. Since the drop sizes in condensation before coalescence or instability are expected to be in the range of 1-2 millimeters, surface tension is expected to be uniformly important and the third definition is not considered further. The characteristic velocity for water using $\sigma/\mu$ is 87.7 m/s. This is quite large and has not been seen in experiments of the present work. The velocity scale based on $(\sigma/\rho R)^{0.5}$ is around 0.2 m/s and matches experimental observations. Hence, it has been adopted in further analysis. In addition, the choice of this scale makes Weber number unity and $Oh = 1/Re$. The dimensionless numbers arising in the present experiments are summarized in Table 2.

Since $(\sigma/\rho R)^{0.5}$ is an appropriate velocity scale, Table 2 reveals that drop oscillations during coalescence are expected to be driven by inertia and surface tension. Here, Reynolds number is high and viscosity is expected to play a secondary role. Between surface tension and gravity, the former is of greater significance. In agreement with this observation, Froude number is also seen to be on the higher side. Thus, the experiments of the present study are controlled by inertia and surface tension as long as velocities generated are concerned, viscosity serving the purpose of damping fluid motion over a longer time span. Additional scales of interest are the following. For small liquid drops in a gaseous environment, the importance of gravity over surface tension is determined by the capillary length

$$l = \sqrt{\frac{\sigma}{\rho g}}$$

(10)

For the properties of water listed earlier, the capillary length is 2.6 mm. Hence, for values of $R > 2.6$ mm, it may be understood that surface tension becomes progressively less important. As the maximum length scale in Table 2 is 1.969 mm, all reported experiments are surface tension dominated as compared to the gravity force, as noted earlier.

The relevant timescales associated with pairs of forces are listed below.

Inertia - Surface tension : $t_{IS} = (\rho R^3/\sigma)^{0.5}$

Viscous - Surface tension : $t_{VS} = \mu R/\sigma$

(11)

Inertia - Viscous: $t_{IV} = R^2/\nu$

Timescales arising in the present study are summarized in Table 3.
Table 3: Timescales estimated for coalescence of water drops under ambient conditions; IS inertia-surface tension, VS viscous-surface tension, IV inertia-viscous.

<table>
<thead>
<tr>
<th>Volume, μl</th>
<th>Base radius, mm</th>
<th>$t_{IS}$, ms</th>
<th>$t_{VS}$, ms</th>
<th>$t_{IV}$, ms</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.06</td>
<td>0.306</td>
<td>0.639</td>
<td>$3.4 \times 10^{-3}$</td>
<td>$0.11 \times 10^{3}$</td>
</tr>
<tr>
<td>0.60</td>
<td>0.659</td>
<td>2.020</td>
<td>$7.4 \times 10^{-3}$</td>
<td>$0.53 \times 10^{3}$</td>
</tr>
<tr>
<td>1.0</td>
<td>0.782</td>
<td>2.610</td>
<td>$9.0 \times 10^{-3}$</td>
<td>$0.75 \times 10^{3}$</td>
</tr>
<tr>
<td>2.0</td>
<td>0.985</td>
<td>3.680</td>
<td>$11.0 \times 10^{-3}$</td>
<td>$1.20 \times 10^{3}$</td>
</tr>
<tr>
<td>4.0</td>
<td>1.240</td>
<td>5.200</td>
<td>$14.0 \times 10^{-3}$</td>
<td>$1.90 \times 10^{3}$</td>
</tr>
<tr>
<td>6.0</td>
<td>1.420</td>
<td>6.370</td>
<td>$16.0 \times 10^{-3}$</td>
<td>$2.50 \times 10^{3}$</td>
</tr>
<tr>
<td>16.0</td>
<td>1.969</td>
<td>10.430</td>
<td>$22.0 \times 10^{-3}$</td>
<td>$4.75 \times 10^{3}$</td>
</tr>
</tbody>
</table>

The (inertia-surface tension) timescale relates to the moment when large velocities are generated within the drop and is expected to relate to the initial phase of coalescence. The inertia-viscous timescale is quite large and is associated with the bulk dissipation of kinetic energy of the fluid, leading to long-term relaxation of the drop towards equilibrium. Additional dissipation occurring at the three-phase contact line is not considered here. The timescale of viscous-surface tension interaction is seen to be small and not of any specific relevance to the present study.

As described in Section 3.1, timescales evaluated in Table 3 are small in comparison to experimental observations where a slow relaxation of the drop is seen over duration of seconds to minutes. A plausible explanation is finite viscous dissipation driven by the movement of the three-phase contact line at the solid surface. Such long timescales are quantitatively not relevant for the global mathematical model of dropwise condensation because of changes brought about by additional condensation over the merging drops. Of interest is the time duration over which two drops become geometrically a single entity, though velocities within may not have fully reduced to zero. These timescales specific to pendant drops have been determined in the present study from imaging experiments described in Section 3.2.

3.1 Literature review

Andrieu et al. [20] studied the time required for coalescence of sessile water drops over a plane surface in the contact angle range of 30º-50º. Drops were imaged in the plan view and the footprint of the merged drops was followed in time. The authors reported bridge formation early-on over a timescale of a few milliseconds, that was followed by a long transient lasting a few minutes, that was proportional to the base radius. Dissipation at the three-phase contact line during the process of condensation was suggested as the mechanism responsible for the long-lived transient. Narhe et al. [21] compared coalescence induced in sessile drops deposited by a syringe as opposed to drops grown by condensation. In the former, free surface oscillations were identified as the reason behind a greater degree of dissipation, leading to short relaxation times. Aryafar et al. [22] studied coalescence of a moving drop of radius $R$ at the interface of two immiscible liquids. Coalescence timescales were found to scale as $R^{1.5}$, being of the order of a few milliseconds for fractional millimeter sized drops of water in air. The Ohnesorge number (Oh) was found to play an important role in the possibility of the drop break-up. For Oh > 1, full coalescence, without break-up was found to take place. Thoroddsen et al. [23] studied bridge formation at the interface of a pendant and a sessile drop using ultra-high speed imaging. The flow dynamics was seen to depend on Reynolds number based on the average radius of curvature and a velocity scale of surface tension coefficient and viscosity. The speed of bridge formation was seen to depend on Reynolds number, slowing down due to viscosity. Bordoloi and Longmire [24] studied coalescence of a liquid drop with a flat liquid interface, both liquids being identical. The Bond number was large while the Ohnesorge number was very small, suggesting that gravity effects were substantial in comparison to surface tension, both being larger than viscosity. In all experiments, the thin film at the periphery of the drop was seen to rupture at its thinnest point leading to rapid collapse of the drop. The coalescence process of a single drop was compared with one where a perturbation was introduced in the neighborhood. The role of external perturbation was to control asymmetry of the drop geometry. Though viscosity was mostly negligible, it was felt within the film as well as a perturbation involving a solid particle. Film rupture resulted in the propagation of surface waves of gravitational origin over the 2-liquid interface.
Wu et al. [25] studied surface-tension controlled coalescence of drops facing each other and examined bridge formation at the interface. The initial growth of bridge radius was seen to scale as $t^{0.5}$ with a constant of proportionality depending on the fluid properties. Graham et al. [26] studied impact of a falling drop with another placed on a horizontal surface. The surface texture was varied, ranging from hydrophilic to hydrophobic. The overall coalescence process revealed the importance of various forces, surface tension, gravity, inertia, and gravity. The study reported the maximum spreading of the drop for a wide range of parameters. Yeh et al. [27] studied coalescence of small droplets over a textured surface of variable wettability. The corresponding Bond numbers were quite small and the process was surface tension dominated. The wettability gradient led to an initial drop impact, following which strong convection patterns were initiated within. The authors identified a short convection regime during which strong fluid velocities were created, followed later by a long diffusion tail. Lower surface tension was seen to reduce the extent of fluid mixing within the drops. Bhardwaj and Attinger [28] studied the spreading of large impacting drops on textured surfaces, numerically as well as by performing experiments. Wetting characteristics at the surface were found to be of secondary importance in comparison to viscous effects.

Blanchette and Bigioni [29] simulated a liquid drop coalescing with a reservoir of an identical liquid. The possibilities of pinch-off, formation of a satellite drop, and partial coalescence were examined in terms of the dimensionless parameters. These phenomena were found to be likely in low viscosity – high surface tension (low Oh) fluids. From experiments over superhydrophobic surfaces, Rykaczewski et al. [30] showed that coalescence can lead to the formation of micro- and nano-scaled satellite drops that are swept away by the primary drops before they can occupy nucleation sites during dropwise condensation. Sprittles and Shikhmurzaev [31] suggested that the initial coalescence mechanism of two freely suspended drops involves trapping of the free surface within, followed by a gradual disappearance of the internal interface, as opposed to a bridge formation. The authors showed that each mechanism is accompanied by its own scaling law. Apart from coalescence, experiments and analysis of single drops falling and spreading on textured surfaces have been reported in the literature [32].

Research in this domain is principally through experiments though some simulation has been reported. Results obtained by various authors show lack of repeatability on one hand, but more importantly, results are configuration specific. Studies of spreading, coalescence, and impact are restricted to sessile drops while pendant drops and those on inclined surfaces are under-represented. These configurations are important from a viewpoint of dropwise condensation. The data of sessile drops cannot be carried forward to the pendant since gravitational forces can upset the relative importance of other forces present. The following section describes experiments on the coalescence of pendant water drops on a horizontal surface when the equilibrium contact angle is close to $90^\circ$.

3.2 Apparatus and instrumentation

Figure 8 shows the schematic drawing of the experimental set-up developed as a part of this work. It is used to form and image one or more pendant drops on a horizontal surface. The newly formed pendant drop is imaged using a high speed camera from a direction normal to the vertical plane. A micro liter syringe with 100 $\mu$L capacity and a least count of 0.02 $\mu$L is used to deposit pendant drops on the underside of the substrate. The substrate is Teflon coated Plexiglas and equilibrium contact angle for water on this surface was measured to be $90^\circ$ (±2°). The surface was cleaned with ethanol and vacuum dried before each experiment. In this manner, the contact angle and the coalescence sequence were found to be repeatable. The light source was a halogen lamp. A monochrome high speed camera (Photron® FASTCAM SA-3), 4000 frames per second, 6 bit digitization, was used for imaging the coalescing drops in time (Figures 9-12).

Precautions required in experiments include maintaining surface quality, alignment, relatively high humidity in the laboratory-space, and cleanliness so that the initial and final equilibrium contact angles are identical for all drop volumes, equal to $\sim 90^\circ$ in the present discussion. For this purpose, surfaces were not re-used and a new coating was applied for each experiment. Coalescence was carefully initiated without impact by increasing the volume of one of the drops using the syringe at a very slow rate. For certain under-prepared surfaces, pinning was observed and the data recorded in such experiments were discarded.

Figure 9 shows an image sequence of coalescing pendant drops of equal volume ($\approx 2.3 \mu$L), as seen in the front-view for drops. Figure 10 shows the image sequence in the front-view for drops of larger but equal
volumes (= 8 µl). Figure 11 is an image sequence recorded for drops of unequal volumes, as indicated. When viewed from the side, the movement of the liquid interface was barely noticeable and therefore side-view images have not been considered further. Direct imaging was resorted to for drop volumes of 1-16 µl. A drop of 0.5 µl volume was not large enough for this purpose. It was difficult to work with drops of volume greater than 16 µl in the pendant mode. The footprint of two coalescing pendant drops smaller than ~ 0.5 µl was therefore imaged using a high resolution confocal microscope (Leica® CTR6500). A time sequence of the footprint thus recorded is shown in Figure 12. In all experiments reposted here, the original contact angle was recovered over a period of several seconds to a minute after the coalescence process eventually equilibrated; this long-time data is not shown in the images of Figures 9-12. For distance measurement, the camera was calibrated in both the $x$- and the $y$-directions before starting the experiment. Since the measurements were intended to track the movement of the centroid, the focusing plane was selected as the mid-plane of the merging drops.

![Figure 8: Schematic drawing of the experimental set-up](image)

**Figure 8:** Schematic drawing of the experimental set-up

![Figure 9: Image sequence of drop coalescence in the front-view for drops of equal volume = 2.3 µl each.](image)

**Figure 9:** Image sequence of drop coalescence in the front-view for drops of equal volume = 2.3 µl each.

![Figure 10: Image sequence of coalescence in the front-view for larger drops of equal volume = 7.8 µl each.](image)

**Figure 10:** Image sequence of coalescence in the front-view for larger drops of equal volume = 7.8 µl each.
3.3 Data analysis

Using an image sequence, the centroid of the newly formed drop can be followed in terms of its $x$- and $y$-coordinates. Here, $x$ is the horizontal direction and $y$, the vertical downward direction. The plot of the coordinates as a function of time can be differentiated to yield the scaling of the two velocity components. As the image movement in the side view is small, it can be concluded that the third component of velocity, though non-zero, is uniformly small during coalescence. The movement of the footprint of the drop provides information on the speed with which the three phase contact line moves over the solid surface. As seen in the image sequence (Figures 9-11), large velocities are generated during early time, decaying slowly over a longer time span. In addition, there is a distinct dependence of magnitude of the resulting velocity on the initial drop sizes. The nature of unsteadiness can also be quite characteristic of the chosen velocity component, the vertical velocity component reflecting the longer lasting gravitational oscillations.

Figures 9-12 show clearly the appearance of two timescales within the transient coalescence process (exemplified by the slow-motion video sequence). The first is rapid and lasts for around ~10 ms at early times. The second is longer and persists over a timescale of ~ 100 ms. This observation is in-line with the time estimates of Table 3, where the initial transience is clearly attributable to the inertia-surface tension coupling while the longer arises from the viscous-inertia relaxation.

Figure 11: Image sequence of coalescence in the front-view for drops of unequal volumes, 1.7 and 4.7 μl.

Figure 12: Image sequence recorded using a confocal microscope for equal drop volumes of (a) 0.04 μl and (b) 0.4 μl each. Early time corresponds to a period less than 20 ms; late time spreads to around 200 ms.
Positions (x- and y-) and the corresponding velocity traces (u and v) of the centroid of the merged drop are shown in Figure 13. The individual drop volumes are taken to be equal (= 2.3 μl on the left and 7.8 μl on the right). The peak velocity and peak displacement in the x-direction are greater initially for the larger drop and settle to comparable levels very soon. However, the y-component behavior is similar for the two. In addition, sustained oscillations can be seen over the time period covered. This trend is a consequence of gravitational oscillations specific to coalescence of drops in the pendant configuration. The vertical component of oscillations in terms of centroidal velocity, as well as time, is practically independent of the drop size. The vertical oscillations have a time period of 15-20 ms corresponding to a frequency of 50-66Hz. The oscillation-timescale matches the ratio \( \frac{U}{g} \), where, \( U = \sqrt{\sigma/\rho R} \). The vertical velocity fluctuations lasted to an order of nearly a full second in experiments.

As per the video sequence, coalescence is seen to progress in two distinct stages. The first is one of bridge formation for a few milliseconds when the two drops lose their distinctive identities and become one indistinguishable liquid mass. Velocities are generated due to difference of internal pressures between the two drops. Since the changes in the x-velocity define changes in the shape of the drop and affect the species gradients, it is the most relevant for dropwise condensation calculations. The x-component velocity decays rapidly with time while the one in the y-direction oscillates for a longer duration. The initial transients are followed by a longer tail of a few 100 ms where velocities initially generated decay with time owing to viscous dissipation. While an effective coalescence time cannot be precisely defined, the timescale of interest will be that in the x-component velocity and is seen to increase with drop volume (and hence the base radius). With reference to Table 3, it is clear that the inertia-viscous timescale \( (\nu=\nu^R/\nu) \) is the most appropriate to estimate the coalescence timescale. Here, \( R \) is the equilibrium radius of the footprint of the merged drop. For the volumes considered, the base radius is not sufficiently differentiated to reveal a clear difference in the x-velocities. These are, however, close to the estimate based on \( (\sigma/\rho R)^{0.5} \) as a velocity scale (Table 2).

For the condensation model, the quantities of interest are the characteristic time of coalescence and velocity components generated. The former will extend the condensation cycle as compared to the model where coalescence is taken to be instantaneous. Velocity data contributes to heat transfer and wall shear stress, and have been used for wall heat flux and shear stress calculations in the following discussion.

**Figure 13:** Quantities of interest for merger of equal drop volumes of 2.3 μl (left column) and 7.8 μl (right column), respectively. (a) Variation of x-coordinate of the centroid with time. (b) Variation of y-coordinate of the centroid with time. (c) Variation of velocity component of centroid along x-coordinate with time. (d) Variation of velocity component of centroid along y-coordinate with time for drops of equal volumes.
Figure 14: Left column: Quantities of interest for merger of unequal drop volumes of 1.7 μl and 4.7 μl, respectively. (a) Variation of x-coordinate of centroid with time. (b) Variation of y-coordinate of centroid with time. (c) Variation of velocity component of centroid along x-coordinate with time. (d) Variation of velocity component of centroid along y-coordinate with time. Right column - (a) variation of shear rate with time for (a) drops of equal volumes, both 2.3 μl, (b) drops of equal volumes, both 7.8 μl, (c) drops of unequal volume, 1.7 μl and 4.7 μl.

Figure 15: Wall heat flux and wall shear stress at coalescence sites compared with surface-averaged values during condensation of water at 303K with a subcooling of 5K. (a) Data for a horizontal surface with condensation on the underside. Shear stress is developed here only by coalescence. (b) Data for a vertical surface where wall shear stress for coalescence is greater than that for a sliding drop. For both instances, heat fluxes during coalescence are substantially higher compared to the surface averaged-values.

Centroid positions and velocity components for coalescence of drops of unequal volumes (1.7 and 4.7 μl) are shown in Figure 14 (left column). The internal pressures of the two drops are separated to a greater degree and hence the initial velocities generated are substantially large. However, the short-term and long-term behavior is similar to experiments with coalescing drops of equal volumes. Gravitational oscillations are again visible in the vertical component of position and velocity. The right column of Figure 14 summarizes...
the shear rates determined from the videographic experiments. Shear rate, defined as $\dot{\gamma} = (\partial v/\partial x) - (\partial u/\partial y)$ is estimated as $u/(R/2)$. As seen, initial shear rates for unequal volume drops are large. The data set, however, clearly reveals that shear rates are the lowest for drops of large volume (see Figure 14(b), right side).

3.4 Effect of coalescence on dropwise condensation

Using the coalescence timescales, the condensation process was extended and the entire process-cycle calculation was repeated. Since the overall cycle time is of the order of seconds-minutes, the millisecond scales associated with coalescence did not create visible changes in the overall condensation patterns. However, there were substantial changes in local wall heat flux and shear stress at the coalescence sites, as are shown in Figure 15. Since coalescence can occur at several sites simultaneously, the maximum values are reported in the figure. Only a part of the condensation cycle is shown. Wall shear stress is obtained from the shear rate discussed in Figure 14. Wall heat flux is obtained from the condensation rate for the characteristic size of the respective coalescing drops and the associated coalescence timescale (see Table 3 and the related discussion). It can be seen that the local instantaneous heat flux as well as wall shear stress is substantially higher during coalescence, as compared to the average taken over all the drops for a horizontal surface. These are also higher than the values obtained for a sliding drop over a vertical surface. In view of the fact that sustenance of dropwise condensation process on textured surfaces is one of the major hurdles in applications, the results of the present study suggest that local values of transport coefficients will impact the life of the promoter layer over the condensing surface. In the light of the momentary but repetitive transients associated with droplet coalescence, the design of the promoter layer must ensure its long term durability.

4. CONCLUSIONS

The sensitivity of a previously developed hierarchical model of dropwise condensation has been tested with respect to details of the drop shape and coalescence dynamics. A numerical model based on open domain software is used to solve the force equilibrium equation and obtain three-dimensional shapes of static drops at criticality. With the correct shape determined, drop motion is independently simulated by a 3D-Navier-Stokes and energy equations solver. The wall shear stress and heat flux information is included in the condensation model in the form of correlations. Quantities of interest such as condensation patterns, wall shear stress, and wall heat transfer have been predicted. These are compared with predictions of the 2-circle approximation. Coalescence is understood from scale analysis and high speed imaging experiments with drops of equal and unequal volumes. The characteristic velocity and timescales of coalescence are included in the condensation cycle to determine its effect on local wall shear stresses and heat transfer. The following results have been obtained in the present study:

i. The new approach of determining the drop shape shows up realistic drop deformation in the two orthogonal directions and a non-circular footprint. Consequently, it predicts larger drops at criticality. Hence, the time taken to reach instability is greater for the drop, while the conduction resistance offered is greater as well. The cycle-averaged Nusselt number with the present approach is smaller than that obtained with a 2-circle approximation. Similarly, skin friction coefficient during drop motion is found to be smaller in comparison.

ii. Coalescence is a short-lived process during which large velocities are generated. During coalescence in pendant mode, persistent oscillations are seen in the vertical velocity component. The horizontal velocity component shows a shorter timescale that arises from an interaction of inertia and surface tension. A longer timescale arises from viscous dissipation but the velocities generated during this stage are negligibly small. Local wall heat flux and wall shear stress during coalescence are substantially larger than the time-averaged values seen during the overall dropwise condensation process.

ACKNOWLEDGMENT

The authors are grateful to the Board of Research in Nuclear Sciences (BRNS), Department of Atomic Energy, Government of India, for financial assistance.
NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>Surface area, $A_b$: base area of a drop, m$^2$</td>
</tr>
<tr>
<td>$c_p$</td>
<td>Specific heat at constant pressure, J/kg-K</td>
</tr>
<tr>
<td>$F$</td>
<td>Force acting on the drop, N</td>
</tr>
<tr>
<td>$h$</td>
<td>Heat transfer coefficient, $q_{wall}/(T_{sat} - T_w)$, W/m$^2$-K</td>
</tr>
<tr>
<td>$h_{lv}$</td>
<td>Latent heat of vaporization, J/kg</td>
</tr>
<tr>
<td>$k$</td>
<td>Thermal conductivity of condensate, W/m-K</td>
</tr>
<tr>
<td>$N$</td>
<td>Nucleation site density, cm$^{-2}$</td>
</tr>
<tr>
<td>$q$</td>
<td>Surface heat transfer, W</td>
</tr>
<tr>
<td>$q_{wall}$</td>
<td>Surface heat flux, W/m$^2$</td>
</tr>
<tr>
<td>$r$, $r_b$</td>
<td>Radius of the drop and base radius at criticality, m</td>
</tr>
<tr>
<td>$r_{cap}$</td>
<td>Capillary length, $\sqrt{\sigma / g (\rho_l - \rho_v)}$, m</td>
</tr>
<tr>
<td>$r_{max}$</td>
<td>Radius of the drop at instability due to fall-off, m</td>
</tr>
<tr>
<td>$r_{crit}$</td>
<td>Radius of the drop at instability due to slide-off, m</td>
</tr>
<tr>
<td>$T$</td>
<td>Temperature, K</td>
</tr>
<tr>
<td>$T_{sat}$</td>
<td>Saturation temperature, K</td>
</tr>
<tr>
<td>$T_w$</td>
<td>Substrate temperature, K</td>
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<td>$\Delta T$</td>
<td>Subcooling, $(T_{sat} - T_w)$, K</td>
</tr>
<tr>
<td>$\Delta t$</td>
<td>Time step, s</td>
</tr>
<tr>
<td>$u$, $v$, $w$</td>
<td>Velocity components in $x$, $y$ and $z$ directions, m/s</td>
</tr>
<tr>
<td>$U$</td>
<td>Terminal velocity of drop, m/s</td>
</tr>
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Dimensionless quantities

<table>
<thead>
<tr>
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<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_f$, $\overline{C_f}$</td>
<td>Local and average skin friction coefficient, $\tau_w/(1/2)\rho U^2$</td>
</tr>
<tr>
<td>$Nu$, $\overline{Nu}$</td>
<td>Local and average Nusselt number, $h r_{cap}/k$</td>
</tr>
<tr>
<td>$\overline{(Nu)}<em>{ad}$, $(\overline{Nu})</em>{ad}$</td>
<td>Local and average Nusselt number of a single sliding drop, $h r_{ad}/k$</td>
</tr>
<tr>
<td>$Pr$</td>
<td>Prandtl number, $\mu C_p/k$</td>
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<tr>
<td>$Re$</td>
<td>Reynolds number, $\rho U(2r_b)/\mu$</td>
</tr>
<tr>
<td>$(T - T_w)/\Delta T$</td>
<td>Dimensionless temperature</td>
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Greek symbols

<table>
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<th>Description</th>
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<tbody>
<tr>
<td>$\alpha$</td>
<td>Inclination angle, degrees</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Dynamic viscosity, Pa-s</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Azimuthal angle measured along the three-phase contact line (degrees)</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Liquid density, kg/m$^3$</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Surface tension at liquid-vapor interface, N/m</td>
</tr>
<tr>
<td>$\tau_w$, $\overline{\tau_w}$</td>
<td>Local and average wall shear stresses, N/m$^2$</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Contact angle, degrees; $\theta_{adv}$ for advancing and $\theta_{rec}$ for receding, $\Delta \theta = (\theta_{adv} - \theta_{rec})$ is hysteresis, and $\theta_{avg}$ is the average contact angle</td>
</tr>
</tbody>
</table>

REFERENCES


