END-SEMESTER EXAMINATION
MTH-201, MTH-201A
LINEAR ALGEBRA
Summer-2014
8th JULY, 2014, 4 PM-7 PM

Time Allowed: 3 hrs Max. Marks: 40

1. Give complete and precise definitions with one example each for the following. [5]

a. Annihilator

b. Minimal polynomial

c. Cyclic vector

d. Invariant Subspace

e. Inner Product Space

2. Solve the following system of equations by using the Gauss-Jordan elimination method. [4]

$$4y + z = 2$$

$$2x + 6y - 2z = 3$$

$$4x + 8y - 5z = 4$$

3. Let V be the vector space of all $n \times n$ matrices over $\mathbb{R}$. Let W_1 be the subset of symmetric matrices, $A^t = A$ and let W_2 be the subset of all skew-symmetric matrices, $A^t = -A$. Prove that [5]

(a) W_1 and W_2 are subspaces of V.

(b) $V = W_1 \oplus W_2$.

(c) Find bases for W_1 and W_2.

4. State and prove the Rank-Nullity Theorem. [5]

5. Prove that the eigenvectors corresponding to distinct eigenvalues of a linear operator are linearly independent. [4]

6. State the following theorems. [3]

a. Caley-Hamilton Theorem

b. Jordan Decomposition Theorem

c. Spectral Theorem

7. Find a matrix for which the minimal polynomial is $x^4 - 5x^2 + 3x - 7$. [2]

8. Determine all 2×2 matrices A such that $A^2 - 3A + 2I = 0$. [3]

9. Find a unitary matrix U such that U^*AU is a diagonal matrix where [4]

$$A = \begin{pmatrix} 1 & 1+i \\ 1-i & 2 \end{pmatrix}, \quad i = \sqrt{(}-1).$$

10. Write down all possible Jordan Cannonical forms for a matrix with characteristic polynomial $(x+3)^3(x-2)^2$. [5]

11. Let $V = Span(S)$, where $S = \{1, t, sin(t), cos(t)\}$. Then V is an inner product space over $\mathbb{R}$ with respect to the inner product [5]

$$(f, g) = \int_0^\pi f(t)g(t)dt$$

Use the Gram-Schmidt orthogonalization process to S to obtain an orthonormal basis for V.