MID-SEMESTER EXAMINATION
MTH-201, MTH-201A
LINEAR ALGEBRA
Summer-2014
Date: 7th June 2014

Time Allowed: 2 hrs Max. Marks: 30

1. Give complete and precise definitions for the following. [5]

a. Vector Space b. Linear Independence c. Basis d. Linear Transformation e. Nullity

2. Give an example of each of the following. [5]

a. A nonempty subset W of $\mathbb{R}^2$ such that W is closed under scalar multiplication but W is not a subspace of $\mathbb{R}^2$.

b. An infinite dimensional vector space.

c. Two subspaces W_1 and W_2 of $\mathbb{R}^2$ such that $W_1 \cup W_2$ is not a subspace.

d. A linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ which is not one-one.

e. A rank 2 linear transformation $T : \mathbb{R}^5 \to \mathbb{R}^3$.

3. Find all solutions to the following system of equations by Gauss-Jordan elmination method (row reduced echelon form): [4]

$$x + y + z = 3$$

$$x + 2y + 2z = 5$$

$$3x + 4y + 4z = 11$$

4. Let V be a vector space over the field $\mathbb{R}$ of real numbers. Prove that V is not equal to the union of a finite number of proper subspaces. [5]

5. Let V be the vector space of all functions from $\mathbb{R}$ to $\mathbb{R}$. Let W_1 be the subset of even functions, $f(-x) = f(x)$ and let W_2 be the subset of odd functions, $f(-x) = -f(x)$. [4]

(a) Prove that W_1 and W_2 are subspaces of V.

(b) Prove that $V = W_1 \oplus W_2$.

6. Let V be a finite-dimensional vector space. Then a linear transformation $T : V \to W$ is an isomorphism if and only if it maps bases to bases. [5]

7. Let V be the vector space of all polynomial functions from $\mathbb{R}$ into $\mathbb{R}$ which have degree less than or equal to 2. Let t_1, t_2 and t_3 be any three distinct real numbers, and let $L_i(p) = p(t_i)$. [5]

a. Prove that L_1, L_2 and L_3 are linear functionals on V.

b. Prove that $B = \{L_1, L_2, L_3\}$ is a basis of V^*.

c. Find a basis of V of which B is the dual basis.